Discrete Math Homework 7

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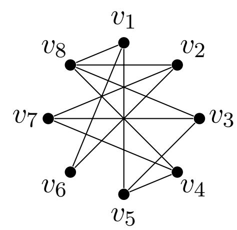
due Friday, April 18, 2025

1 Short answer

1. Let G be the graph below:



   (a) List the degrees $\deg(v_1), \deg(v_2), \dots, \deg(v_8)$ in this graph.

   (b) Use your answer to (a) to find the number of edges in G without counting.

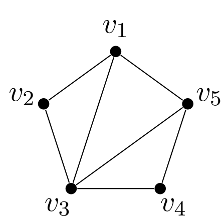
2. Let G be the graph which has six vertices $\{a_1, a_2, b_1, b_2, b_3, b_4\}$ and all 8 edges of the form $a_i b_j$.

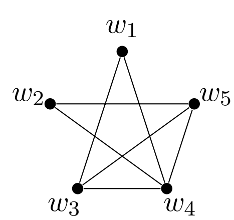
   (a) Draw a diagram of G.

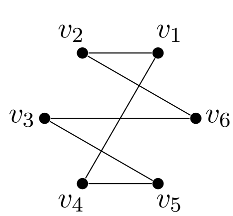
   (b) Label all vertices in the diagram with their degree.

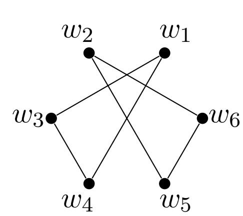
   (c) Verify the handshake lemma for G.

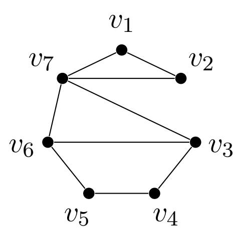
3. For each pair below, determine whether the two graphs are isomorphic. If they are, give an isomorphism; if not, explain why not.

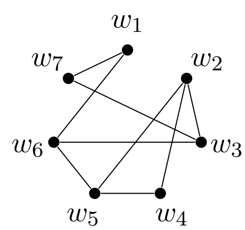
   (a) 

and


   (b) 

and


(c) 

and


2 Proofs

4. *You have already written a rough draft of this problem; now, read my feedback and write a final draft.*

   Prove by induction on n that
   $$\sum_{k=0}^{n}(2^k + 1) = 2^{n+1} + n$$
   for all positive integers n.

   (Note: lots of people on the first homework assignment wanted to start their proof with $n = 0$. That's fine: if something is true for all integers $n \geq 0$, it's also true for all positive integers n, after all. If you do this, though, you should make sure that your induction step starts at the right place.)

5. *For this problem, write a rough draft of a proof; any reasonable attempt will be given full credit. I will give you feedback, and you will write a final draft on the next homework assignment.*

   Prove that for all integers $n \geq 2$, there is a simple[1] graph with $2n$ vertices in which every vertex has degree 3.

[1] As a reminder, a graph is *simple* if it has no loops or parallel edges.