Probability Theory Homework 8

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due Friday, November 22, 2024

1 Course evaluations

Course evaluations for this class (and other classes) should now be accessible on D2L. The deadline for filling out course evaluations is December 2nd: after classes are over, but before our final exam on December 5th.

These are very useful to me in deciding how to teach this class in the future. (Comments are especially helpful, but even if you don't leave those, numerical ratings are helpful, and they're more reliable if more of you give them.) The evaluations are anonymous, and are not released until after final grades are due, so there are no drawbacks to being honest.

As an incentive, if at least 50% of the class fills out a course evaluation, I will wear a funny hat for the final exam.

2 Homework problems

1. Let $\mathbf{X} \sim \text{Exponential}(\lambda = \frac{1}{2})$, let $\mathbf{Y} \sim \text{Uniform}(a = 1, b = 3)$, and let $\mathbf{Z}$ be the random variable with PDF
$$f_{\mathbf{Z}}(t) = \begin{cases} 2/t^3 & t \geq 1, \\ 0 & t < 1. \end{cases}$$

    These have something in common: $\mathbb{E}[\mathbf{X}] = \mathbb{E}[\mathbf{Y}] = \mathbb{E}[\mathbf{Z}] = 2$. For each random variable, find the probability that it's less than this expected value.

2. Find the variance of each random variable in question 2.

3. If $\mathbf{\Theta} \sim \text{Uniform}(a = 0, b = 3)$, find the probability density function of $\sin(\mathbf{\Theta})$.

4. Let $\mathbf{W}$ be normally distributed with $\mu = 0$ and unknown standard deviation σ. Find $\mathbb{E}[\mathbf{W} \mid \mathbf{W} > 0]$ in terms of σ.

5. You are a birdwatcher listening for bird calls in a quiet park on a peaceful day in November.

    Based on your experience, you hear an average of one bird call per minute. Also, when you hear a bird call, it's a robin 50% of the time, a woodpecker 30% of the time, and a sparrow 20% of the time.

Assuming that three types of bird calls are independent Poisson processes, identify the distributions of the following random variables: give the name of the distribution and its parameters.

(a) **R** is the number of robin calls you hear over the course of an hour.

(b) **T** is the time it takes you to hear 10 bird calls total.

(c) **W** is the time it takes you to hear a woodpecker call.

(d) **C** is the total number of bird calls you've heard by the time you've heard your first woodpecker call.