Probability Theory Homework 1

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due Friday, January 27, 2023

1. As part of the game "Scrabble", you draw tiles with letters on them from a bag. Let's consider a scenarion in which the game is nearly over; there are four tiles in the bag, labeled "A", "C", "G", and "I", and you are about to draw *two* of those tiles.

   (a) Describe the sample space of this random experiment.

   (b) You think you can win the game if you draw a vowel from the bag. Express the event "you draw a vowel" as a set of outcomes.

   (c) What is the probability that you draw a vowel?

2. If a number between 1000 and 9999 is chosen uniformly at random, there is a $\frac{1}{100}$ chance that it is a palindrome (like 2772, for example) and a $\frac{1}{15}$ chance that it is divisible by 15 (like $2190 = 15 \cdot 146$, for example). Only three numbers in this range are both palindromes and divisible by 15: they are 5115, 5445, and 5775.

   What is the probability that a number between 1000 and 9999 chosen uniformly at random is either a palindrome or divisible by 15 (or both)?

3. You are in a game show in which you're asked a sequence of true/false questions. If you answer two of them wrong, you're out!! Unfortunately, you have no idea what the answers are, so you guess randomly each time, and you have a $\frac{1}{2}$ chance of guessing the correct answer.

   Let A_3 be the event that your first wrong answer is to the third question you are asked.

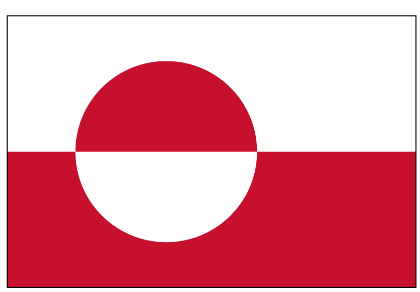
   Let B_4 be the event that your *second* wrong answer is to the fourth question you are asked.

   (a) Compute $\Pr[A_3]$.

   (b) Compute $\Pr[A_3 \cap B_4]$.

   (c) Compute $\Pr[B_4]$.

4. The flag of Greenland is shown below:



It is an 18×12 rectangle; if placed on a coordinate plane with bottom left corner at $(0, 0)$ and top right corner at $(18, 12)$, it is divided in half by a line at $y = 6$ and has a circle of radius 4 centered at $(7, 6)$.

The top half of the flag is white and the bottom half is red; within the circle, the two colors are swapped.

Suppose that a point on this flag is chosen uniformly at random.

(a) Find the probability that the chosen point is red.

(b) Find the probability that the chosen point is red **or** inside the circle. (As usual, "or" in mathematics includes the possibility that both things happen.)

5. Suppose that you know that your friend was born sometime in the year 2001, but not which day. Given this state of ignorance, we can model your friend's birthday as being chosen uniformly at random from the 365 days of a non-leap year. (In reality, these are not entirely uniform, but they're pretty close, and we can ignore this effect.)

   (a) What is the probability that your friend was born in January?

   (b) Suppose you remember that your friend's birthday is on the 20^{th} (of some month). Conditioned on this fact, what is the probability that your friend was born in January?

   (c) Now suppose you remember that your friend's birthday is on the 30^{th} (of soe month). Conditioned on this fact, what is the probability that your friend was born in January?