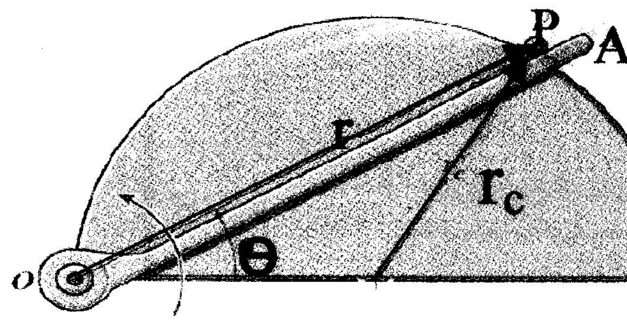
Given: The ball (P) is guided along the vertical circular path.

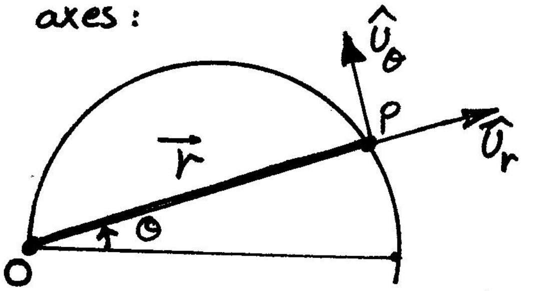
$W = 0.5$ lb, $\dot{\theta} = 0.4$ rad/s, $\ddot{\theta} = 0.8$ rad/s^2, $r_c = 0.4$ ft

Find: Force of the arm OA on the ball when $\theta = 30°$.



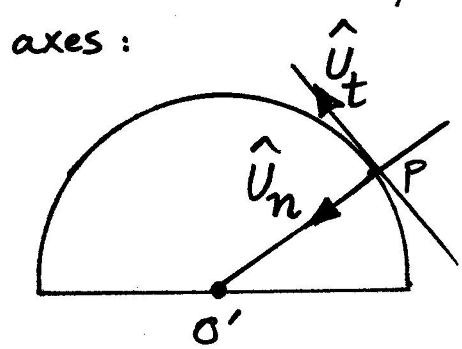
Solution:

1- Lets show the positive directions of radial and transverse axes:



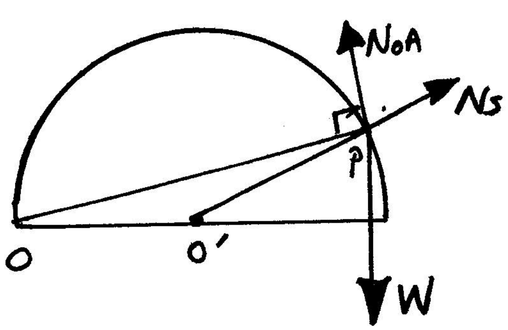
$\hat{U}_r$ is along the position vector $\vec{r}$.
$\hat{U}_\theta$ is $\perp$ $\hat{U}_r$ and ccw. (Showing the positive direction of θ)

2- Lets show the positive directions of normal and tangential axes:



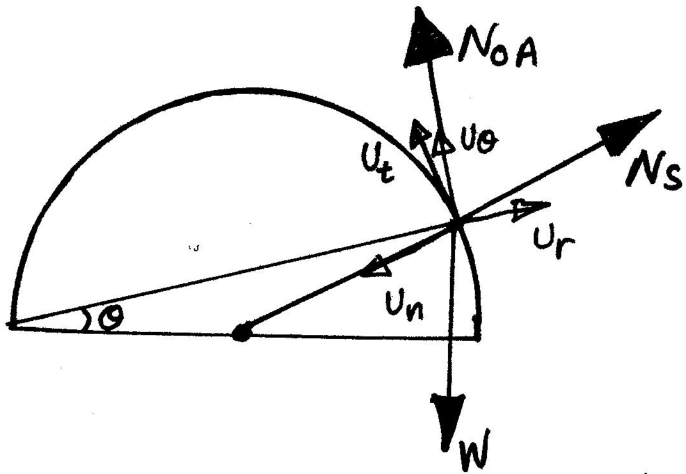
$\hat{U}_t$ is the unit vector along t, which is tangent to the path and in the direction of motion.
$\hat{U}_n$ is $\perp$ to the tangent to the path pointing toward the Center of curvature.

3- Now lets show all the forces we have here:



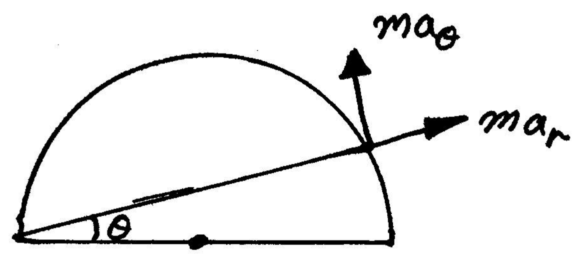
N_s = Normal force $\perp$ Path or in the direction of n and outside of the circular surface.
NoA is $\perp$ OP ($\perp$ to the $\hat{U}_r$)
W is weight and is downward.

4- Now lets combine the FBD and the Coordinates systems:

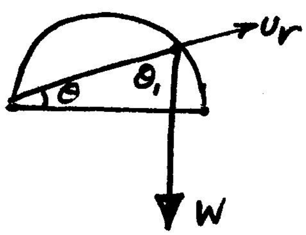


So only N_{OA} is along $\hat{U}_\theta$ and other forces make angles with $\hat{U}_r$ or $\hat{U}_\theta$.

5- Lets show the kinetic diagram. They are along $\hat{U}_r$ and $\hat{U}_\theta$ as shown.

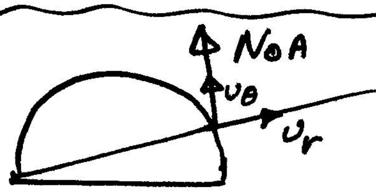


Now the task is finding the angles between the forces we have and $\hat{U}_r$ and $\hat{U}_\theta$:

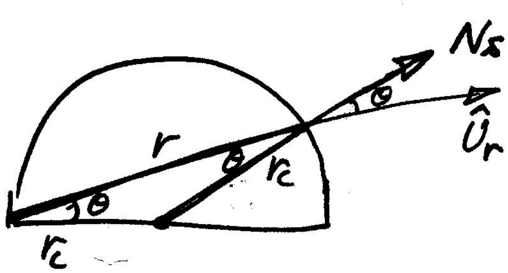


The angle that W makes with U_r is θ_1 and it is the complimentary angle:

$\theta + \theta_1 = 90 \qquad \theta_1 = 90 - 30 = 60$



N_{OA} is along $\hat{U}_\theta$ so it has no component along U_r.



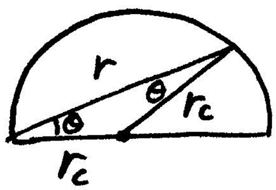
N_s makes an angle θ with $\hat{U}_r$ as shown.

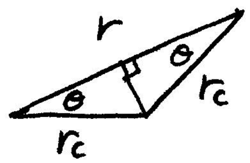
Lets solve for the unknowns after writing the equations of motion:

$$\Sigma F_r = m a_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$\Sigma F_\theta = m a_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

How do we write an equation describing r in terms of θ?





$r = 2 r_c \cos\theta$

$r_c = 0.4 \Longrightarrow \boxed{r = 0.8\cos\theta}$

General Eqs for all θs	@ $\theta = 30°$, $\dot{\theta} = 0.4$, $\ddot{\theta} = 0.8$
$r = 0.8\cos\theta$	$r = 0.693$ ft (at $\theta = 30°$)
$\dot{r} = -0.8\sin\theta \cdot \dot{\theta}$	$\dot{r} = -0.16$ ft/s
$\ddot{r} = -0.8\left[\cos\theta\,\dot{\theta}^2 + \sin\theta\,\ddot{\theta}\right]$	$\ddot{r} = -0.431$ ft/s²
$a_r = \ddot{r} - r\dot{\theta}^2$	$a_r = -0.542$ ft/s²
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$	$a_\theta = 0.426$ ft/s²
	$m = \frac{W}{g} = \frac{0.5}{32.2} = 0.0155$ slug

Eqs of motion:

$$\Sigma F_r = m a_r = N_s\cos\theta - W\overbrace{\cos\theta_1}^{\sin\theta} = m a_r \rightarrow \text{find } N_s$$

$$\Sigma F_\theta = m a_\theta = N_{OA} + N_s\sin\theta - W\cos\theta = m a_\theta \rightarrow \text{find } N_{OA}$$

$\boxed{N_s = 0.279 \text{ lb}}$ $\boxed{N_{OA} = 0.3 \text{ lb}}$

ψ is not required here. But it is $120°$. See if you can verify this!