Putnam practice - Probability problems

1. What is the probability that a 5-digit number starts or ends with a 1?

2. A fair coin is flipped 5 times, what is the probability that there is no consecutive heads?

3. (2001 A2) You have coins $C_1, C_2, \ldots, C_n$. For each k, C_k is biased such that, when tossed, it has probability $\frac{1}{2k+1}$ of falling heads. If n coins are tossed, what is the probability that the number of heads is odd?

4. (2004 A1) Basketball star Shanille O'Keal's team statistician keeps track of the number, $S(N)$, of successful free throws she has made in her first N attempts of the season. Early in the season, $S(N)$ was less than 80% of N, but by the end of the season $S(N)$ was more than 80% of N. Was there necessarily a moment in between when $S(N)$ was exactly 80% of N?

5. (2014 A4) Suppose X is a random variable that takes on only non-negative integer values with expect values $E[X] = 1$, $E[X^2] = 2$, and $E[X^3] = 5$. Determine the smallest possible value of the probability of the event $X = 0$.