Calculus IV Homework 6

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due Friday, March 31, 2023

1 Exercises

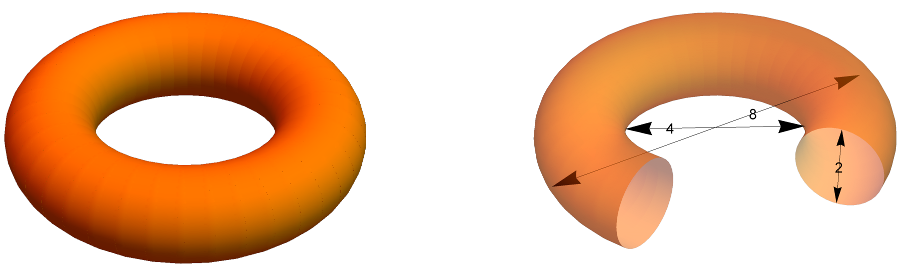
1. For each of the following shapes, describe their "shadows" or projections onto each of the xy-, xz-, and yz-planes. If you prefer to draw a picture, that is sufficient, but be sure your proportions are roughly correct.

   For example, if I gave you a vertical cylinder, then its projection onto the xy-plane would be a disk, and its projections onto the other two planes would be rectangles.

   (a) A cone of height 2, standing upright, whose base is a circle of radius 1.

   (b) The flat triangle with corners at the points $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 3)$.

   (c) The torus from the first homework assignment, whose diagram I am reusing:



2. Integrate $f(x, y, z) = z$ over the surface parameterized by

$$\mathbf{r}(u, v) = (\cos v, \sin v, u(1 + \cos v))$$

   where $0 \le u \le 1$ and $0 \le v \le 2\pi$.

3. Find the flux of $\mathbf{F} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ across the portion of the surface $z = xy$ where $0 \le x \le y \le 1$.

   (Hint: look for a parameterization with $x = u$ and $y = v$, but keep in mind that the domain of the parameterization is not rectangular.)

2 Harder problems

4. Find the center of mass of a thin conical shell whose shape is given by the equation $z = \sqrt{x^2+y^2}$ for $0 \le z \le 1$, and whose density (mass per unit area) is given by $\delta(x,y,z) = 1+z$.

5. Let M be the surface with the parameterization

$$\mathbf{r}(u,v) = \left((2+v\cos\tfrac{u}{2})\cos u, (2+v\cos\tfrac{u}{2})\sin u, v\sin\tfrac{u}{2}\right)$$

   where $0 \le u \le 2\pi$ and $-1 \le v \le 1$.

   (a) Use a computer tool to graph the surface; attach a screenshot as a separate file to your homework submission.

   (If you do not have a preferred computer tool, Wolfram Alpha is fairly straightforward to use; ask it for a "3d parametric plot" and then include the necessary details. Stop by office hours or send me an email if you've been trying for a few minutes and can't get it to work.)

   (b) Let $\mathbf{F} = \mathbf{k}$ (the vector field that points directly upward). Compute the flux of $\mathbf{F}$ across M using the formula

$$\iint_R \mathbf{F} \cdot \left(\frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v}\right) du\,dv.$$

   (Hints: a lot of the work in computing $\frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v}$ can be skipped; a lot of terms simplify or cancel; choose wisely between integrating $du\,dv$ and $dv\,du$.)

   (c) Explain what's wrong with the calculation in (b).