Math 2390 Homework 2

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due Friday, February 4, 2022

1 Short answer

1. Build a truth table for the statement $P \iff (Q \implies P)$. What's a shorter way to say the same statement?
2. To find out if
$$P(x) : x \text{ is a rational number}$$
is true, you come up with the test
$$Q(x) : x\text{'s decimal expansion terminates.}$$
   (a) Is $Q(x)$ a necessary condition for $P(x)$ (for all $x \in \mathbb{R}$)?
   (b) Is $Q(x)$ a sufficient condition for $P(x)$ (for all $x \in \mathbb{R}$)?
3. Put $\implies$, $\impliedby$, or $\iff$ between the two open sentences in each part to make them true for all $x \in \mathbb{R}$. (Use $\iff$ if you can.)
   (a) $P(x) : x^2 + 1 > 5$ and $Q(x) : \frac{1}{x^2+1} < \frac{1}{5}$.
   (b) $P(x) : x + 5 \geq 1$ and $Q(x) : (x + 5)^2 \geq 1^2$.
4. For statements P and Q, determine whether the compound statement $(P \wedge {\sim}P) \implies Q$ is a tautology, a contradiction, or neither.
5. Is the statement "$\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, (x \text{ is even}) \implies (y \text{ is even})$" true or false?
6. Write the statement "There are arbitrarily large integers n such that $P(n)$" using quantifiers.

2 Slightly longer answer

1. Let S be a region in the plane: a subset of $\mathbb{R} \times \mathbb{R}$. For each of the following open sentences (they are not statements because we didn't specify S), **explain in plain words** what they are saying about S.
   (a) $\exists x \in \mathbb{R}, \exists y \in \mathbb{R}, (x, y) \in S$.
   (b) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, (x, y) \in S$.

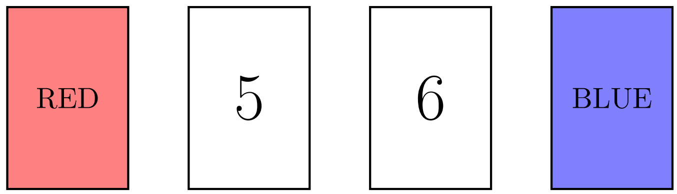
(c) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, (x, y) \in S$.

(d) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, (x, y) \in S$.

(Three of these have simple descriptions. The fourth one is trickier; do your best.)

2. (This is a well-known puzzle about logical statements.)

   You have four cards; each card has a number on one side, and a color on the other side. They are arranged in front of you so that all you can see is the following:



   I make the claim: "Every card with an even number on one side is red on the other side." Which cards do you need to flip over to find out if my claim is true?

   Explain your answer.