Calculus IV Homework 2

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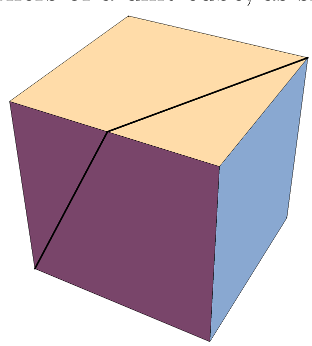
due Friday, February 3, 2023

1 Exercises

1. Find parametrizations for the following 3-dimensional curves:

   (a) A circle of radius 2, centered at the origin, in the plane $x = y$.

   (b) A piecewise linear curve from $(0, 0, 0)$ to $(\frac{1}{2}, 0, 1)$ to $(1, 1, 1)$. (This curve is the shortest path between opposite corners of a unit cube, as shown in the diagram below.)



   (c) A curve that goes from the north pole to the south pole of a sphere of radius 1 so that, in spherical coordinates, $\theta = \phi$ at every point of the curve.

2. Integrate $f(x, y, z) = xyz$ over the curve $\mathbf{r}(t) = (t, t^2, \frac{2}{3}t^3)$, where $0 \le t \le 1$.

3. Let $\mathbf{F}$ be the vector field $\mathbf{i} - xy\,\mathbf{j}$ and let C be the oriented curve parametrized by $\mathbf{r}(t) = (t, 1 - t^2)$ as t goes from -1 to 1.

   (a) Draw a diagram of the curve C and of the vector field $\mathbf{F}$ over a reasonable region that contains C.

   (b) Based on your diagram, make a prediction of whether

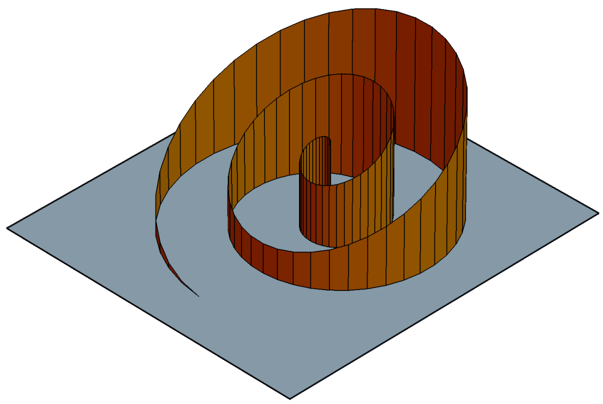
$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

   will be positive or negative. Then, find the integral.

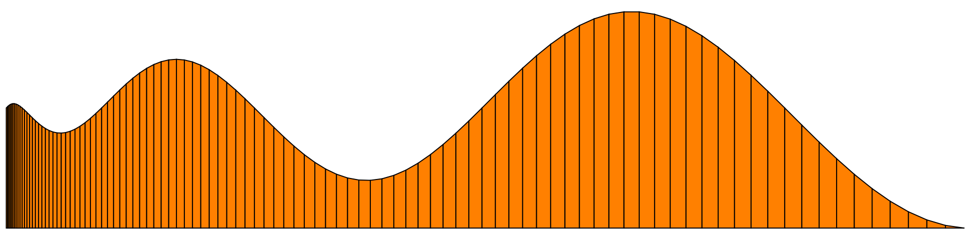
2 Harder problems

1. Let C be the quarter of the ellipse $\frac{x^2}{4} + y^2 = 1$ for which $x, y \geq 0$. Find $\int_C xy\, ds$.

4. The spiral $r = \theta$ where $0 \leq \theta \leq 5\pi$ in the xy-plane is drawn; a strip of paper is placed vertically on that spiral, and cut so that the height of the spiral at each point (x, y) will be exactly equal to $x + 5\pi$. This is shown in the diagram below, where $\nearrow$ is the positive-x direction and $\nwarrow$ is the positive-y direction:



Now suppose we take this strip of paper and unwrap it into a flat shape:



Give a description of the curve forming the top edge of the strip of paper.

(Fine print: I don't need it to be in the form $y = f(x)$, it will be easier to give it a parametric description $x = g(t), y = h(t)$. You may leave integrals unevaluated.)