Calculus IV Homework 4

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due Friday, March 3, 2023

1 Exercises

1. Find the circulation density and the flux density of each of the following vector fields:
   (a) The vector field $\frac{\mathbf{i}+\mathbf{j}}{x+y}$.
   (b) The vector field $e^x(1+\sin y)\,\mathbf{i} + e^x \cos y\,\mathbf{j}$.
2. Find the flux density of $\mathbf{F} = x^2y\,\mathbf{i} + xy^2\,\mathbf{j}$, and use it to find the outward flux of $\mathbf{F}$ across the boundary of the unit square $\{(x,y) : 0 \le x, y \le 1\}$.
3. On the practice exam, I asked you to find the (counterclockwise) circulation of $\mathbf{F} = x\,\mathbf{i} + xy\,\mathbf{j}$ around the closed curve C parameterized by

$$\mathbf{r}(t) = \begin{cases} (-t,-t) & -1 \le t \le 0 \\ (t,t^2) & 0 \le t \le 1 \end{cases}$$

   Now that we know Green's theorem, we can solve the same problem more easily.
   (a) Find the circulation density of $\mathbf{F}$, as a function of x and y.
   (b) Describe the region bounded by C by giving bounds on x and y.
   (c) Find the circulation around C by integrating the circulation density over that region.

2 Harder problems

4. For each part of this problem, come up with an example of a 2-dimensional vector field with the required properties.
   (a) A nonzero vector field for which both the circulation density and the flux density are 0.
   (b) A vector field for which the circulation density and flux density are both a positive constant at every point.
   (c) A conservative vector field for which the flux density at (x,y) is proportional to x^2+y^2.
5. The vector field $\mathbf{F} = 4y\,\mathbf{i} + x^2\,\mathbf{j}$ is *not* conservative. However, the circulation around *some* closed curves will still be 0.

   Find a circle of radius 1 (a circle parameterized by $\mathbf{r}(t) = (a+\cos t, b+\sin t)$ as $0 \le t \le 2\pi$) such that $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$. (This can be done the hard way, but it is also possible to solve this problem without taking any integrals.)