Discrete Math Homework 6

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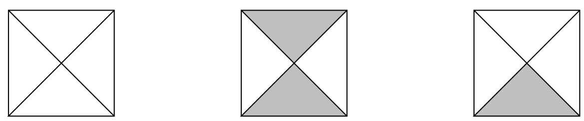
due Friday, April 4, 2025

1 Short answer

1. Divide a square into 4 triangles by drawing the diagonals. Let A be the set of all $2^4 = 16$ ways to draw the square with some subset of the triangles shaded (including all or none of them). A few elements of A are shown below:

   Let R be the relation on A defined as follows: $x \mathrel{R} y$ if x can be rotated to turn it into y. This is an equivalence relation.

   (a) If x_1, x_2, and x_3 are the three elements of A drawn above, list the elements of the equivalence classes $[x_1]$, $[x_2]$, and $[x_3]$.

   (b) Find the total number of equivalence classes A has. (No combinatorial formula is necessary; you should be able to do this by counting.)

2. Evaluate the following expressions, Simplify as much as possible, but do not approximate.

   (a) $\displaystyle\sum_{j=-1}^{1} \frac{j^2 - j + 1}{j^2 + j + 1}$.

   (b) $\displaystyle\sum_{k=1}^{10} a_k$ where $a_k = \begin{cases} 1 & \text{if } k \text{ is even,} \\ 0 & \text{if } k \text{ is odd.} \end{cases}$.

   (c) $\displaystyle\prod_{i=1}^{5} \frac{i}{i+3}$.

   (d) $\displaystyle\sum_{n=4}^{9} \sqrt{n}$.

3. Define a sequence $x_1, x_2, x_3, \dots$ by the rule that $x_1 = 1$ and, for all $n \geq 1$, $x_{n+1} = 2n - x_n + 1$.

   (a) Compute the first 6 terms of the sequence.

   (b) Guess a formula for x_n in terms of n.

2 Proofs

4. *You have already written a rough draft of this problem; now, read my feedback and write a final draft.*

   Let $f\colon \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be defined by the formula $f(x, y) = 2x + 3y$. Prove that f is a surjective (onto) function.

5. *For this problem, write a rough draft of a proof; any reasonable attempt will be given full credit. I will give you feedback, and you will write a final draft on the next homework assignment.*

   Prove by induction on n that
   $$\sum_{k=0}^{n} (2^k + 1) = 2^{n+1} + n$$
   for all positive integers n.