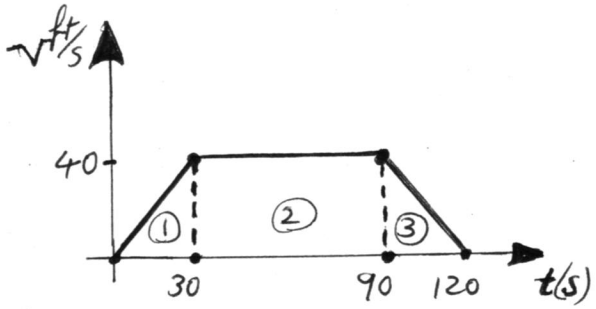
Example: v-t graph for a train moving between two stations is given. Find the a-t graph and s-t graph over this time interval.

12-47



When $0 \leq t < 30$

period ①:

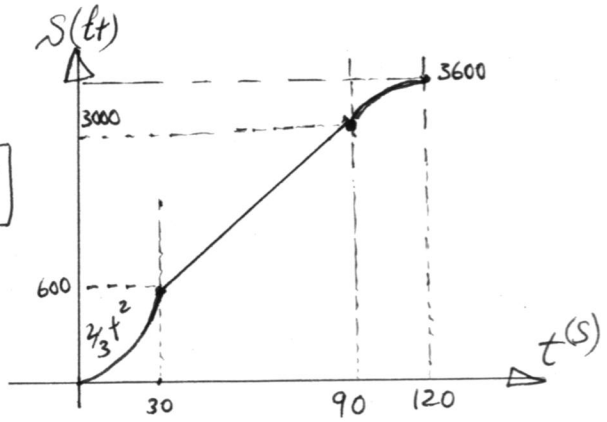
$$\int_0^s ds = \int_0^t v\,dt$$

You need to find the equation of the line:

$$v = \frac{40}{30}t = \frac{4}{3}t \quad (\text{slope} = 4/3) \Longrightarrow$$

$$\int_0^s ds = \int_0^t \frac{4}{3}t\,dt \Rightarrow s = \frac{4}{3}\frac{t^2}{2} \Rightarrow \boxed{s = \frac{2}{3}t^2}$$

Notice that when v-t graph is linear, the s-t graph is quadratic.



$$s_{(\text{at } t=30)} = \frac{2}{3}(30)^2 = 600 \text{ ft}$$

When $30 < t < 90$ → period ② $\Longrightarrow$ $\int_{600}^{s} ds = \int_{30}^{t} v\,dt$

equation of v for this period:

$$v = 40 \Longrightarrow \int_{600}^{s} ds = \int_{30}^{t} 40\,dt \Rightarrow s - 600 = 40(t-30)$$

$$\Rightarrow \boxed{s = 40t - 600}$$

when $t = 90 \Rightarrow s = 40(90) - 600 = 3000$ ft

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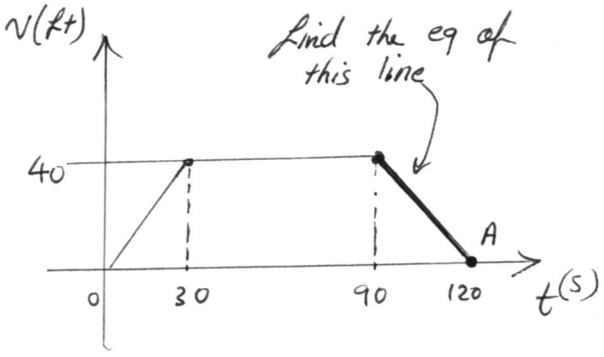
when $90 < t < 120$ or period ③ $\int_{3000}^{s} ds = \int_{90}^{t} v\,dt$

equation of v for this period:

$$v = -\frac{4}{3}t + \frac{480}{3}$$

Let's see how we get this equation:

Example - continue



Eq. of line for period ③

Point A coordinates $\begin{cases} 120 = x_1 \\ 0 = y_1 \end{cases}$ $\quad m = -4/3$

in a x-y coordinate system $\longrightarrow y - y_1 = m(x - x_1)$

$\longrightarrow y - 0 = -\frac{4}{3}(x - 120) \Longrightarrow$ $\boxed{v = -\frac{4}{3}t + \frac{480}{3}}$

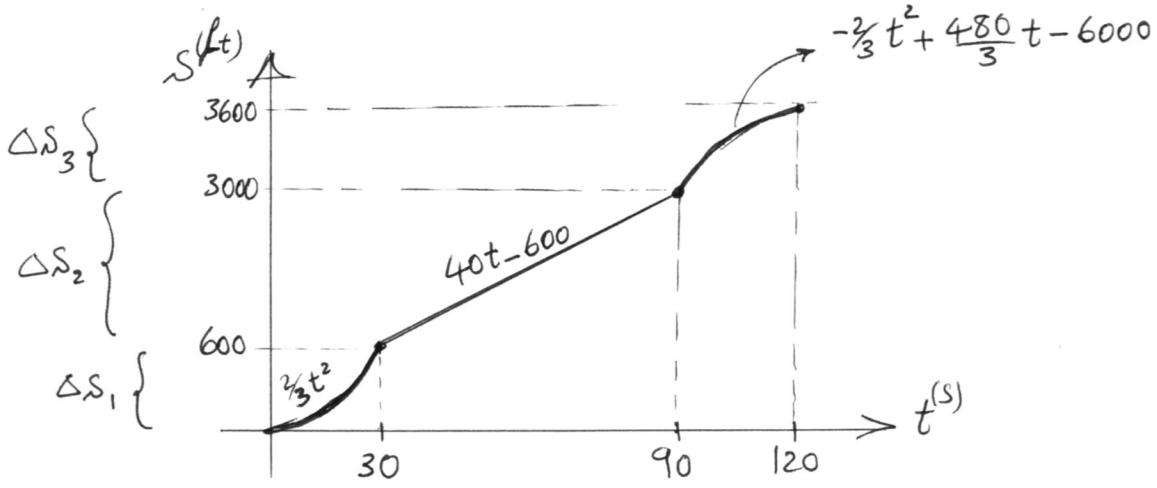
So $\int_{3000}^{s} ds = \int_{90}^{t} v\,dt \Rightarrow s - 3000 = \int_{90}^{t} \left(-\frac{4}{3}t + \frac{480}{3}\right)dt = \left[-\frac{4}{3}\frac{t^2}{2} + \frac{480}{3}t\right]_{90}^{t}$

$s - 3000 = -\frac{2}{3}t^2 + \frac{480}{3}t - \left(-\frac{2}{3}(90)^2 + \frac{480}{3}(90)\right) \Longrightarrow$ $\boxed{s = -\frac{2}{3}t^2 + \frac{480}{3}t - 6000}$

Check when $t = 90 \Rightarrow s = 3000$ ft correct ✓

when $t = 120 \Rightarrow s = 3600$ ft (final position)

Check one more time:



area ① $= \frac{40 \times 30}{2} = 600 = \Delta s_1$

area ② $= 60 \times 40 = 2400 = \Delta s_2$

area ③ $= \frac{40 \times 30}{2} = 600 = \Delta s_3$

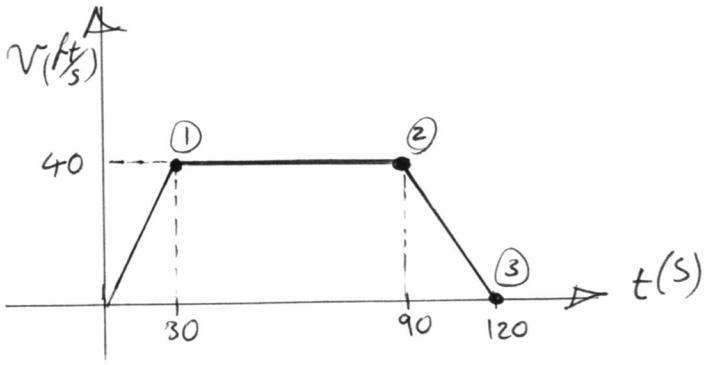
Continue

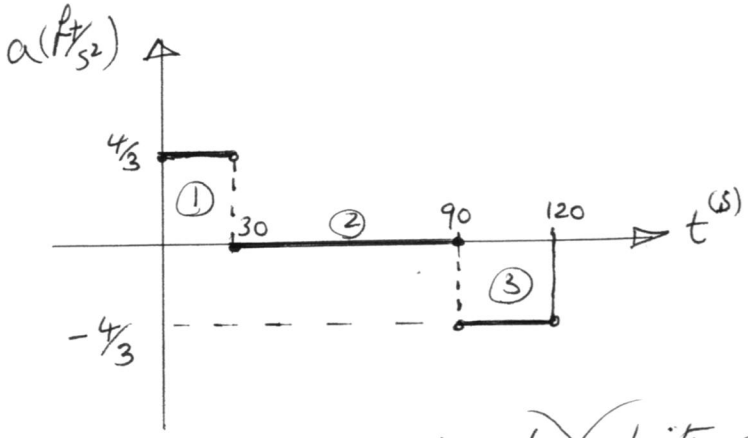
Finding the equations for acceleration:

period ① $0 < t \leq 30 \rightarrow v = \frac{4}{3}t \Rightarrow a = \frac{dv}{dt} = \frac{4}{3}$

Period ② $30 < t \leq 90 \rightarrow v = 40 \Rightarrow a = \frac{dv}{dt} = 0$

period ③ $90 < t \leq 120 \rightarrow v = -\frac{4}{3}t + \frac{480}{3} \Rightarrow a = \frac{dv}{dt} = -\frac{4}{3}$





Check area under a-t graph is the change in velocity at the end of that interval:

Area ① $= \frac{4}{3}(30) = 40$ ✓ $= v(\text{at } t=30)$

Area ② $= 0 \Rightarrow v(\text{at } t=90) = 40+0 = 40$ ✓

Area ③ $= -\frac{4}{3}(30) = -40 \Rightarrow v(\text{at } t=120) = 40+0-40 = 0$ ✓