Math 2390 Homework 4

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1 Short answer

1. In this problem, assume that $A, B \subseteq \mathbb{R}$. Write the following claims as logical statements beginning "$\forall x \in \mathbb{R}, \dots$" and involving "$x \in A$" and "$x \in B$".
   (a) $A \cap B \subseteq A \cup B$.
   (b) $(A \cap B) \cup (A \cap \overline{B}) = A$.
2. Let $\mathbb{P}$ denote the set of all prime numbers (and $\mathbb{N}$, as usual, the set of natural numbers).
   (a) Write the statement "$\forall n \in \mathbb{N}, (n > 1 \implies \exists p \in \mathbb{P}, p \mid n)$" in twelve words or less, and without using any mathematical symbols.
   (b) Explain why the $\forall n \in \mathbb{N}$ statement from part (a) is true in the particular cases $n = 1$, $n = 3$, and $n = 6$.
3. The triangle inequality (Theorem 4.17 in the textbook) says that for all $x, y \in \mathbb{R}$, we have $|x + y| \le |x| + |y|$.

   Show how you can prove each of the following claims simply by substituting the right expressions for x and y in the triangle inequality (and maybe rearranging the result a bit).
   (a) $\forall a \in \mathbb{R}, |a - 1| \ge |a| - 1$.
   (b) $\forall a, b, c \in \mathbb{R}, |a - c| \le |a - b| + |b - c|$.

2 Proof

4. *You have already written a rough draft of this proof; now, write a final draft.*

   Prove that for all $a, b \in \mathbb{Z}$, if $a + b = 100$, then a and b are both even or both odd.
5. *For this problem, write a rough draft of an actual proof. I will give you feedback, and you will write a final draft for Homework 5.*

   Prove that for all sets A and B, $(A \times B) \cap (B \times A) = (A \cap B) \times (A \cap B)$.