

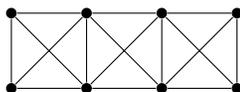
Graph Theory Homework 2

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due Friday, February 4, 2022

1 Short answer

1. Let G be the graph shown below. (It can be defined as the graph with vertex set $\{1, 2\} \times \{1, 2, 3, 4\}$, in which vertices (x, y) and (x', y') are adjacent whenever $|x - x'| \leq 1$ and $|y - y'| \leq 1$.)



- (a) Is G bipartite? Either give a bipartition, or find a reason why it cannot be bipartite.
 - (b) Is \overline{G} bipartite? Either give a bipartition, or find a reason why it cannot be bipartite.
2. H is a bipartite graph. On one side of the bipartition, there are n vertices; their degrees are $1, 2, 3, \dots, n$. On the other side of the bipartition, there are also n vertices; all of them have degree 4.

What is n ?

3. For each sequence below, find a graph with that degree sequence.

In theory, you can do these using an algorithm we will discuss in class, but I intend them to be possible just by thinking about what these graphs should look like, and about graphs you are familiar with.

- (a) $2, 2, 2, 2, 2, 2, 1, 1$.
- (b) $2, 2, 2, 2, 2, 2, 1, 1, 1, 1$.
- (c) $7, 1, 1, 1, 1, 1, 1, 1$.
- (d) $7, 7, 7, 7, 7, 7, 6, 6$.

2 Proof

4. Prove that, for $n \geq 5$, the complement of the cycle graph C_n is connected. Explain why your proof does not work for $n = 3$ and $n = 4$.

You have already written a rough draft of this problem. Now, write the final draft.

5. Prove by induction on d that the cube graph Q_d has $d \cdot 2^{d-1}$ edges.

There are direct proofs without the use of induction as well, but for this problem, I specifically want you to practice writing a proof by induction.

Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 3.