

Putnam practice - Calculus-related problems

1. (2006 A1) Find the volume of the region of points (x, y, z) such that $(x^2 + y^2 + z^2 + 8)^2 \leq 36(x^2 + y^2)$.
2. (2015 A1) Let A and B be points on the same branch of the hyperbola $xy = 1$. Suppose P is a point lying between A and B on this hyperbola such that the area of triangle APB is as large as possible. Show that the region bounded by the hyperbola and the chord AP has the same area as the region bounded by the hyperbola and the chord PB .

3. (2002 A1) Let k be a fixed positive integer. The n -th derivative of $\frac{1}{x^k-1}$ has the form $\frac{P_n(x)}{(x^k-1)^{n+1}}$ where $P_n(x)$ is a polynomial. Find $P_n(1)$.

4. (2007 B2) Suppose $f : [0, 1] \rightarrow \mathbb{R}$ has a continuous derivative and $\int_0^1 f(x)dx = 0$. Prove that for every $0 < \alpha < 1$, we have $\left| \int_0^\alpha f(x)dx \right| \leq \frac{1}{8} \max_{0 \leq x \leq 1} |f'(x)|$.