

# Probability Theory Homework 2

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due Friday, January 31, 2025

1. A fair coin is flipped repeatedly until one of two things happens: Heads has come up twice (not necessarily in a row) or Tails has come up three times (not necessarily in a row). The sequence of results is recorded.
  - (a) Draw a branching diagram to describe the ways this experiment could end. (There should be ten “leaves” to the diagram, which are the ten possible outcomes of the random experiment.)
  - (b) Are the events “the first coin flipped is Heads” and “the last coin flipped is Heads” independent?

2. An insurance company offers home insurance policies and car insurance policies. Of their clients, 25% just have home insurance, 40% just have auto insurance, and 35% have both.

Based on past data, each client with home insurance has a 60% chance of renewing that policy next year, and each client with car insurance has a 50% chance of renewing that policy next year. (For clients with both types of insurance, the decisions appear to be independent.)

Assuming these trends continue:

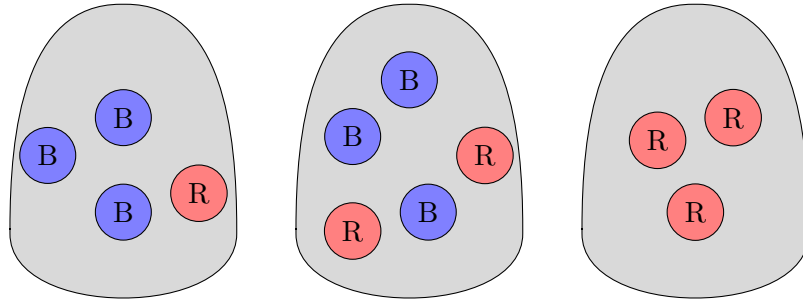
- (a) What is the probability that a client with both types of insurance renews at least one policy?
  - (b) What is the probability that a randomly chosen client renews at least one policy?
3. On the previous homework assignment, we looked at a random experiment which had the sample space  $S = \{1, 2, 3, 4, \dots\}$  with  $\Pr[\{k\}] = \frac{1}{k(k+1)}$  for all  $k$ .

Let  $A$  be the event  $\{1, 2, 3, 4\}$  (the outcome is less than or equal to 4) and let  $B$  the event  $\{1, 3, 5, 7, \dots\}$  (the outcome is odd).

- (a) Find  $\Pr[A | B]$ .
  - (b) Find  $\Pr[B | A]$ .
4. Suppose you live in an extremely unfortunate climate where it always rains on weekends (Saturday and Sunday). For that matter, there is always a  $\frac{1}{2}$  chance of rain on a weekday (Monday–Friday) as well.

If you have completely forgotten which day of week it is (so that it is equally likely to be any of the seven days), but you go outside and see that it’s raining, what is the probability that it’s the weekend?

5. You have three bags of marbles, as shown below: the first has 3 blue marbles and 1 red marble, the second has 3 blue marbles and 2 red marbles, and the third has 3 red marbles.



You pick one of the three bags, uniformly at random. Then, you draw a marble from the bag you picked, also uniformly at random.

- (a) What is the probability that you draw a red marble?
- (b) Given that you draw a red marble, what is the conditional probability that the bag you picked was the third bag?