

## In-class problems for Lecture 3

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1. The system of equations below has infinitely many solutions. Solve for  $y$  and  $z$  in terms of  $x$ .

$$\begin{cases} 3x + 2y - 3z = -1 \\ 3x - y + 2z = 2 \end{cases}$$

2. The following system of equations has already been solved for  $x_1, x_2, x_3$  in terms of  $x_4, x_5$ :

$$\begin{cases} x_1 + 3x_2 - 2x_3 + x_4 - x_5 = 1 \\ -2x_1 + x_2 + 2x_4 - x_5 = 1 \\ x_1 + x_2 - x_3 - x_4 = -1 \end{cases} \rightsquigarrow \begin{cases} x_1 = 2 - x_4 \\ x_2 = 5 - 4x_4 + x_5 \\ x_3 = 8 - 6x_4 + x_5 \end{cases}$$

- (a) Find two different particular solutions  $(x_1, x_2, x_3, x_4, x_5)$  to this system of equations.
- (b) Solve for  $x_1, x_2, x_5$  in terms of  $x_3, x_4$  instead. Try to do as little additional work as possible.
3. Consider the following system of equations:

$$\begin{cases} 3x_1 + 5x_2 + x_3 - 2x_4 = 4 \\ x_1 + 2x_2 + x_3 - x_4 = -1 \end{cases}$$

- (a) Write this system of equations in matrix form: as  $A\mathbf{x} = \mathbf{b}$ , where  $A$  is a  $2 \times 4$  matrix,  $\mathbf{x}$  is the column vector of our variables  $x_1, \dots, x_4$ , and  $\mathbf{b}$  is a  $2 \times 1$  column vector.
- (b) Take the first two columns of  $A$  only. Find the inverse of this  $2 \times 2$  matrix.
- (c) Left-multiply both sides of the matrix equation  $A\mathbf{x} = \mathbf{b}$  by the inverse matrix you've found.
- (d) Your result should now be row-reduced. Use it to solve for  $x_1, x_2$  in terms of  $x_3, x_4$ .
4. Consider the following system of equations, already written in matrix form:

$$\begin{bmatrix} 2 & 1 & -5 \\ 0 & 1 & -1 \\ -2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

- (a) Left-multiply both sides of this matrix equation by the row vector  $\begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$ .
- (b) What does the result tell you about the system of equations?

<sup>1</sup>This document comes from the Math 3272 course webpage: <https://facultyweb.kennesaw.edu/mlavrov/courses/3272-fall-2022.php>