

Graph Theory Homework 8

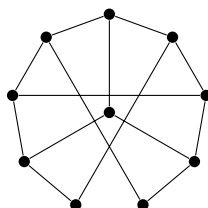
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due Monday, May 2, 2022

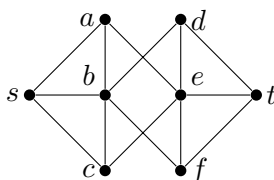
(Note the unusual due date. This is due on Monday at 11:59pm.)

1 Short answer

- Find graphs with the following properties:
 - A graph with degree sequence $3, 3, 3, 3, 2$.
 - A 3-regular graph with at least one cut vertex. (*Hint: the graph you found in part (a) might help.*)
- Find an ear decomposition of the graph below.



- In the graph below, determine $\kappa(s, t)$; explain why the value cannot be larger or smaller.



2 Proof

- The Kneser graph $K(n, 2)$ is a graph whose vertices are 2-element subsets of $\{1, 2, \dots, n\}$ (that is, the pairs $\{1, 2\}, \{1, 3\}, \{1, 4\}, \dots, \{n-1, n\}$) with an edge between two vertices that **do not** overlap. The Petersen graph is the Kneser graph $K(5, 2)$; if you want to make sure you understand the definition, you can draw $K(5, 2)$ and check that your result is isomorphic to the Petersen graph.

Prove that when n is even, $\omega(K(n, 2)) = n/2$ and $\alpha(K(n, 2)) \geq n - 1$. (Think about the difference between proving $=$ and \geq .)

You have already written a rough draft of this problem. Now, write the final draft.