

Math 2390 Homework 8

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due Monday, May 2, 2022

(Note the unusual due date. This is due on Monday at 11:59pm.)

1 Short answer

1. Give an example of a function $f: \{1, 2, 3, 4\} \rightarrow \{x, y, z\}$ that is:
 - (a) neither surjective (onto) nor injective (one-to-one).
 - (b) surjective, but not injective.
2. Let $A = \{0, 1, \dots, 9\} \times \{0, 1, \dots, 9\}$ and let $B = \{0, 1, \dots, 99\}$.
 - (a) Find a bijection $f: A \rightarrow B$ for which $f(x, y)$ is given by some short arithmetic formula involving x and y .
 - (b) Find a formula for f^{-1} . It may help to use the mod operator: recall that $a \bmod 10$ is the remainder when a is divided by 10.

2 Proof

3. You have already written a rough draft of this proof; now, write a final draft.

Let x_n be a sequence that starts at $x_1 = 4$, and continues by the rule $x_{n+1} = \frac{2}{x_n} + \frac{x_n}{2}$ for all $n \in \mathbb{N}$.

Prove that that the sequence is always decreasing and always stays greater than 2.

4. Consider the incomplete proof below:

Theorem. For all functions $f: A \rightarrow B$ and subsets $C, D \subseteq A$, $f(C) - f(D) \subseteq f(C - D)$.

Proof. Let y be an arbitrary element of $f(C) - f(D)$. Then (a) _____, so there is an $x \in C$ such that $y = f(x)$. Moreover, $x \notin D$, since (b) _____. Therefore $x \in C - D$, so by definition $y \in f(C - D)$. \square

- (a) How do we know that there is an $x \in C$ such that $y = f(x)$?
- (b) How do we know that $x \notin D$?
- (c) Give an example showing that $f(C) - f(D)$ is not necessarily equal to $f(C - D)$.