Graph Theory Homework 7

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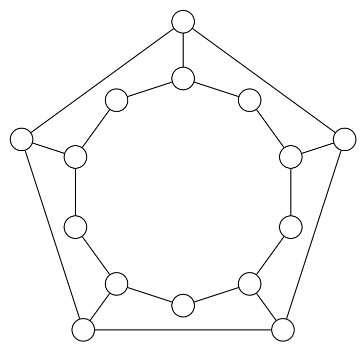
due Friday, April 22, 2022

1 Short answer

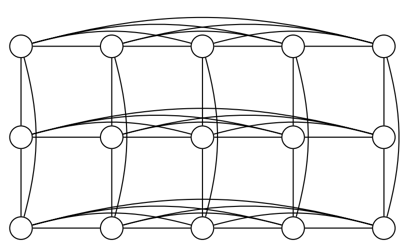
1. Complete the drawing below to a plane embedding of the dodecahedral graph (you are trying to get a 3-regular, 20-vertex graph with 12 faces of length 5).



   Then, find a 3-coloring of the graph.

2. Draw a graph with 10 vertices that has clique number 2 and independence number 4.

3. The 3 by 5 **rook graph** has 15 vertices that can be arranged in a grid such that each vertex is adjacent to all vertices in the same row or column. It's illustrated below:



   (a) Find a 5-coloring of the rook graph.

   (b) Give a reason why 5 is the minimum number of colors possible.

2 Proof

4. Do **one** of the following; for extra credit, do both.

   (a) Prove that for all $n \geq 6$, if a connected n-vertex graph has $n+2$ edges or fewer, then it is planar.

   (b) Prove that for all $n \geq 6$, there is a connected n-vertex graph with $n+3$ edges which is not planar.

   You have already written a rough draft of this problem. Now, write the final draft.

5. The Kneser graph $K(n,2)$ is a graph whose vertices are 2-element subsets of $\{1,2,\dots,n\}$ (that is, the pairs $\{1,2\},\{1,3\},\{1,4\},\dots,\{n-1,n\}$) with an edge between two vertices that **do not** overlap. The Petersen graph is the Kneser graph $K(5,2)$; if you want to make sure you understand the definition, you can draw $K(5,2)$ and check that your result is isomorphic to the Petersen graph.

   Prove that when n is even, $\omega(K(n,2)) = n/2$ and $\alpha(K(n,2)) \geq n-1$. (Think about the difference between proving $=$ and $\geq$.)

   Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 8.