

Putnam practice - Algebra problems

1. Prove that the number $\frac{5^{125}-1}{5^{25}-1}$ is not prime.

2. Find the zeros of the polynomial $x^4 - 6x^3 + 18x^2 - 30x + 25$, given that the sum of two of them is 4.

3. (2019 A1) Determine all possible values of the expression

$$A^3 + B^3 + C^3 - 3ABC$$

for any integers A , B , and C .

4. (2020 A2) Let k be a non-negative integer. Evaluate

$$\sum_{j=0}^k 2^{k-j} \binom{k+j}{j}.$$

5. Show that for an odd integer $n \geq 5$,

$$\binom{n}{0}5^{n-1} - \binom{n}{1}5^{n-2} + \binom{n}{2}5^{n-3} - \cdots + \binom{n}{n-1}$$

is not prime.