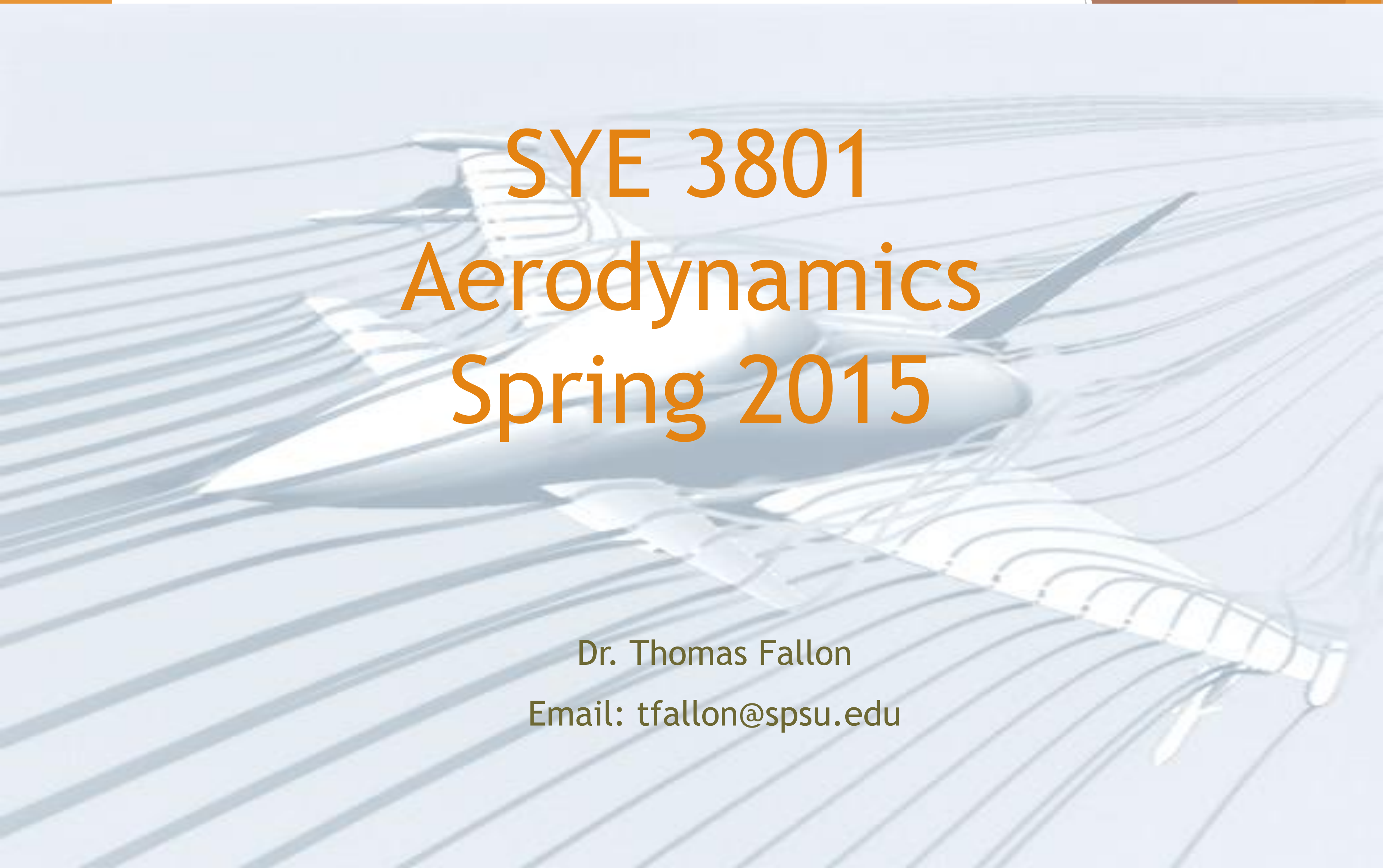

SYE 3801 Aerodynamics Spring 2015

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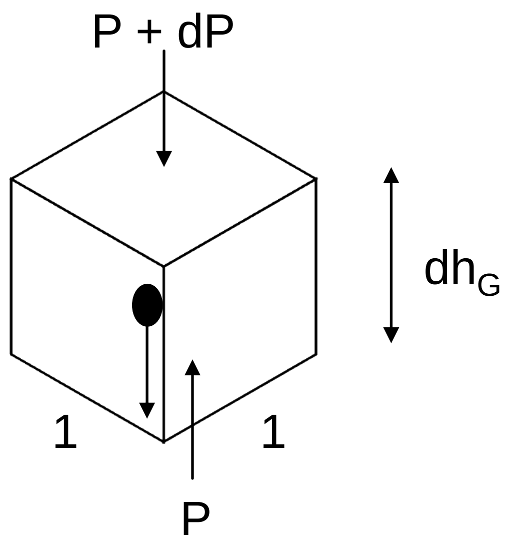
Hydrostatic Equation: Review

- Absolute Altitude (h_a)is important especially for space flight, because the local acceleration due to gravity at *sea level*:

$g = g_0[r/h_a]^2 = g_0[r/(r + h_G)]^2$ **(1)** ← Hydrostatic Equation

Consider a stationary fluid element:



Volume of element: $1x1xdh_G$
Mass of element: $\rho x1x1xdh_G$
Weight of element: $gx\rho xdh_G$

$F = PxA = Px1x1$

Three forces are balanced:

$P = P + dP + g\rho dh_G$

$dP = - g\rho dh_G$ (2) ← Hydrostatic Equation (valid for gases and liquids)

Geopotential Altitude

- The value of g changes with altitude. In calculations, a constant value of g_0 is used (g_0 = g at sea level). To compensate for changing g with altitude, we use geopotential altitude.

- Geopotential Altitude (h) is given by: **$h = \{r/(r + h_G)\}h_G$ (3)**
- **$h \simeq h_G$ for low altitudes** (< 65km, 213,000 ft)
- Divide the Hydrostatic Equation by the Equation of State:
- $\frac{dP}{P} = -\frac{\rho g_0 dh}{\rho RT} = -\frac{g_0}{RT}\mathrm{dh}$
- We can obtain pressure P at h by integrating between h_1 and h:

$$\int_{P_1}^{P} \frac{dP}{P} = -\frac{g_0}{RT}\int_{h_1}^{h} dh$$

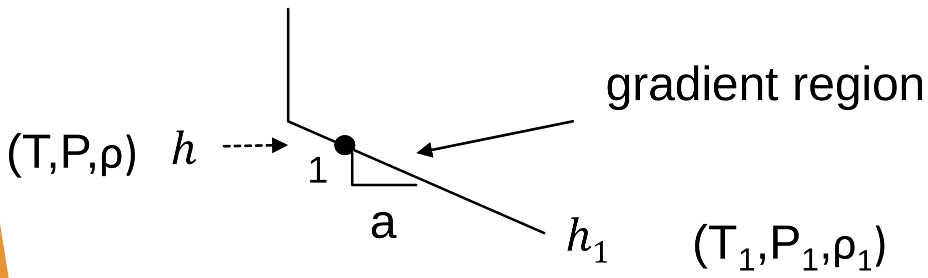
$$\ln\left(\frac{P}{P_1}\right) = -\frac{g_0}{RT}(h - h_1)$$

$$\frac{P}{P_1} = e^{-\frac{g_0}{RT}(h-h_1)}, \text{for isothermal region} \quad \mathbf{(4)}$$

- From the Equation of State:

$$\frac{P}{P_1} = \frac{\rho T}{\rho_1 T_1} = \frac{\rho}{\rho_1}$$

$$\frac{\rho}{\rho_1} = e^{-\frac{g_0}{RT}(h-h_1)}, \text{ for isothermal region} \quad \textbf{(5)}$$



$$\frac{T - T_1}{h - h_1} = \frac{dT}{dh} = a$$

- We know that

$$\frac{dP}{P} = -\frac{\rho g_0 dh}{\rho RT} = -\frac{g_0}{RT}\text{dh} = -\frac{g_0}{aR}\frac{dT}{T}$$

$$\int_{P_1}^{P} \frac{dP}{P} = -\frac{g_0}{aR}\int_{T_1}^{T} \frac{dT}{T}$$

$$ln\left(\frac{P}{P_1}\right) = -\frac{g_0}{aR} ln(\frac{T}{T_1})$$

Recall: *ln(A) – ln(B) = ln(A-B), and q*ln(A) = ln(A^q)*

$$ln\left(\frac{P}{P_1}\right) = -\frac{g_0}{aR} ln(\frac{T}{T_1}) \rightarrow l\,n\left(\frac{P}{P_1}/\frac{T}{T_1}^{-\frac{g0}{aR}}\right) = 0$$

Recall: $\ln(A) = B \rightarrow e^B = A$

$$e^0 = \frac{P}{P_1}/\frac{T}{T_1}^{-\frac{g0}{aR}} = 1 \rightarrow \frac{P}{P_1} = [\frac{T}{T_1}]^{-\frac{g0}{aR}}$$ **(6)** For gradient region

- From the Equation of State:

$$\frac{P}{P_1} = \frac{\rho RT}{\rho_1 RT_1} = [\frac{T}{T_1}]^{-\frac{g0}{aR}}$$

$$\frac{\rho}{\rho_1} = \left[\frac{T}{T_1}\right]^{-[\frac{g0}{aR}+1]}$$ **(7)** For gradient region

- Variation of T is linear with altitude:

$$T = T_1 + a(h - h_1)$$ **(8)**

Recall: P, ρ, and T under standard conditions (sea level):

$P_s = 1.01325 \times 10^5\ N/m^2 = 1atm$
$\rho_s = 1.225\ kg/m^3$
$T_s = 288.15K$

- Recall: P, ρ, and T under standard conditions (sea level):

$P_s = 1.01325 \times 10^5\ N/m^2 = 1atm$
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$T_s = 288.15K$

- Using equations 1 – 8 the standard atmosphere tables are generated.
- Note: the actual atmosphere data (T,P,ρ) at any given altitude may be different from the data given in tables depending on the day.

Example: Calculate the standard atmosphere values of T,P, and ρ at A geopotential altitude of 14 km.

Beginning at sea-level, the first region is a gradient region from $h = 0$ to $h = 11$ km.

$$a = \frac{dT}{dh} = \frac{216.66 - 288.15}{11 - 0} = -6.5\frac{K}{km} = -0.0065\frac{K}{m}$$

At sea-level P_1 = 1.01x10^5 N/m^2, ρ_1 = 1.23 kg/m^3, g_0= 9.8 m/s^2
R = 287 J/(KgK)

Using equation (6) for the gradient region:

$$P = P_1 x \left[\frac{T}{T_1}\right]^{-\frac{g0}{aR}} = (1.01x105)\left[\frac{216.66}{288.15}\right]^{\frac{-9{\cdot}8}{-0{\cdot}0065x287}}$$

$$P(at\ 11\ km) = 2.26x104\frac{N}{m^2}$$

Using equation (7) for the gradient region:

$$\rho = \rho_1 x \left[\frac{T}{T_1}\right]^{-\left\{\frac{g0}{aR}+1\right\}} = (1.23)\left[\frac{216.66}{288.15}\right]^{-\left[\frac{9{\cdot}8}{-0{\cdot}0065x287}+1\right]}$$

$$= 0.367\frac{kg}{m^3}(at\ 11\ km)$$

- The values of P and ρ form the base for the first isothermal region. For the isothermal region use equations (4) and (5):

$$P = P_1 e^{-\frac{g_0}{RT}(h-h_1)} = (2.26x104)e^{-\left[\frac{9.8}{(287)(216.66)}\right](14{,}000-11{,}000)} = 1.41x104\,\frac{N}{m^2}$$

$$\rho = \rho 1 * \frac{P}{P_1} = 0.367 * \frac{1.41x104}{2.26x104} = 0.23\,\frac{kg}{m^3}$$