

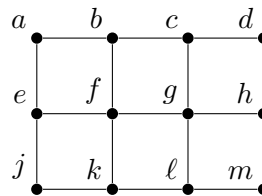
# Graph Theory Homework 4

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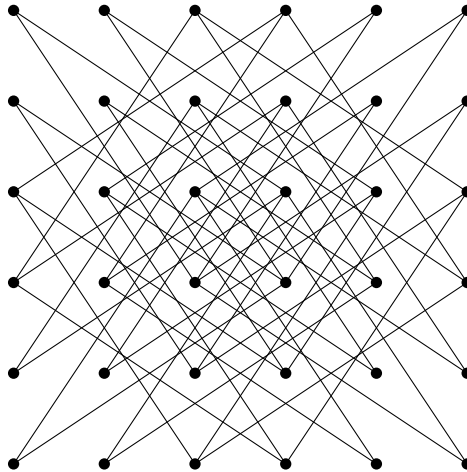
due Friday, March 4, 2022

## 1 Short answer

1. How many labeled trees have a Prüfer code of the form  $12x14$  for some value  $x$ ?
2. In the graph below, find an augmenting path for the matching  $M = \{bf, cg, ej, hm, k\ell\}$ . Then, augment  $M$  by that path to get a bigger matching.



3. The graph below has 36 vertices arranged in a  $6 \times 6$  grid. There is an edge between two vertices if they are either 2 steps apart horizontally and 3 vertically, or else 3 steps apart horizontally and 2 vertically.



Find a vertex cover in this graph that demonstrates that the graph does not have a perfect matching.

## 2 Proof

4. Let  $G$  be a connected graph with  $n$  vertices and exactly one cycle. Prove that  $G$  has  $n$  edges.

*You have already written a rough draft of this problem. Now, write the final draft.*

5. Find the number of trees with vertex set  $\{v_1, v_2, \dots, v_n\}$  in which vertices  $v_3, v_4, v_5, \dots, v_n$  are leaves.

Give **two** proofs of your answer: one using Prüfer codes, and one directly by reasoning about the structure of these trees.

*Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 5.*