

# Probability Theory Homework 3

Mikhail Lavrov

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1. How many sequences of 5 letters satisfy **all three** of the following conditions?

- All 5 letters must be distinct.
- At least one letter must be a consonant.
- At least one letter must be a vowel.

*(There are 21 consonants and 5 vowels in the alphabet.)*

2. A bag of marbles contains 3 red, 4 blue, and 5 green marbles.

Suppose I tell you that I took 3 marbles out of the bag (without replacement), and they were all the same color. What is the probability that the color was blue?

3. In Georgia, a car license plate has 3 letters, followed by 4 numbers. There are enough license plates issued that the sequence of numbers can be assumed to be uniformly random. What is the probability that at least one of the digits 0–9 appears on the license plate more than once?

4. You draw 4 cards from a standard 52-card deck. *(26 of the cards in the deck are red, and 26 are black.)*

What is the probability that you draw two red cards and two black cards if:

- (a) You draw a hand of 4 cards, so you are holding them all at once?
- (b) You draw 4 cards one at a time, putting each card back and shuffling before you draw the next?

5. Suppose that you have a biased coin with an unknown probability  $p$  of coming up Heads.

- (a) Which value of  $p$  maximizes the probability that if the coin is tossed 4 times, it comes up Heads 3 of those times?
- (b) Is part (a) the same question as “If the coin is tossed 4 times and comes up Heads 3 of those times, what is the likeliest value of  $p$ ?” Why or why not?