Probability Theory Homework 3

Mikhail Lavrov

due Friday, February 17, 2023

1. Suppose you pick a subset of $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ uniformly at random. What is the probability that your subset contains **exactly one** of the numbers 1, 2, or 3?

2. A bag of marbles contains 6 red marbles and 7 blue marbles.

   You take 4 marbles out of the bag, without replacement. What is the probability that at least one of the marbles is red?

3. You roll a fair six-sided die three times. What is the probability that you roll three different numbers in ascending order?

   (For example, $1, 3, 6$ counts, because $1 < 3 < 6$, but $3, 6, 1$ does not count, because 6 is not less than 1, and $2, 2, 5$ does not count, because 2 is not less than 2.)

4. You draw 4 cards from a standard 52-card deck. What is the probability that you draw one card of each suit if:

   (a) You draw a hand of 4 cards, so you are holding them all at once?

   (b) You draw 4 cards one at a time, putting each card back and shuffling before you draw the next?

   (There are 4 suits in the deck; there are 13 cards of each suit.)

5. You buy a box of chocolates for Valentine's Day. There are 9 chocolates in the box. The store has 3 kinds of chocolates available (milk, dark, and white chocolate) in unlimited amounts. You decide to get a random assortment of chocolates.

   (a) How many random assortments are possible? You do not care how the chocolates are arranged in the box—you only care about the number of chocolates of each kind.

   (b) If "random assortment of chocolates" means that each chocolate is (independently) equally likely to be any of the three kinds, what is the probability that you get 3 chocolates of each kind in the box?