

# Discrete Math Homework 1

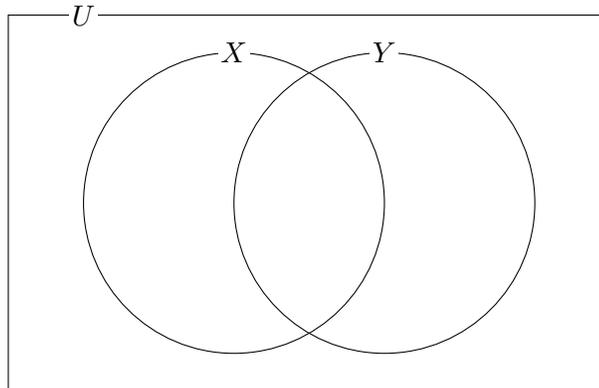
Mikhail Lavrov

due Friday, January 20, 2023

## 1 Sets and statements about sets

1. Consider the statement “The empty set is a subset of every set”.
  - (a) Write it in the following form: “For all sets  $A$ , \_\_\_\_\_.”
  - (b) Expand it further to: “For all sets  $A$ , if  $x$  \_\_\_\_\_, then  $x$  \_\_\_\_\_.”  
(Use the definition of subsets.)
  - (c) Do you think the statement is true or false? Explain your answer briefly.
2. In this problem, let  $U$  (our universal set) be the set of integers. Let  $X$  be the set of all integers less than 10, and let  $Y$  be the set of all perfect squares (numbers of the form  $n^2$ , where  $n$  is an integer).

Fill in the Venn diagram below by writing down, in each of the four regions, a few example elements. (*Three of the four regions have infinitely many elements; you should just give a few examples.*)



3. Use the sets  $A = \{1, 2, 3\}$ ,  $B = \{2, 3, 4\}$ , and  $C = \{3, 4, 5\}$  with the operations  $\cup$  (union),  $\cap$  (intersection), and  $-$  (relative complement) to write down an expression that simplifies to  $\{1, 3, 4\}$ .
4. Let  $S = \{1, 2, 3\}$  and  $T = \{1\}$ .
  - (a) Write down all the elements of the power set  $\mathcal{P}(S - T)$ .
  - (b) Write down all the elements of  $\mathcal{P}(S) - \mathcal{P}(T)$ .

- (c) You should have gotten different results for part (a) and part (b). Explain, for general  $S$  and  $T$ , what sort of elements you expect to appear in one of these results but not the other.

5. Draw the set

$$\bigcup_{n=1}^{\infty} [2n - 1, 2n)$$

on a number line. Be sure to use filled and empty circles to indicate which endpoints are/are not in the set.

## 2 Counting problems

6. An author submitted 10 short stories to an anthology. You're the editor, and you would like to include between 3 and 5 of them.

Two of the stories are clearly the best, and you've decided to put one of them first and the other last (but you're not sure which should be first and which should be last). In between, you want to put between 1 and 3 of the other stories, in some order.

How many possible ways are there for you to pick out and order the stories while obeying these conditions?

7. (a) There are 9000 four-digit numbers. How many of them are palindromes—written the same forwards and backwards? An example is 2772.
- (b) If a random four-digit number is chosen, there is exactly a  $\frac{1}{15}$  probability that it is divisible by 15. What's more, only 3 four-digit numbers are both palindromes and divisible by 15: they are 5115, 5445, and 5775.

How many four-digit numbers are palindromes **or** divisible by 15 (or both)?