Discrete Math Homework 7

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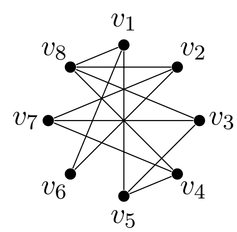
due Friday, April 14, 2023

1 Short answer

1. Let G be the graph below:



   (a) List the degrees $\deg(v_1), \deg(v_2), \dots, \deg(v_8)$ in this graph.

   (b) Use your answer to (a) to find the number of edges in G without counting.

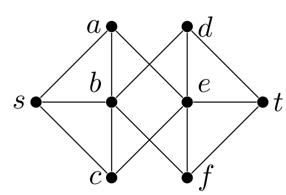
2. Let G be the graph which has six vertices $\{a_1, a_2, b_1, b_2, b_3, b_4\}$ and all 8 edges of the form $a_i b_j$.

   (a) Draw a diagram of G.

   (b) Label all vertices in the diagram with their degree.

   (c) Verify the handshake lemma for G.

3. Let G be the graph below:



   (Note: I haven't labeled the edges in this graph; every answer in this problem should just be given as a sequence of vertices.)

   (a) Find a path from s to t of the shortest length possible. (Briefly explain why the path cannot be shorter.)

(b) Find a path from s to t that is as long as possible. You don't have to explain why the path cannot be longer.

(A path cannot repeat vertices or edges.)

(c) Find a trail from s to t of length 8. (A trail can repeat vertices, but not edges.)

2 Proofs

4. *You have already written a rough draft of this problem; now, read my feedback and write a final draft.*

   Let $a_1, a_2, a_3, \ldots$ satisfy the property that $a_n = \frac{a_{n-1}+a_{n+1}}{2}$ for all integers $n \geq 2$.

   Prove that if $a_1 = a_2$, then the sequence is constant (all terms of the sequence are equal).

5. *For this problem, write a rough draft of a proof; any reasonable attempt will be given full credit. I will give you feedback, and you will write a final draft on the next homework assignment.*

   Prove by induction on n that for all $n \geq 1$,

$$\sum_{k=1}^{n} \frac{1}{2^k} = 1 - \frac{1}{2^n}.$$