# Discrete Math Homework 4

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due Friday, February 28, 2025

## 1 Short answer

1. (a) Write the statement "Some integers $n$ are divisible by 12, but not divisible by 18" using quantifiers (and the definition of divisibility).

   (b) The statement "$\forall n \in \mathbb{Z}, ((\exists k \in \mathbb{Z}, n = 12k) \implies (\exists k \in \mathbb{Z}, n = 4k))$" is a claim about a property of divisibility. Write it in words.

2. Identify the mistake in each of the following proofs that "The sum of any two odd integers is even."

   (a) If we consider some cases like $1 + 1 = 2$, $1 + 3 = 4$, $3 + 3 = 6$, $3 + 5 = 8$, and so on, the sum is even in all of them.

   (b) Let $x$ and $y$ be any two odd integers; then $x = 2k + 1$ and $y = 2k + 1$ for some $k$. Therefore $x + y = 4k + 2 = 2(2k + 1)$, which is twice the integer $2k + 1$, so $x + y$ is even by definition.

   (c) Let $x$ and $y$ be any two odd integers, and suppose that their sum $x + y$ is even. Then there exist integers $a$, $b$, and $c$ such that $x = 2a + 1$, $y = 2b + 1$, and $x + y = 2c$. This means that $(2a+1)+(2b+1) = 2c$, which simplifies to $c = a+b+1$, which is an integer.

3. Suppose we want to prove the claim "For all positive integers $n$, if $n$ is even, then $2^n - 1$ is divisible by 3."

   Classify each of the following as the beginning of a direct proof, proof by contrapositive, proof by contradiction, or a mistake.

   (a) Let $n$ be a positive integer. Suppose that $n$ is even; we want to show that $2^n - 1$ is divisible by 3.

   (b) Let $n$ be a positive integer. Suppose that $2^n - 1$ is divisible by 3; we want to show that $n$ is even.

   (c) Let $n$ be a positive integer. Suppose that $2^n - 1$ is not divisible by 3; we want to show that $n$ is odd.

   (d) Let $n$ be a positive integer. Suppose that $n$ is even, but $2^n - 1$ is not divisible by 3.

## 2 Proofs

4. *You have already written a rough draft of this problem; now, read my feedback and write a final draft.*

   Prove that for any two odd positive integers $r$ and $s$, $3r - 5s$ is even.

5. *For this problem, write a rough draft of a proof; any reasonable attempt will be given full credit. I will give you feedback, and you will write a final draft on the next homework assignment.*

   Prove that for any two integers $x$ and $y$, if $x$ is divisible by 3 and $xy$ is not divisible by 6, then $y$ is odd.