

# Probability Theory Homework 8

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due Monday, April 28, 2025

1. If  $\mathbf{X} \sim \text{Exponential}(\lambda)$ , find  $\mathbb{E}[e^{\mathbf{X}}]$  in terms of  $\lambda$ .  
For which values of  $\lambda$  does this expected value converge?
2. If  $\Theta \sim \text{Uniform}(a = 0, b = 3)$ , find the probability density function of  $\sin(\Theta)$ .  
(Use radians, not degrees.)
3. Let  $\mathbf{W}$  be normally distributed with  $\mu = 0$  and unknown standard deviation  $\sigma$ . Find the conditional expected value  $\mathbb{E}[\mathbf{W} \mid \mathbf{W} > 0]$  in terms of  $\sigma$ .
4. An IT help center receives an average of 100 calls per hour. Of these calls:
  - 60% are simple problems that can be fixed by turning something off then on again.
  - 30% are difficult problems that the help center can't address.
  - 10% are prank calls.

Assuming that three types of phone calls are independent Poisson processes, identify the distributions of the following random variables: give the name of the distribution and its parameters.

- (a)  $\mathbf{S}$  is the time (in hours) until the first call about a simple problem is received.
- (b)  $\mathbf{T}$  is the time (in hours) until 500 calls are received (of any type).
- (c)  $\mathbf{P}$  is the number of prank calls received in the first hour.
- (d)  $\mathbf{Q}$  is the number of prank calls among the first 100 calls.

(Note that  $\mathbf{P}$  and  $\mathbf{Q}$  look very similar, since there are 100 phone calls per hour on average, but they do have different distributions!)