

Graph Theory Homework 1

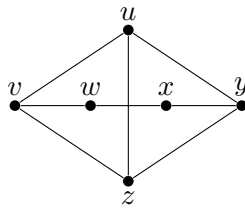
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due Friday, January 21, 2022

1 Short answer

In this section, all I expect you to do is find the answer, possibly accompanied by a diagram or a one-sentence explanation if I ask for it.

1. Let G be the graph below:



Let H_1 be the complement of G : the graph with all the same vertices, and exactly the edges which **do not** exist in G .

- (a) How many edges does H_1 have?

(Knowing that K_6 has 15 edges, you can answer this question without doing part (b) first. But I suppose I will not know if you do.)

- (b) Draw a diagram of H_1 . You do not have to keep the vertices in the same places.

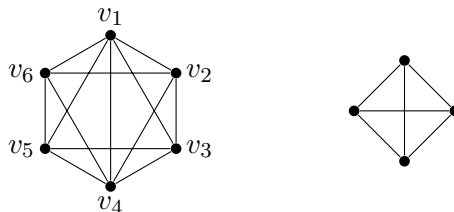
- (c) Find a cycle of length 6 in G .

2. Let H_2 be the graph whose vertices are the numbers $1, 2, \dots, 15$, with an edge between a and b if $a + b$ is a multiple of 7. (For example, vertex 3 is adjacent to vertex 4, because $3 + 4 = 7$, and also to vertex 11, because $3 + 11 = 14$.)

- (a) Draw a diagram of H_2 .

- (b) What are the connected components of H_2 ?

3. Let H_3 be the graph shown below on the left. Inside H_3 , we want to find subgraphs that look like the graph on the right: subgraphs that have 4 vertices and 6 edges.



How many such subgraphs are there? List them all, by telling me which vertices are part of each subgraph.

4. Find a connected 6-vertex graph with 10 edges which has diameter 3. Point out the two vertices at distance 3 from each other.

2 Proof

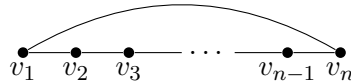
In this section, you should write a proof. Write in complete sentences and justify your logic. I am not grading the length of your proof, only its correctness, but a typical solution can be a paragraph long.

5. Prove that, for $n \geq 5$, the complement of the cycle graph C_n is connected. Explain why your proof does not work for $n = 3$ and $n = 4$.

Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 2.

Because having a Monday off is messing with my schedule, we will not have defined complements or cycle graphs yet by the time you're doing this assignment. So let me summarize.

The cycle graph C_n is the graph that has only the vertices and edges we need for an n -vertex cycle. Here is a diagram:



The complement (which we can write $\overline{C_n}$) has exactly the edges that C_n does not have. For example, in $\overline{C_7}$, v_1 is adjacent to v_3, v_4, v_5, v_6 , but not to v_2 or v_7 : because v_1 was adjacent to v_2 and v_7 in C_7 .