

Discrete Math Homework 1

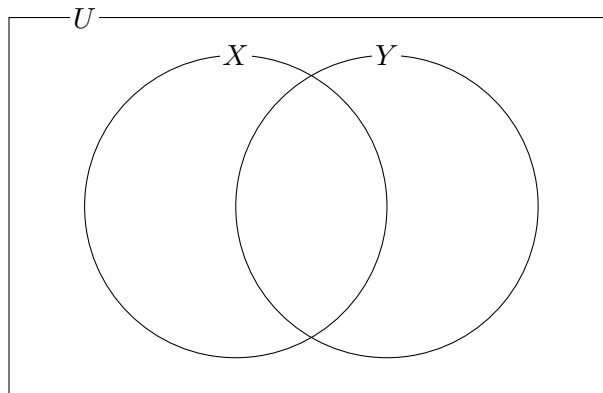
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due Friday, January 17, 2025

1 Sets and statements about sets

1. Consider the statement “The empty set is a subset of every set”.
 - (a) Write it in the following form: “For all sets A , _____.”
 - (b) Expand it further to: “For all sets A , if x _____, then x _____.”
(Use the definition of subsets.)
 - (c) Do you think the statement is true or false? Explain your answer briefly.
2. In this problem, let U (our universal set) be the set of integers. Let X be the set of all integers less than 10, and let Y be the set of all perfect squares (numbers of the form n^2 , where n is an integer).

Fill in the Venn diagram below by writing down, in each of the four regions, a few example elements. (*Three of the four regions have infinitely many elements; you should just give a few examples.*)



3. Use the sets $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, and $C = \{3, 4, 5\}$ with the operations \cup (union), \cap (intersection), and $-$ (relative complement) to write down an expression that simplifies to $\{1, 3, 4\}$.

4. Let $S = \{1, 2, 3\}$ and $T = \{3, 4\}$.
- Write down all the elements of the power set $\mathcal{P}(S - T)$.
 - Write down all the elements of $\mathcal{P}(S) - \mathcal{P}(T)$.
 - You should have gotten different results for part (a) and part (b). Explain, for general S and T , what sort of elements you expect to appear in one of these results but not the other.

5. Draw the set

$$\bigcup_{n=1}^{\infty} [2n - 1, 2n)$$

on a number line. Be sure to use filled and empty circles to indicate which endpoints are/are not in the set.

2 Counting problems

- There are 9000 four-digit numbers. How many of them are palindromes—written the same forwards and backwards? An example is 2772.
- How many ways are there to scramble the letters of the word “FRIDAY”? (Valid ways include “FRIDAY”, “ADFIRY”, and “YADIRF”; the six letters in the word, in any order.)
- At Universally Quantified University, there are two coffee shops in the student center. The first coffee shop sells 3 different drinks and 5 different pastries. The second coffee shop sells 4 different drinks and 6 different pastries.

In how many different ways is it possible to order a drink and a pastry from the same coffee shop?

- Between 1 and 100, there are exactly 25 primes, and exactly 33 numbers divisible by 3. How many numbers between 1 and 100 are either prime *or* divisible by 3 (or both)?
- A password generator app claims to generate “memorable” passwords, such as “fromycy” or “pocodugi”.

The passwords it generates consist of eight lowercase letters, alternating consonants and vowels (starting with a consonant). For this purpose, “a”, “e”, “i”, “o”, “u”, and “y” are considered vowels and the other 20 letters are considered consonants.

How many passwords are possible under this scheme?