Probability Theory Homework 5

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1. For each random variable described below, give its distribution (which will be Binomial, Geometric, Hypergeometric, Pascal, or Poisson) and the parameters of that distribution.

   (a) You are working on a difficult homework assignment with 5 problems. Every minute, you have an idea for the problem you are working on, but it only has a 5% chance of working. If it doesn't work, you keep working on that problem, and if it works, you move on to the next problem.

   The random variable $\mathbf{W}$ is the time (in minutes) that it takes you to finish the homework assignment.

   (b) A standard 52-card deck is split equally between two players. The random variable $\mathbf{X}$ is the number of aces received by the first player. (There are four aces in the deck.)

   Note: this is a key parameter when playing the "card game" "War".

   (c) Every time you turn on the light, the light bulb has a 1% chance of burning out and you have to replace it. The random variable $\mathbf{Y}$ is the number of times you turn on the light before you have to replace the light bulb.

   (d) A hitchhiker standing by the side of a rural highway sees, on average, one car drive by every 10 minutes. (None of the cars stop to pick up the poor hitchhiker.) The random variable $\mathbf{Z}$ is the number of cars that the hitchhiker sees over the next hour.

2. You take a 20-question multiple choice exam on which every correct answer is worth 1 point, and every incorrect answer is worth $-\frac{1}{4}$ points to discourage guessing. Each question has five options: (A) through (E). On each question, you are able to eliminate one of the options as certainly wrong (leaving four), and then guess randomly between the other four options.

   (a) The number of correct answers you give is a random variable $\mathbf{X}$ with a binomial distribution. What are the parameters (n and p) of that distribution?

   (b) Express the number of points you receive as a linear transformation $a\mathbf{X} + b$ of $\mathbf{X}$, the random variable from part (a).

   (c) Find the probability that you get exactly 12.5 points.

3. Alice and Bob buy a bag of apples, which contains 10 good and 4 bad apples. They split the apples in half evenly: Alice and Bob each take 6 apples, chosen at random. Let $\mathbf{A}_{\text{good}}$ be the number of good apples Alice gets, and define $\mathbf{A}_{\text{bad}}$, $\mathbf{B}_{\text{good}}$, and $\mathbf{B}_{\text{bad}}$ similarly.

Describe the distribution of $\mathbf{A}_{\text{good}}$ as a hypergeometric distribution (give its parameters). Then, write $\mathbf{A}_{\text{bad}}$, $\mathbf{B}_{\text{good}}$, and $\mathbf{B}_{\text{bad}}$ as functions of $\mathbf{A}_{\text{good}}$.

4. On a game show, you start with a prize pool of \$1 000 000, but before you get it, you must answer trivia questions. Every time you get a question wrong, the current prize pool is cut in half, and you must keep answering questions. The game ends when you get a question right—and you receive whatever is left of the prize pool.

   (For example, if you get the first two questions wrong, the prize pool is cut in half twice, so it becomes \$250 000*. If you then get the third question right, you go home with* \$250 000 *as your winnings.)*

   (a) If you have a $\frac{1}{3}$ chance of getting each question right, what is the distribution of $\mathbf{Q}$: the total number of questions you have to answer? (Give the name of the distribution and its parameters.)

   (b) Express the amount of money you win as a function of $\mathbf{Q}$.

   (c) Find the expected amount of money you win.

5. A fair 10-sided die is rolled; let $\mathbf{D}_{10}$ be the number that comes up. Find $\mathbb{E}[\min\{\mathbf{D}_{10}, 6\}]$.

   (If the notation is not clear: $\min\{x, y\}$ *just refers to the smaller of the two numbers,* x *and* y*. This notation is not unique to probability, but it comes up more often in probability problems.)*