

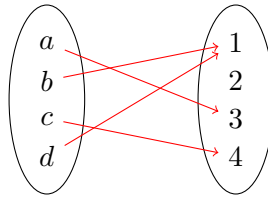
Discrete Math Homework 5

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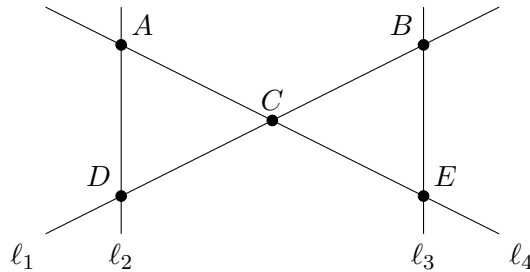
due Friday, March 21, 2025

1 Short answer

1. Here is an arrow diagram for a function f from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$.



- (a) Write the following three sets: the domain of f , the co-domain of f , and the range of f .
- (b) For each element of the co-domain, find its inverse image (a subset of the domain).
- (c) The function f is neither injective (one-to-one) nor surjective (onto). For each of these properties, say how f violates it.
- (d) Represent f as a set of ordered pairs.
2. Find the following quantities (to get practice with functions and relations at the same time as reviewing counting techniques):
- (a) The number of *injective* (one-to-one) functions from $\{1, 2, 3\}$ to the power set $\mathcal{P}(\{1, 2, 3\})$.
- (b) The number of *surjective* (onto) functions from $\{0, 1\}^3$ to $\{0, 1\}$.
- (c) The number of *reflexive* relations on the set $\{a, b, c, d\}$.
3. The diagram below shows five points A, B, C, D, E and four lines $\ell_1, \ell_2, \ell_3, \ell_4$, each of which passes through some of the five points:



Define a relation Q between the sets $\{A, B, C, D, E\}$ and $\{\ell_1, \ell_2, \ell_3, \ell_4\}$ as follows: for a point $p \in \{A, B, C, D, E\}$ and a line $\ell \in \{\ell_1, \ell_2, \ell_3, \ell_4\}$, $p Q \ell$ if and only if ℓ passes through p .

- (a) Draw an arrow diagram for Q .
 - (b) Describe the inverse relation Q^{-1} as a set of ordered pairs.
4. Consider a relation on the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ represented by the symbol \bowtie and defined by the rule that $x \bowtie y$ if and only if $x + y$ is divisible by 3. (For example, $1 \bowtie 5$, because $1 + 5 = 6$, which is a multiple of 3.)
- (a) Is \bowtie reflexive? (If not, give a counterexample.)
 - (b) Is \bowtie symmetric? (If not, give a counterexample.)
 - (c) Is \bowtie transitive? (If not, give a counterexample.)

2 Proofs

5. *You have already written a rough draft of this problem; now, read my feedback and write a final draft.*

Prove that for any two integers x and y , if x is divisible by 3 and xy is not divisible by 6, then y is odd.

6. *For this problem, write a rough draft of a proof; any reasonable attempt will be given full credit. I will give you feedback, and you will write a final draft on the next homework assignment.*

Let $f: \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ be defined by the formula $f(x, y) = 2x + 3y$. Prove that f is a surjective (onto) function.