

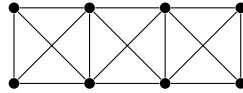
# Graph Theory Homework 2

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due Friday, February 4, 2022

## 1 Short answer

1. Let  $G$  be the graph shown below. (It can be defined as the graph with vertex set  $\{1, 2\} \times \{1, 2, 3, 4\}$ , in which vertices  $(x, y)$  and  $(x', y')$  are adjacent whenever  $|x - x'| \leq 1$  and  $|y - y'| \leq 1$ .)



- (a) Is  $G$  bipartite? Either give a bipartition, or find a reason why it cannot be bipartite.
  - (b) Is  $\overline{G}$  bipartite? Either give a bipartition, or find a reason why it cannot be bipartite.
2.  $H$  is a bipartite graph. On one side of the bipartition, there are  $n$  vertices; their degrees are  $1, 2, 3, \dots, n$ . On the other side of the bipartition, there are also  $n$  vertices; all of them have degree 4.

What is  $n$ ?

3. For each sequence below, find a graph with that degree sequence.

In theory, you can do these using an algorithm we will discuss in class, but I intend them to be possible just by thinking about what these graphs should look like, and about graphs you are familiar with.

- (a)  $2, 2, 2, 2, 2, 2, 1, 1$ .
- (b)  $2, 2, 2, 2, 2, 2, 1, 1, 1, 1$ .
- (c)  $7, 1, 1, 1, 1, 1, 1, 1$ .
- (d)  $7, 7, 7, 7, 7, 7, 6, 6$ .

## 2 Proof

4. Prove that, for  $n \geq 5$ , the complement of the cycle graph  $C_n$  is connected. Explain why your proof does not work for  $n = 3$  and  $n = 4$ .

*You have already written a rough draft of this problem. Now, write the final draft.*

5. Prove by induction on  $d$  that the cube graph  $Q_d$  has  $d \cdot 2^{d-1}$  edges.

There are direct proofs without the use of induction as well, but for this problem, I specifically want you to practice writing a proof by induction.

*Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 3.*