Discrete Math Homework 2

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1 Counting problems

1. A website asks you to create a password using 8 lowercase letters. It has a "security" requirement: all 8 letters must be different. How many strings of 8 lowercase letters are *eliminated* by the security requirement?

2. At Existentially Quantified University, there are two coffee shops in the student center. The first sells 5 kinds of pastries and the second sells 8 kinds of pastries; all 13 kinds of pastries are different.

   In how many ways can you order 3 pastries of different kinds, not all from the same shop?

3. You got a box of chocolates for your birthday: 3 pieces of dark chocolate, 3 pieces of milk chocolate, and 3 pieces of white chocolate. The pieces of chocolate of each kind are identical. In how many distinguishable orders can you eat your chocolate?

4. You have to choose 4 out of the 7 days of the week to work on building a robot for the robotics club. You don't want to give up your entire weekend to do this, so you can choose at most one of Saturday or Sunday (not both). How many ways are there to choose 4 days?

5. How many 5-digit numbers contain exactly two instances of the digit 5?

6. Alice, Bob, and Carol are sharing 20 dumplings at a restaurant. How many ways are there to decide how many dumplings each of them gets?

   The dumplings are all identical, and even though it is rude, we include the cases where someone does not get any dumplings at all.

7. My fridge has fridge magnets on it; each magnet has a word on it, and they can be put together to form a sentence. Unfortunately, I've lost most of the magnets, and now I only have six:

   $$\boxed{\text{my}}\ \boxed{\text{cat}}\ \boxed{\text{meowed}}\ \boxed{\text{at}}\ \boxed{\text{the}}\ \boxed{\text{dog}}$$

   How many different sentences can I make using at least 4 of the magnets? (The sentence doesn't have to be coherent: something like $\boxed{\text{dog}}\ \boxed{\text{at}}\ \boxed{\text{meowed}}\ \boxed{\text{the}}$ counts.)

8. How many 8-bit sequences (sequences of 8 symbols that are either 0 or 1) have an odd number of 1's?

2 Logical statements

9. Consider the statement "If either Pauli or Quinn goes to see a movie, then both of them go."

   (a) Write the statement above in terms of p, q, and logical connectives, where p is "Pauli goes to see a movie" and q is "Quinn goes to see a movie".

   (b) Write a truth table for the statement (with rows for all possible truth values of p and q). You can fill in the table given below, or draw your own.

p	q	
T	T	
T	F	
F	T	
F	F	

10. Assuming the statement "If you haven't learned the meaning of friendship, you haven't learned anything" is true, is "You have learned something" a necessary condition for "You have learned the meaning of friendship", a sufficient condition, or both? Briefly explain why.