Calculus IV Homework 8

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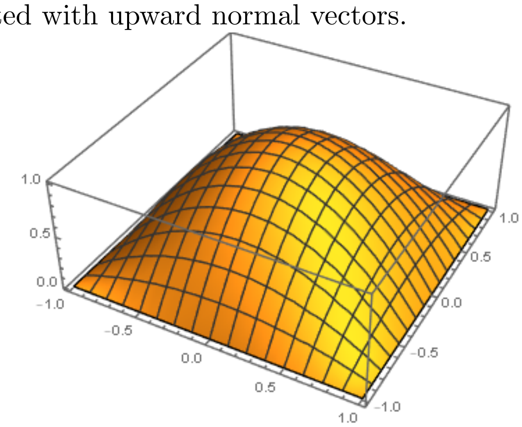
due Friday, April 28, 2023

1 The divergence theorem

1. Find the outward flux of $\mathbf{F} = (x+2y)\,\mathbf{i} + (3z+4x)\,\mathbf{j} + (5y+6z)\,\mathbf{k}$ across the sphere of radius 2 (that is, given by the equation $x^2+y^2+z^2=4$) by using the divergence theorem.

2. Let S be the portion of the surface $z = (x^2-1)(y^2-1)$ bounded by $-1 \le x \le 1$ and $-1 \le y \le 1$, which is graphed below; note that when $x = \pm 1$ or $y = \pm 1$ on this surface, $z = 0$. Let S be oriented with upward normal vectors.



   Let Q be the square bounded by $-1 \le x \le 1$ and $-1 \le y \le 1$ in the plane $z = 0$, and let R be the three-dimensional region that lies between Q and S.

   Suppose we want to measure the flux of $\mathbf{F} = x^3\,\mathbf{i} + z^2\,\mathbf{j} + x^2 z\,\mathbf{k}$ across S. Use the divergence theorem to express this flux in terms of a double integral over Q and a triple integral over R.

   You do not have to evaluate these integrals, but you should simplify them to straightforward iterated integrals.

2 Review exercises

3. Find parameterizations of the following curves:

   (a) A line segment from the point $(1, 0, -2)$ to the point $(0, 1, 3)$.

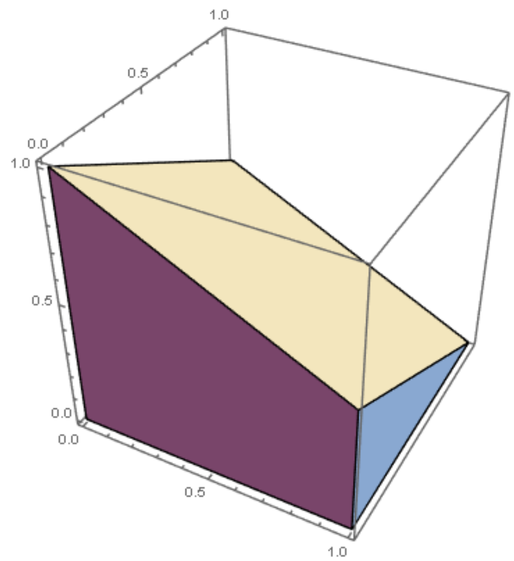
   (b) A circle of radius 2 in the plane $y = 0$ centered at the point $(-1, 0, 1)$.

(c) The curve that follows the upside-down paraboloid $z = 2 - x^2 - y^2$ in the plane $x = y$ from $(-1, -1, 0)$ to $(1, 1, 0)$.

4. (a) Find the gradient of $f(x, y, z) = xe^{x+y+z}$.

   (b) Find the flow of the gradient field you found in part (a) over *each* of the curves parameterized in the previous problem.

5. The solid cube with boundaries $0 \leq x \leq 1$, $0 \leq y \leq 1$, and $0 \leq z \leq 1$ is cut in half by the plane $z = 1 - \frac{1}{2}x - \frac{1}{2}y$ (that is, we consider only the half satisfying $z \leq 1 - \frac{1}{2}x - \frac{1}{2}y$), as shown below.



   Find the centroid of the remaining half of the cube.