Discrete Math Homework 8

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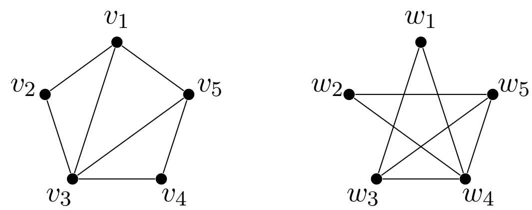
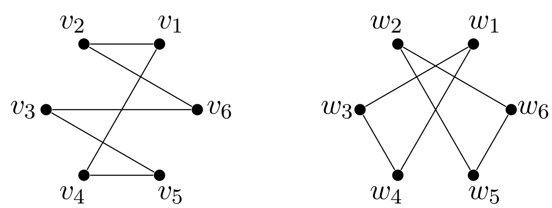
due Friday, April 28, 2023

1 Short answer

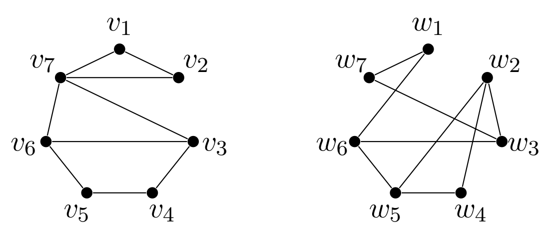
1. (a) Determine whether the two graphs below are isomorphic. If they are, give an isomorphism; if not, explain why not.



   (b) Determine whether the two graphs below are isomorphic. If they are, give an isomorphism; if not, explain why not.



   (c) Determine whether the two graphs below are isomorphic. If they are, give an isomorphism; if not, explain why not.

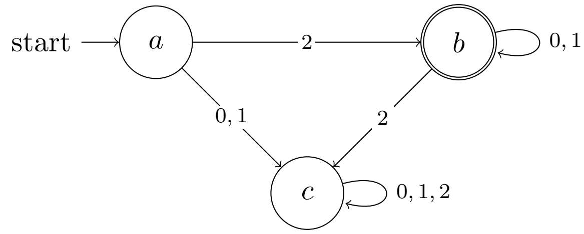


2. Draw lines from the regular expressions in the first row to the strings they match in the second row. A regular expression can match more than one string.

$\boxed{2(0 \mid 1)^*}$ $\quad$ $\boxed{(0 \mid 10 \mid 20)^*}$ $\quad$ $\boxed{2^*0^*1^*2^*}$ $\quad$ $\boxed{(0 \mid 1)^* \mid (1 \mid 2)^*}$

22122 $\quad$ 00000001 $\quad$ λ $\quad$ 20010010 $\quad$ 10200

3. Consider the finite-state automaton in the diagram below:



(a) The language that this automaton accepts is exactly the language matched by one of the regular expressions in question 2. Which one?

(b) Briefly explain the purpose of each of the states a, b, and c; what is the automaton "doing" in each of those states?

(c) Suppose that we change the automaton so that state c is accepting, but state b is not. How would the language accepted by the automaton change? You may explain in words, or by giving a regular expression.

2 Proofs

4. *You have already written a rough draft of this problem; now, read my feedback and write a final draft.*

Prove by induction on n that for all $n \geq 1$,

$$\sum_{k=1}^{n} \frac{1}{2^k} = 1 - \frac{1}{2^n}.$$

5. (a) Let G be an arbitrary graph with vertices $v_1, v_2, \ldots, v_n$. Suppose we know that exactly k vertices have degree 1, all other vertices have degree at least 2, and at least one vertex has degree 10.

What is the minimum possible value of the sum

$$\deg(v_1) + \deg(v_2) + \cdots + \deg(v_n)?$$

(b) Forgetting part (a) for the moment, let G be a tree with vertices $v_1, v_2, \ldots, v_n$. What must the sum

$$\deg(v_1) + \deg(v_2) + \cdots + \deg(v_n)$$

be equal to?

(c) (Bonus problem!)

Use parts (a) and (b) to write a proof that if a tree has a vertex of degree 10, then it has at least 10 leaves.