**Proceedings of 2004 DETC:**
**28th Biennial Mechanisms and Robotics Conference**
**28 Sep – 2 Oct 2004, Salt Lake City**

# DETC2004-57189

# DESIGN AND ANALYSIS OF A HYBRID PARALLEL PLATFORM THAT INCORPORATES TENSEGRITY

**Matthew Marshall**
Center for Intelligent Machines and Robotics
University of Florida
Gainesville, Florida 32611

**Carl D. Crane III**
Center for Intelligent Machines and Robotics
University of Florida
Gainesville, Florida 32611

## ABSTRACT

A new hybrid parallel platform device that is based on tensegrity is introduced in this paper. A tensegrity structure is one that is comprised of members that are either in tension (ties) or compression (struts). The device studied in this paper replaces the 'upper' and 'lower' set of ties of a 3-strut tensesgrity system with rigid bodies. Further, the three struts are replaced by three leg connectors whose lengths can be changed via prismatic actuators. The three remaining ties are replaced by the series combination of a spring and a non-compliant tie where the length of the non-compliant tie can be controlled. An analysis is presented that shows how the connector leg lengths and non-compliant tie lengths can be determined so as to position and orient the upper platform at a desired pose and at a desired total potential energy level. It is the control of the potential energy in the system that makes this new hybrid parallel-platform unique.

## INTRODUCTION

An extensive amount of research has been done with respect to the analysis of in-parallel platforms. Typical in-parallel platforms are comprised of a top and base platform that are connected by six leg connectors. Each connector is in general modeled as an S-P-H kinematic chain. In other words, the leg connector is connected to the bottom platform by a spherical joint and the top platform by a Hooke joint. The leg connector itself can change length due to the prismatic joint. This paper will not attempt to document the large volume of work related to in-parallel platforms, but rather will provide a brief introduction to tensegrity as tensegrity concepts will be incorporated in the new hybrid platform to be presented here.

The word *tensegrity* is a combination of the words *tension* and *integrity* (Edmondson, 1987 and Fuller, 1975). Tensegrity structures are spatial structures formed by a combination of rigid elements in compression (struts) and connecting elements that are in tension (ties). No pair of struts touch and the end of each strut is connected to three non-coplanar ties (Yin et al, 2002). The entire configuration stands by itself and maintains its form solely because of the internal arrangement of the struts and ties (Tobie, 1976). Figure 1 shows a tensegiry structure with three struts and nine ties.

The development of tensegrity structures is relatively new and the related works have only existed for approximately twenty five years. Kenner, 1976, established the relation between the rotation of the top and bottom ties. Tobie, 1976, presented procedures for the generation of tensile structures by physical and graphical means. Yin, 2002, obtained Kenner's results using energy considerations and found the equilibrium position for unloaded tensegrity prisms. Stern, 1999, developed generic design equations to find the lengths of the struts and elastic ties needed to create a desired geometry for a symmetric case. Knight, 2000, addressed the problem of stability of tensegrity structures for the design of deployable antennae.

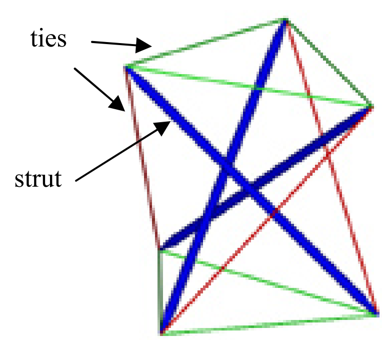


Figure 1: Three Strut Tensegrity Structure

This paper presents a new hybrid parallel device that incorporates tensegrity principles, i.e. the members are internally stressed. The term 'hybrid' is used since the parallel mechanism will incorporate both rigid and compliant members. A description of the device is presented in the next section followed by a reverse position analysis that allows for the positioning and orienting the top platform of the new device at a specified potential energy state of the system. Additional dynamic analyses are beyond the scope of the current paper.

## DESCRIPTION OF HYBRID PLATFORM

Figure 2 shows one version of the hybrid platform. Comparing this device with the tensegrity structure shown in Figure 1, it can be seen that the top and bottom sets of triangular ties have now been replaced by rigid bodies. The struts shown in Figure 1 have been replaced by typical S-P-H kinematic chain leg connectors. The three remaining ties now consist of a compliant element (a spring with known spring constant and free length) in series with a non-compliant tie. The effective length of each non-compliant tie can be changed by having an actuator wind the tie about a drum which is primitively illustrated on the base platform in Figure 2.

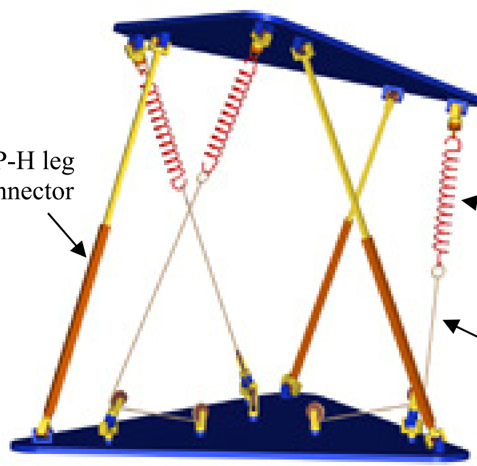


Figure 2: Hybrid Platform

The hybrid platform is a six degree of freedom device. Three actuators are used to control the lengths of the three leg connectors. Three additional actuators are used to change the effective lengths of the three non-compliant ties. A reverse position analysis will be presented whereby the desired position and orientation of the top platform is specified together with the desired potential energy of the system. The lengths of the three leg connectors (whose detemination is trivial) and the lengths of the three non-compliant ties will be calculated to meet the pose and energy requirements.

The force balance equation based on a free body diagram of the top platform introduces one important fact with regards to the hybrid device. If no external wrench (force/torque) is applied to the top platform, then the lines along the six legs must be linearly dependent. Rather than having the user specify positions and orientations of the top platform where this linear dependency condition occurs, the problem statement will be modified. In the analysis presented here, the user will specify the desired position and orientation and the desired potential energy for the system together with the screw coordinates (but not the magnitude) of the external wrench that will hold the top platform in equilibrium. The lengths of the three leg connectors, the lengths of the three non-compliant ties, and the magnitude of the external wrench that maintains equilibrium will be determined. A more detailed problem statement is presented in the next section.

## PROBLEM STATEMENT

The analysis will proceed by attaching coordinate system 1 to the base platform and coordinate system 2 to the moving top platform. The problem statement can now be stated as follows:

given:

- the coordinates of the connection points of the six leg elements to the top platform as measured with respect to coordinate system 2
- the coordinates of the connection points of the six leg elements to the bottom platform as measured with respect to coordinate system 1
- the desired position and orientation of the top platform with respect to the base which is defined by the 4×4 transformation matrix ${}^{1}_{2}\mathbf{T}$ that transforms points known in coordinate system 2 to their coordinates in coordinate system 1
- the three spring constants ($k_1$, $k_2$, $k_3$) and free lengths ($l_{01}$, $l_{02}$, $l_{03}$)
- the potential energy stored in the springs, U
- the screw along which an external wrench acts that holds the system in equilibrium at the desired position and orientation, $\mathbf{\$}_{ext}$

find:

- the length of each S-P-H leg connector ($L_1$, $L_2$, $L_3$)
- the effective length of the each non-compliant tie ($l_{c1}$, $l_{c2}$, $l_{c3}$)

It should be noted that solution method to be presented does not guarantee that the S-P-H connectors are in compression nor that the compliant elements are in tension. Future work will focus on the necessary conditions required to ensure that the resulting system is in tensegrity, but this is beyond the scope of the current paper.

## DETERMINATION OF S-P-H LEG LENGTHS

The first step of the analysis is to determine the coordinates of the six points that connect the leg connectors to the top platform in terms of coordinate system 1. This is a trivial task since the coordinates of these points are known in terms of coordinate system 2 together with the transformation matrix ${}^{1}_{2}\mathbf{T}$ that transforms points from coordinate system 2 to coordinate system 1. At this point the coordinates of the lower and upper end points of all six leg connectors are assumed to be known in coordinate system 1. The length of each of the three S-P-H leg connectors can now readily be determined as the distance between its lower and upper end points.

## DETERMINATION OF LINE COORDINATES

The analysis will proceed by determining the Plücker coordinates of the lines along each of the six leg connectors. Figure 3 shows two distinct points, $\mathbf{r}_1$ and $\mathbf{r}_2$, the join of which determine a line. The unitized direction of the line can be written as

$$\mathbf{S} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|} \ . \tag{1}$$

Letting **r** designate a vector from the origin to any general point on the line, it is apparent that $\mathbf{r}$-$\mathbf{r}_1$ is parallel to **S**. Thus it may be written that

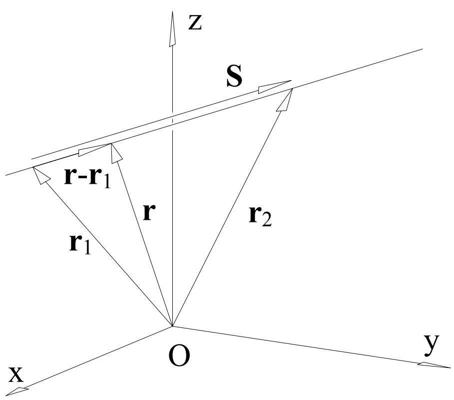


Figure 3: Determination of a Line

$$(\mathbf{r}-\mathbf{r}_1) \times \mathbf{S} = \mathbf{0} \ . \tag{2}$$

This can be expressed in the form

$$\mathbf{r} \times \mathbf{S} = \mathbf{S}_{0L} \tag{3}$$

where

$$\mathbf{S}_{0L} = \mathbf{r}_1 \times \mathbf{S} \tag{4}$$

is clearly the moment of the line about the origin of the reference coordinate system. It is also apparent that $\mathbf{S} \cdot \mathbf{S}_{0L} = 0$.

The coordinates of a line will be written as $\{\mathbf{S} ; \mathbf{S}_{0L}\}$ and will be referred to as the Plücker coordinates of the line. The semi-colon is introduced to signify that the dimensions of **S** and $\mathbf{S}_{0L}$ are different, i.e. **S** is dimensionless while $\mathbf{S}_{0L}$ has units of length. Further, the coordinates $\{\mathbf{S} ; \mathbf{S}_{0L}\}$ are homogeneous since from (3) it is apparent that the coordinates $\{\lambda\mathbf{S} ; \lambda\mathbf{S}_{0L}\}$, where $\lambda$ is any non-zero scalar, determine the same line.

At this point, the Plücker coordinates of the six leg connectors can readily be determined since the end points of each connector are known. The Plücker coordinates of the lines along the three S-P-H connectors will be referred to as $\{\mathbf{S}_1 ; \mathbf{S}_{0L1}\}$, $\{\mathbf{S}_2 ; \mathbf{S}_{0L2}\}$, and $\{\mathbf{S}_3 ; \mathbf{S}_{0L3}\}$, and of the lines along the three compliant connectors will be referred to as $\{\mathbf{S}_{c1} ; \mathbf{S}_{0Lc1}\}$, $\{\mathbf{S}_{c2} ; \mathbf{S}_{0Lc2}\}$, $\{\mathbf{S}_{c3} ; \mathbf{S}_{0Lc3}\}$.

## DETERMINATION OF SPRING ELONGATIONS

It was shown by Ball (1900) that a force can be modeled as the magnitude of the force, f, times the Plücker line coordinates of its line of action, where the direction of the line, **S**, is a unit vector. It is also shown that a pure moment applied to a body can be written as the magnitude of the moment, m, times the Plücker coordinates of a line at infinity. A line at infinity can be thought of as the intersection of the plane at infinity with a second plane that passes through the origin. The coordinates of a line at infinity can be written as $\{\mathbf{0} ; \mathbf{S}\}$ where **S** is a dimensionless direction vector that is perpendicular to this second plane.

Ball showed that this representation of forces and moments allowed for the direct summation of the coordinates of forces and moments acting on a rigid body to result in the net equivalent force and moment acting on the body. For example, the summation of two forces and one moment can be written as

$$f\,\{\mathbf{S} ; \mathbf{S}_0\} = f_1\,\{\mathbf{S}_1 ; \mathbf{S}_{0L1}\} + f_2\,\{\mathbf{S}_2 ; \mathbf{S}_{0L2}\} + m_3\,\{\mathbf{0} ; \mathbf{S}_3\} \ . \tag{5}$$

The resultant is computed as $f\,\{\mathbf{S} ; \mathbf{S}_0\}$ where **S** is a unit vector and where

$$f\,\mathbf{S} = f_1\,\mathbf{S}_1 + f_2\,\mathbf{S}$$
$$f\,\mathbf{S}_0 = f_1\,\mathbf{S}_{0L1} + f_2\,\mathbf{S}_{0L2} + m_3\,\mathbf{S}_3 \ . \tag{6}$$

In general, $\mathbf{S} \cdot \mathbf{S}_0 \neq 0$ so the resultant cannot be interpreted as a pure force, i.e. a force magnitude along a line of action. Rather the resultant is in general a wrench which is a force magnitude, f, multiplied by the coordinates of a screw, $\{\mathbf{S} ; \mathbf{S}_0\}$. The screw can be interpreted as a line with a pitch and the wrench can thus be interpreted as a force along the line of action of the screw together with a moment whose axis is parallel to that of the force. The pitch of the screw relates the magnitude of the force to the magnitude of the moment. The notation **\$** is introduced to represent the coordinates of a screw as a length six vector as

$$\mathbf{\$} = \begin{bmatrix} \mathbf{S} \\ \mathbf{S}_0 \end{bmatrix} \ . \tag{7}$$

It is important to recognize that the units of the first three elements of **\$** are dimensionless while the last three elements have units of length.

Returning to the case of the hybrid mechanism, for the top platform to be in equilibrium, the sum of the forces exerted by the six leg connectors must equal the external wrench that is applied to the top platform. Using the notation presented thus far, the force balance equation may be written as

$$f_1\,\mathbf{\$}_1 + f_2\,\mathbf{\$}_2 + f_3\,\mathbf{\$}_3 + f_{c1}\,\mathbf{\$}_{c1} + f_{c2}\,\mathbf{\$}_{c2} + f_{c3}\,\mathbf{\$}_{c3} + f_{ext}\,\mathbf{\$}_{ext} = \mathbf{0} \ . \tag{8}$$

In this equation, the line coordinates of the six legs have been determined. The screw coordinates of the external wrench, $\mathbf{\$}_{ext}$, are given in the problem statement. None of the force magnitudes are yet known.

Rearranging (8) yields

$$f_1\,\mathbf{\$}_1 + f_2\,\mathbf{\$}_2 + f_3\,\mathbf{\$}_3 + f_{c2}\,\mathbf{\$}_{c2} + f_{c3}\,\mathbf{\$}_{c3} + f_{ext}\,\mathbf{\$}_{ext} = -\,f_{c1}\,\mathbf{\$}_{c1} \tag{9}$$

and writing this equation in matrix form yields

$$\left[\mathbf{\$}_1\ \mathbf{\$}_2\ \mathbf{\$}_3\ \mathbf{\$}_{c2}\ \mathbf{\$}_{c3}\ \mathbf{\$}_{ext}\right] \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_{c2} \\ f_{c3} \\ f_{ext} \end{bmatrix} = -f_{c1}\,\mathbf{\$}_{c1} \tag{10}$$

where [$\$_1$ $\$_2$ $\$_3$ $\$_{c2}$ $\$_{c3}$ $\$_{ext}$] is a 6×6 matrix whose columns are screw (or line) coordinates. Dividing both sides by the magnitude $f_{c1}$ gives

$$\left[\$_1\ \$_2\ \$_3\ \$_{c2}\ \$_{c3}\ \$_{ext}\right]\begin{bmatrix} f_1' \\ f_2' \\ f_3' \\ f_{c2}' \\ f_{c3}' \\ f_{ext}' \end{bmatrix} = -\$_{c1} \tag{11}$$

where

$$\begin{array}{l} f_1' = \dfrac{f_1}{f_{c1}},\ f_2' = \dfrac{f_2}{f_{c1}},\ f_3' = \dfrac{f_3}{f_{c1}}, \\ f_{c2}' = \dfrac{f_{c2}}{f_{c1}},\ f_{c3}' = \dfrac{f_{c3}}{f_{c1}},\ f_{ext}' = \dfrac{f_{ext}}{f_{c1}} \end{array}. \tag{12}$$

The terms $f_1'$, $f_2'$, … $f_{ext}'$ can be readily determined from (11) by multiplying $-\$_{c1}$ by the inverse of the 6×6 matrix [$\$_1$ $\$_2$ $\$_3$ $\$_{c2}$ $\$_{c3}$ $\$_{ext}$]. The case where the geometry is such that the matrix is singular is not addressed here.

The next step of the analysis will be to determine the value of the force magnitude $f_{c1}$ such that the potential energy specification is met. The potential energy stored in the three springs can be written as

$$U = \frac{k_1\,\delta_1^{\,2} + k_2\,\delta_2^{\,2} + k_3\,\delta_3^{\,2}}{2} \tag{13}$$

where $\delta_i$ is the elongation of spring *i* from its free length. The force in each spring may be written as

$$f_{ci} = k_i\,\delta_i\ ,\ i = 1..3. \tag{14}$$

Using (14) to substitute for $\delta_i$ in (13) yields

$$U = \frac{1}{2}\left(\frac{f_{c1}^{\,2}}{k_1} + \frac{f_{c2}^{\,2}}{k_2} + \frac{f_{c3}^{\,2}}{k_3}\right). \tag{15}$$

Equation (12) can be used to substitute for $f_{c2}$ and $f_{c3}$ to yield

$$U = \frac{1}{2}\left(\frac{f_{c1}^{\,2}}{k_1} + \frac{f_{c1}^{\,2}\left(f_{c2}'\right)^2}{k_2} + \frac{f_{c1}^{\,2}\left(f_{c3}'\right)^2}{k_3}\right). \tag{16}$$

Equation (16) can be solved for $f_{c1}$ to give

$$f_{c1} = \sqrt{\frac{2\,U\,k_1\,k_2\,k_3}{k_2\,k_3 + k_1\,k_3\,(f_{c2}')^2 + k_1\,k_2\,(f_{c3}')^2}}\ . \tag{17}$$

Since $f_{c1}$ is now known, the magnitudes of the forces along all the legs as well as the magnitude of the force along the equilibrium screw can be determined from (12). Lastly, the elongations of the three springs can be determined since the forces in the compliant leg connectors are known. The elongations can be calculated as

$$\delta_i = \frac{f_{ci}}{k_i},\quad i = 1..3\ . \tag{18}$$

## DETERMINATION OF NON-COMPLIANT TIE LENGTHS

The remaining task is to determine the length of the non-compliant tie in each of the compliant leg connectors. This is a relatively simple task since the total length of and elongation of the spring in each of the compliant connectors is now known.

The term $L_{ci}$ will be used to represent the known total length of compliant leg connector *i*. This total length is equal to the sum of the spring free length, $l_{0i}$, the spring elongation, $\delta_i$, and the length of the non-compliant tie, $l_{ci}$. Thus,

$$l_{ci} = L_{ci} - l_{0i} - \delta_i \tag{19}$$

This completes the analysis as presented in the problem statement. In summary, the lengths of the three S-P-H leg connectors and the length of the non-compliant ties in the three compliant leg connectors have been determined in order to position and orient the top platform as desired with the desired total potential energy. Also, the force magnitude of the external wrench (whose screw coordinates were specified) that holds the system in equilibrium was also determined.

## NUMERICAL EXAMPLE

A 3-3 platform geometry which is similar to the geometry of the three strut tensegrity structure shown in Figure 1 is used in this numerical example. This means that there are three connection points on the top and base platforms and that there are thus two legs at each connection point.

The given information is specified as:

- The top and base platforms are equilateral triangles with sides equal to 20 cm in length. The bottom end points of the leg connectors in terms of coordinate system 1 and the top end points in terms of coordinate system 2 are given as

$$^1\mathbf{P}_{1B} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\quad ^1\mathbf{P}_{2B} = \begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix},\quad ^1\mathbf{P}_{3B} = \begin{bmatrix} 10 \\ 17.32 \\ 0 \end{bmatrix},$$

$$^2\mathbf{P}_{1T} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\quad ^2\mathbf{P}_{2T} = \begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix},\quad ^2\mathbf{P}_{3T} = \begin{bmatrix} 10 \\ 17.32 \\ 0 \end{bmatrix}$$

  where the coordinates are expressed in units of cm and the preceding superscript indicates which coordinate system the point is expressed in. Note that the end points of the three S-P-H leg connectors are respectively ($\mathbf{P}_{1B}$,$\mathbf{P}_{1T}$), ($\mathbf{P}_{2B}$,$\mathbf{P}_{2T}$), ($\mathbf{P}_{3B}$,$\mathbf{P}_{3T}$) and the end points of the three compliant leg connectors are ($\mathbf{P}_{2B}$, $\mathbf{P}_{1T}$), ($\mathbf{P}_{3B}$, $\mathbf{P}_{2T}$), ($\mathbf{P}_{1B}$, $\mathbf{P}_{3T}$).
- The desired position and orientation of the top platform is defined by the following transformation matrix

$$
{}^{1}_{2}\mathbf{T} = \begin{bmatrix} 0.893 & 0.325 & 0.312 & 8 \\ -0.326 & 0.944 & -0.051 & 5 \\ -0.312 & -0.056 & 0.949 & 18 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

where the first three columns are dimensionless and the last column has units of cm.

- The three spring constants and free lengths are given as

$$k_1 = k_2 = k_3 = 20 \text{ N/cm}, \; l_{01} = l_{02} = l_{03} = 3 \text{ cm} .$$

- The desired potential energy is specified as

$$U = 40 \text{ N cm} .$$

- The coordinates of the screw along which the external wrench acts is given as

$$\$_{ext} = [0.516, 0.083, 0.853, 4.924, -8.528, -2.147]^T$$

where the first three components are dimensionless and the last three have units of cm.

The problem at hand is to determine the lengths of the three S-P-H leg connectors and the lengths of the non-compliant ties in each of the compliant connectors. Determining the coordinates of the top platform points in terms of coordinate system 1 gives

$$
{}^{1}\mathbf{P}_{1T} = \begin{bmatrix} 8 \\ 5 \\ 18 \end{bmatrix}, \quad {}^{1}\mathbf{P}_{2T} = \begin{bmatrix} 25.85 \\ -1.52 \\ 11.77 \end{bmatrix}, \quad {}^{1}\mathbf{P}_{3T} = \begin{bmatrix} 22.56 \\ 18.09 \\ 13.92 \end{bmatrix}.
$$

The lengths of the six leg connectors can now be determined as the distance between the endpoints as

$$L_1 = 20.32 \text{ cm}, L_2 = 13.23 \text{ cm}, L_3 = 18.76 \text{ cm},$$

$$L_{c1} = 22.20 \text{ cm}, L_{c2} = 27.29 \text{ cm}, L_{c3} = 32.09 \text{ cm} .$$

The Plücker line coordinates of the six legs are calculated in terms of coordinate system 1 as

$$
\$_1 = \begin{bmatrix} 0.394 \\ 0.246 \\ 0.866 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \$_2 = \begin{bmatrix} 0.442 \\ -0.114 \\ 0.889 \\ 0 \\ -17.790 \\ -2.287 \end{bmatrix}, \$_3 = \begin{bmatrix} 0.669 \\ 0.041 \\ 0.742 \\ 12.85 \\ -7.419 \\ -11.179 \end{bmatrix}
$$

$$
\$_{c1} = \begin{bmatrix} 0.540 \\ -0.225 \\ -0.811 \\ 0 \\ 16.214 \\ -4.504 \end{bmatrix}, \$_{c2} = \begin{bmatrix} -0.581 \\ 0.690 \\ -0.431 \\ -7.470 \\ 4.313 \\ 16.966 \end{bmatrix}, \$_{c3} = \begin{bmatrix} -0.703 \\ -0.564 \\ -0.434 \\ 0 \\ 0 \\ 0 \end{bmatrix} .
$$

Solving equation (11) for the force ratios yields

$$f_1' = 1.583, f_2' = 1.821, f_3' = 1.910,$$

$$f_{2c}' = 1.410, f_{3c}' = 1.386, f_{ext}' = -2.846 .$$

The value for $f_{c1}$ was then found from (17) as

$$f_{c1} = 18.147 \text{ N}$$

and the remaining force magnitudes were found from (12) as

$$f_1 = 28.730 \text{ N}, f_2 = 33046 \text{ N}, f_3 = 34.664 \text{ N},$$

$$f_{2c} = 25.584 \text{ N}, f_{3c} = 24.822 \text{ N}, f_{ext} = -51.649 \text{ N} .$$

Equation (18) was used to find the elongations of the springs in the three compliant legs and equation (19) was used to determine the lengths of the non-compliant ties as

$$l_{c1} = 18.297 \text{ cm}, l_{c2} = 23.007 \text{ cm}, l_{c3} = 27.850 \text{ cm}.$$

## CONCLUSION

A new hybrid parallel platform which incorporates tensegrity has been introduced. Three of the leg connectors are the traditional S-P-H connectors and three are comprised of a spring in series with a non-compliant tie. A reverse position analysis was presented that determines the lengths of the three S-P-H connectors and the lengths of the non-compliant ties that will position and orient the top platform as desired at a specified potential energy state.

The novel aspect of the device is that the potential energy can be controlled at any desired pose. Changing the potential energy state will impact the vibrational properties of the device. Future work is aimed at incorporating additional leg connectors so that the external wrench that is required to hold the top platform at equilibrium is no longer required.


## ACKNOWLEDGMENTS

The authors would like to gratefully acknowledge the support of the Air Force Office of Scientific Research, Grant Number F49620-00-1-0021 and of the Department of Energy, grant number DE-FG04-86NE37967.


## REFERENCES


Ball, R., (1900), A Treatise on the Theory of Screws, Cambridge University Press.

Duffy, J., Rooney, J., Knight, B., and Crane, C., (2000), A Review of a Familly of Self-Deploying Tensegrity Structures with Elastic Ties, *The Shock and Vibration Digest*, Vol. 32, No. 2, March 2000, pp. 100-106.

Edmondson, A., (1987) *A Fuller Explanation: The Synergetic Geometry of R. Buckminster Fuller*, Birkhauser, Boston.

Fuller, R., (1975), *Synergetics: The Geometry of Thinking*, MacMillan Publishing Co., Inc., New York, 1975.

Kenner, H., (1976), *Geodesic Math and How to Use It*, University of California Press, Berkeley and Los Angeles, CA.

Knight, B.F., (2000), Deployable Antenna Kinematics using Tensegrity Structure Design, *Ph.D. thesis*, University of Florida, Gainesville, FL.

Stern, I.P., (1999), Development of Design Equations for Self-Deployable N-Strut Tensegrity Systems, *M.S. thesis*, University of Florida, Gainesville, FL.

Tobie, R.S., (1976), A Report on an Inquiry into The Existence, Formation and Representation of Tensile Structures, *Master of Industrial Design thesis*, Pratt Institute, New York.

Yin, J., Duffy, J., and Crane, C., (2002), An Analysis for the Design of Self-Deployable Tensegrity and Reinforced Tensegrity Prisms with Elastic Ties, *International Journal of Robotics and Automation*, Special Issue on Compliance and Compliant Mechanisms, Volume 17, Issue 1.

 