Discrete Math Homework 4

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due Friday, March 3, 2023

1 Short answer

1. (a) Write the statement "Some integers n are divisible by 12, but not divisible by 18" using quantifiers (and the definition of divisibility).

   (b) The statement "$\forall n \in \mathbb{Z}, ((\exists k \in \mathbb{Z}, n = 12k) \implies (\exists k \in \mathbb{Z}, n = 4k))$" is a claim about a property of divisibility. Write it in words.

2. Suppose we want to prove the claim "For all positive integers n, if n is even, then $2^n - 1$ is divisible by 3."

   Classify each of the following as the beginning of a direct proof, proof by contrapositive, proof by contradiction, or a mistake.

   (a) Let n be a positive integer. Suppose that n is even; we want to show that $2^n - 1$ is divisible by 3.

   (b) Let n be a positive integer. Suppose that $2^n - 1$ is divisible by 3; we want to show that n is even.

   (c) Let n be a positive integer. Suppose that $2^n - 1$ is not divisible by 3; we want to show that n is odd.

   (d) Let n be a positive integer. Suppose that n is even, but $2^n - 1$ is not divisible by 3.

3. Let S be the set $\{4, 5, 6, 7, 8, 9\}$ and let R be the relation from S to S defined as follows: $(x, y) \in R$ if there is some integer $d > 1$ such that both x and y are divisible by d.

   (For example, $(4, 6) \in R$, because 4 and 6 are both divisible by 2.)

   (a) Describe R by a set of ordered pairs.

   (b) Draw an arrow diagram for R.

   (c) Find an example of $x, y, z \in S$ such that $x \; R \; y$ and $y \; R \; z$ are both true, but $x \; R \; z$ is false.

2 Proofs

4. *You have already written a rough draft of this problem; now, read my feedback and write a final draft.*

   Prove that for any two odd positive integers r and s, $3r - 5s$ is even.

5. *For this problem, write a rough draft of a proof; any reasonable attempt will be given full credit. I will give you feedback, and you will write a final draft on the next homework assignment.*

   Prove that for any two integers x and y, if x is divisible by 3 and xy is not divisible by 6, then y is odd.