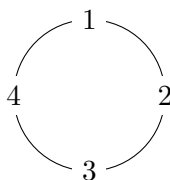


Probability Theory Homework 2

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due Friday, February 10, 2023*

1. Four people are seated around a table as in the diagram below. Another clumsy person accidentally spills water on two **adjacent** people at the table. (For example, the water could be spilled on 1 and 2, or on 1 and 4, but not on 1 and 3.) The clumsy person is equally likely to spill water on any of the four adjacent pairs.



Let W_1 be the event “person 1 had water spilled on them”. Define W_2, W_3, W_4 similarly.

- (a) Which pairs of the events W_1, W_2, W_3, W_4 are **independent**? Explain why.
 - (b) Which pairs of the events W_1, W_2, W_3, W_4 are **disjoint**? Explain why.
2. You operate a polar research station; every morning, you send a radio message indicating which way the wind is blowing. The wind is likely to blow in any of 4 directions, which you indicate with the codewords “000”, “110”, “101”, and “011”.

Due to the extreme weather, each bit (0 or 1) of the message independently has a $\frac{1}{3}$ chance of being flipped in transit: 1 changed to 0 or vice versa.

What is the probability that when you send your message, the recipient receives “111” on their end?

3. You are waiting for a bus at a bus stop. There are actually two buses that stop here. The one you want will arrive at a uniformly random time within the next 30 minutes. There is also a different bus, which will arrive at a uniformly random time within the next 60 minutes.

Given that **exactly one** of the buses arrives within the next 5 minutes, what is the probability that it is the one you want?

(Note: we want a conditional probability here—and in the next two problems, as well.)

4. You find a 6-sided die on the ground. For no reason in particular, you decide to roll it. After you roll it **three times**, you discover something peculiar: every time, the die has landed on the side labeled \boxtimes (4). You get suspicious: what if this is an extremely biased die that always lands on 4?

*Note that your third homework assignment will be due on Friday, February 17th; only a week later than this assignment. Plan accordingly! All the material on this assignment will be covered in class by Monday, February 6th.

- (a) What extra piece of information would you need to calculate, given your experimental data, how likely this die is to be an “extremely biased” die that always lands on 4?
 - (b) Come up with a plausible value for this extra piece of information, and use it to compute that probability.
5. Suppose you live in an extremely unfortunate climate where it always rains on weekends (Saturday and Sunday). For that matter, there is always a $\frac{1}{2}$ chance of rain on a weekday (Monday–Friday) as well.

If you have completely forgotten which day of week it is (so that it is equally likely to be any of the seven days), but you go outside and see that it’s raining, what is the probability that it’s the weekend?