Discrete Math Homework 8

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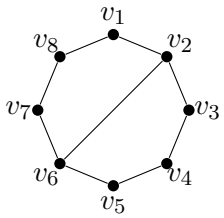
due Friday, April 25, 2025

1 Short answer

1. The final exam will be cumulative, though it will focus more heavily on topics from the last few lectures. On Monday, April 28th, we will have an in-class review in which we can go over any topics you like.

   To answer this homework problem, please suggest a topic you'd like me to go over in class.

2. Let G be the graph below:



   (a) How many edges need to be deleted from G in order to obtain a spanning tree of G?

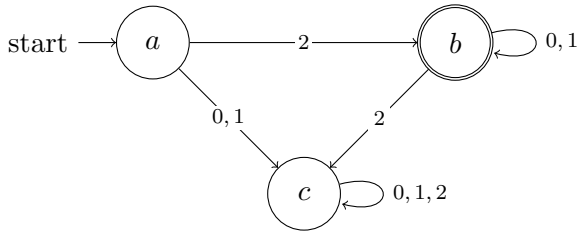
   (b) How many spanning trees does G have? *(Hint: think about how many ways there are to delete the correct number of edges in a way that does not disconnect G.)*

3. Draw lines from the regular expressions in the first row to the strings they match in the second row. A regular expression can match more than one string.

2(0 | 1)*	(0 | 10 | 20)*	2*0*1*2*	(0 | 1)* | (1 | 2)*

22122	00000001	λ	20010010	10201

4. Consider the finite-state automaton in the diagram below:



(a) The language that this automaton accepts is exactly the language matched by one of the regular expressions in question 2. Which one?

(b) Briefly explain the purpose of each of the states a, b, and c; what is the automaton "doing" in each of those states?

(c) Suppose that we change the automaton so that seeing a 2 in state c takes the automaton to state b, instead of staying in state c. How would the language accepted by the automaton change? You may explain in words, or by giving a regular expression.

2 Proofs

5. *You have already written a rough draft of this problem; now, read my feedback and write a final draft.*

   Prove that for all integers $n \geq 2$, there is a simple[1] graph with $2n$ vertices in which every vertex has degree 3.

[1] As a reminder, a graph is *simple* if it has no loops or parallel edges.