Graph Theory Homework 5

Mikhail Lavrov

due Friday, March 25, 2022

1 Short answer

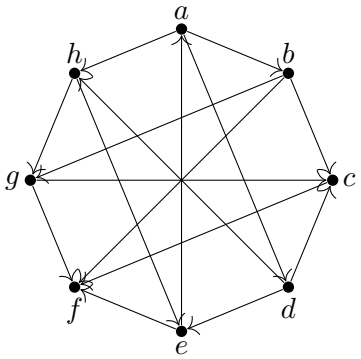
Note: even after we cover multigraphs and directed graphs in this class, it is still the case that when I say "graph", I mean an undirected graph with no loops or parallel edges. If I want to ask about directed graphs or multigraphs, I will specify.

1. Explain why the following assertions, despite looking false, are actually true.

   (a) All graphs with the degree sequence $6, 6, 5, 3, 2, 1, 1$ are Eulerian.

   (b) Not all graphs with the degree sequence $4, 4, 4, 4, 4, 2, 2, 2$ are Eulerian.

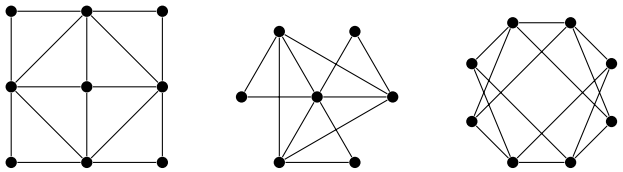
2. In the directed graph below, find an arc (u, v) such that if it is reversed (deleted and replaced by (v, u)), the result is acyclic. Give a topological ordering of the resulting directed graph after (u, v) is reversed.



   If the picture is unclear, the set of arcs is $\{(a, b), (a, d), (a, h), (b, c), (b, f), (b, g), (c, f), (d, c), (d, e), (d, h), (e, a), (e, f), (g, c), (g, f), (h, e), (h, g)\}$.

   (Hint: if you're not sure how to get started, try looking for cycles, or try finding the strongly connected components.)

3. In each of the graphs below, either find a Hamiltonian cycle, or say why it is not Hamiltonian.



2 Proof

4. Find the number of trees with vertex set $\{v_1, v_2, \ldots, v_n\}$ in which vertices $v_3, v_4, v_5, \ldots, v_n$ are leaves.

   Give **two** proofs of your answer: one using Prüfer codes, and one directly by reasoning about the structure of these trees.

   You have already written a rough draft of this problem. Now, write the final draft.

5. Prove that every sequence $d_1, d_2, \ldots, d_n$ such that $d_1 + d_2 + \cdots + d_n$ is even is the degree sequence of some multigraph. (Don't forget that a loop contributes 2 to the degree of a vertex.)

   Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 6.