Discrete Math Homework 5

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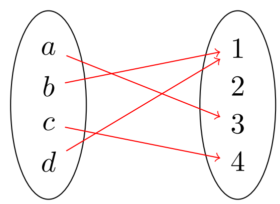
due Friday, March 17, 2023

1 Short answer

1. Here is an arrow diagram for a function f from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$.



   (a) Write the following three sets: the domain of f, the co-domain of f, and the range of f.

   (b) For each element of the co-domain, find its inverse image (a subset of the domain).

   (c) The function f is neither injective (one-to-one) nor surjective (onto). For each of these properties, say how f violates it.

   (d) Represent f as a set of ordered pairs.

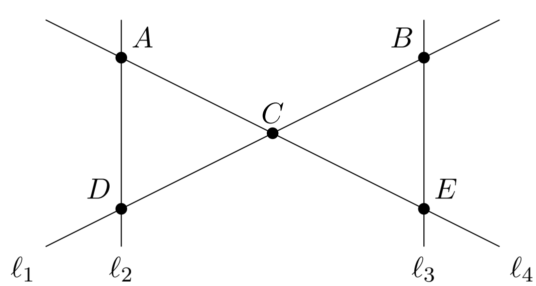
2. Find the following quantities (to get practice with functions and relations at the same time as reviewing counting techniques):

   (a) The number of *injective* (one-to-one) functions from $\{1, 2, 3\}$ to the power set $\mathcal{P}(\{1, 2, 3\})$.

   (b) The number of *surjective* (onto) functions from $\{0, 1\}^3$ to $\{0, 1\}$.

   (c) The number of *reflexive* relations on the set $\{a, b, c, d\}$.

3. The diagram below shows five points A, B, C, D, E and four lines $\ell_1, \ell_2, \ell_3, \ell_4$, each of which passes through some of the five points:



Let Q be the "incidence relation" from points to lines, defined as follows: for a point p and a line ℓ,

$$p \; Q \; \ell \iff \ell \text{ passes through } p.$$

(a) Draw an arrow diagram for Q.

(b) Describe the inverse relation Q^{-1} as a set of ordered pairs.

4. Let R be the relation defined on the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ by the rule that $x \; R \; y$ if and only if $x + y$ is divisible by 3. (For example, $1 \; R \; 5$, because $1 + 5 = 6$, which is a multiple of 3.)

   (a) Is R reflexive? (If not, give a counterexample.)

   (b) Is R symmetric? (If not, give a counterexample.)

   (c) Is R transitive? (If not, give a counterexample.)

2 Proofs

5. *You have already written a rough draft of this problem; now, read my feedback and write a final draft.*

   Prove that for any two integers x and y, if x is divisible by 3 and xy is not divisible by 6, then y is odd.

6. *For this problem, write a rough draft of a proof; any reasonable attempt will be given full credit. I will give you feedback, and you will write a final draft on the next homework assignment.*

   Define a function f from the positive integers $\mathbb{Z}^+ = \{1, 2, 3, 4, 5, \ldots\}$ to the integers $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ by the following rule:

   - If $n \in \mathbb{Z}^+$ is odd, let $f(n) = \frac{n-1}{2}$.
   - If $n \in \mathbb{Z}^+$ is even, let $f(n) = -\frac{n}{2}$.

   Prove that f is injective (one-to-one).