Math 2390 Homework 1

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1. In class, I forgot to mention a variant of the $\{x : \text{property of } x\}$ definition of sets. The part before the colon can be a function of x rather than just x. For example, we can write something like $\{x^2 : x \in \mathbb{N} \text{ and } x \leq 3\}$ to say "for every x such that $x \in \mathbb{N}$ and $x \leq 3$, the set contains x^2".

   (a) Write $\{x^2 : x \in \mathbb{N} \text{ and } x \leq 3\}$ by listing all of its elements.

   (b) Write the set $\{-4, -2, 0, 2, 4\}$ using this notation (there are many possibilities for how to do this; just pick one).

2. Find all subsets of $S = \big\{\emptyset, 1, \{1\}\big\}$ which are **not** also elements of S.

3. Using the sets $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, and $C = \{3, 4, 5\}$ and the operations $\cup$ (union), $\cap$ (intersection) and $-$ (set difference), write down an expression that gives the set $\{2, 3, 5\}$.

4. For this problem, we will work in the universe $U = \mathbb{Z}$. Let $A = \{n \in \mathbb{Z} : |n| \leq 10\}$ and $B = \{k^2 : k \in \mathbb{Z}\}$

   Draw a Venn diagram of A and B in the universe U, putting a few example elements in each region of the Venn diagram.

5. Use the $\bigcup_{i=1}^{n}$ notation to write down the union $[1, 2] \cup [3, 4] \cup [5, 6] \cup [7, 8] \cup [9, 10]$.

6. In this problem, let T be the set of all two-element subsets of $\{1, 2, 3, 4\}$.

   (a) Write down all the elements of T.

   (b) Write down the partition of T in which two elements of T are in the same part of the partition whenever they have the same largest element.

   This is a common way to define a partition. We might say: *"Partition the elements of T by their largest element."*

7. Write down sets A and B, as small as you can make them, such that $\{(1, 3), (3, 1), (5, 1), (3, 2)\}$ is a subset of $A \times B$.

8. Find an open sentence $P(n)$, over a domain that includes at least the integers 1 through 9, which is true for every element of $\{4, 5, 9\}$ and false for every element of $\{1, 3, 6\}$.

9. Two lines are considered parallel if they don't intersect.

   State the negation of the statement "Two lines that are parallel to the same line are always parallel to each other" without using words like "no", "not", "don't", and so forth.

10. Sometimes the notation $P \uparrow Q$ is used to denote the operation "$\sim(P \wedge Q)$".

    Describe when the expression $(P \uparrow P) \uparrow (P \uparrow P)$ is true and when it is false. (You can write down a simpler expression for it, or a truth table; your choice.)