SYE 3801 Aerodynamics Spring 2015

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P, T, and ρ Altitudes

- Pressure altitude: Standard altitude that corresponds to a given pressure
- Temperature altitude: Standard altitude that corresponds to a given temperature
- Density altitude: Standard altitude that corresponds to a given density

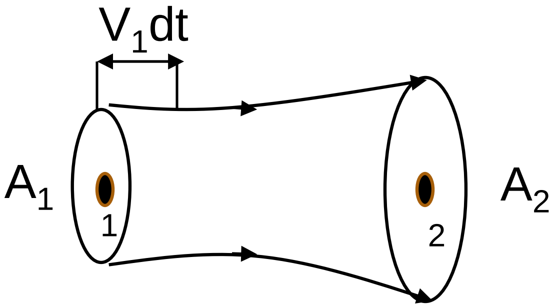
Example: An airplane is flying at an altitude where the actual pressure and temperature are 4.72×10^4 N/m^2 and 255.7 K, respectively. Determine pressure, temperature, and density altitudes.

1) From Appendix D (5th Edition) *Standard Atmosphere.* Standard altitude value Corresponding to $P = 4.72 \times 10^4$ N/m^2 is **6000m**
2) From Appendix D (5th Edition) *Standard Atmosphere.* Standard altitude value Corresponding to $T = 255.7$ K is **5000m**
3) For density altitude value corresponding to $\rho = 0.643$ kg/m^3 is **6240m**

Note: Temperature altitude has limited usefulness – multiple altitudes can have the same temperature.

Basic Aerodynamics

- Airplane flying at 10,000 ft. and 200 knots experiences certain amount of pressure and velocity at a given point near the wing tip
- A Space Shuttle engine experiences certain amount of pressure and temperature during takeoff
- These quantities need to be calculated using the laws of nature
- **Inviscid flow** is flow with no friction (used for simplifying math)
- **Viscous flow** is flow with friction
- **Continuity Equation:**
- Conservation of mass: Mass can be neither created nor destroyed, except when considering $E = mc^2$



- Consider streamlines that go through the circumference of the circle

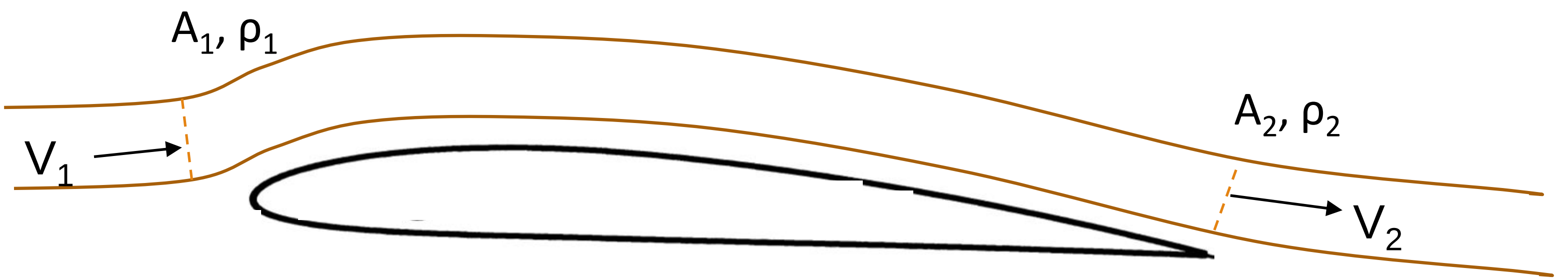
- These streamlines form a tube called a stream tube
- The cross sectional area of the tube may change; e.g. water hose nozzle
- But for steady flow (invariant with time), the mass that flows through the cross section at point 1 is the same as that through point 2

Consider A_1 to be the cross sectional area of the stream tube; V_1 is the velocity at point 1

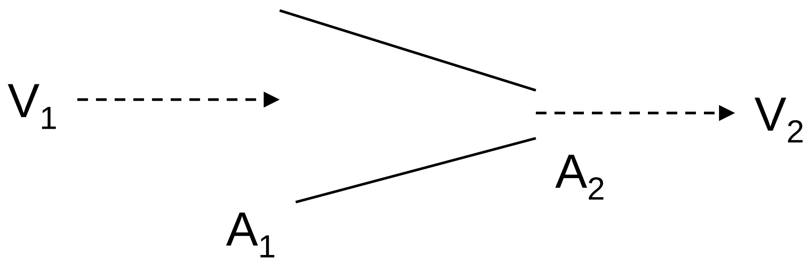
In a lapse of time dt, all the fluid elements move a distance $= V_1 dt$
Volume swept by the fluid elements $= A_1 V_1 dt$
Mass of gas 'dm' in this volume $= \rho_1 A_1 V_1 dt$

- The mass flow $\dot{m}$ through an area A is the mass crossing A per unit time.
- Mass flow = dm/dt = $\dot{m}_1 = \rho_1 A_1 V_1$ (kg/s or slugs/s)
- Similarly the mass flow through $\dot{m}_2 = \rho_2 A_2 V_2$, because mass can be neither created or destroyed.
- Therefore, $\dot{m}_1 = \dot{m}_2 \rightarrow \rho_1 A_1 V_1 = \rho_2 A_2 V_2$ (1)
- (1) is referred to as the continuity equation for steady fluid flow

- Continuity Equation is applicable to flow through ducts, tubes, wind, tunnels, rocket engines, airfoils, and so on.



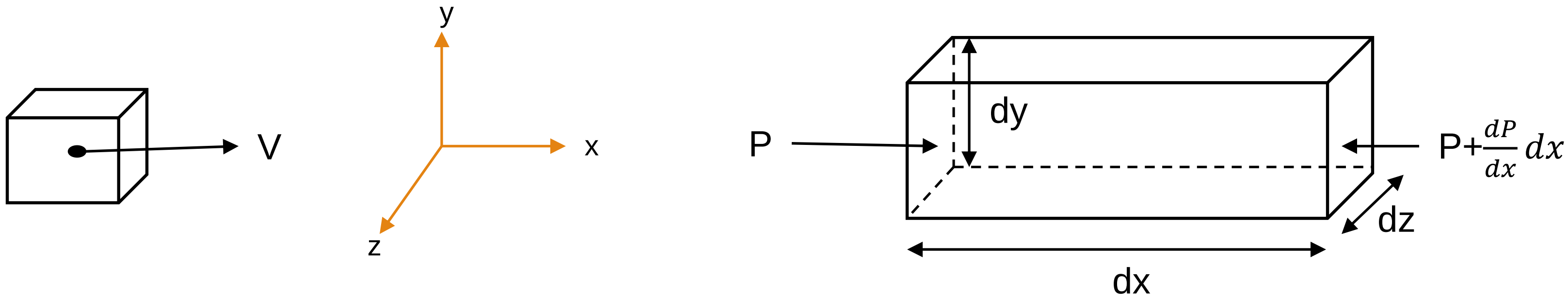
- Compressible Flow:
  - Flow in which the density of the fluid element can change from point to point; e.g. supersonic flow, rocket engines
- Incompressible Flow:
  - Flow in which the density of the fluid element is always constant
- *In real life, gases get compressed during any flow, but the change in density is negligible.
- Incompressible flow assumption is a good approximation of liquids in motion
- Low speed flow of air, where $V < 100$ m/s (225 mi/hr) can be assumed to be incompressible



- For incompressible flow $\rho_1 = \rho_2 = \rho$

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$
$\rho A_1 V_1 = \rho A_2 V_2$
$V_2 = (A_1/A_2) V_1$
Since $A_2 < A_1 \rightarrow V_2 > V_1$

- Momentum Equation
  - Newton's Second Law of Motion applies to fluids
  - $F = ma$

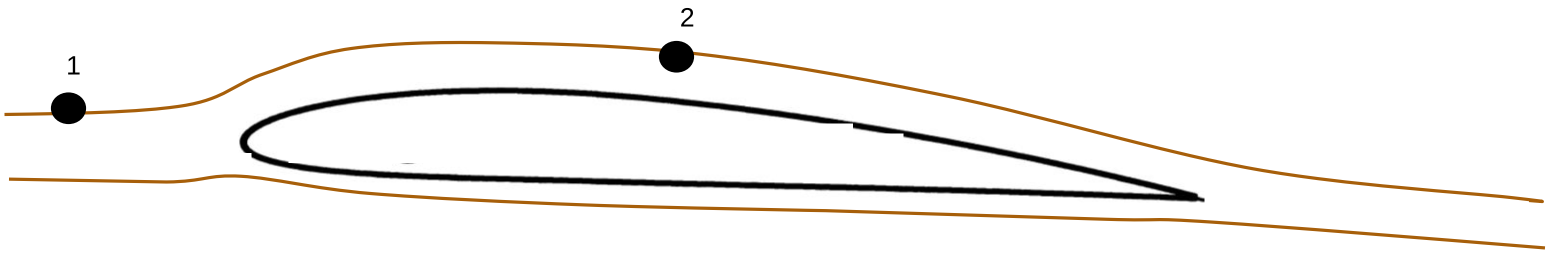


- Pressure on left face = P
- Area of left face = dydz
- Force on left face = Pdydz
- Pressure changes from point to point in the flow
- Change in pressure per unit length is $\frac{dP}{dx}$
- At the distance dx from the left face, the pressure is $\frac{dP}{dx}dx$
- Therefore, the pressure on the right face = P+$\frac{dP}{dx}dx$
- Force on the right face = [P+$\frac{dP}{dx}dx$]dydz {acts in the –x direction}
- The net force in the x-direction is: F = Pdydz - [P+$\frac{dP}{dx}dx$]dydz = -$\frac{dP}{dx}dx$dydz
- *mass of the fluid element m = ρdxdydz
- *acceleration of the fluid element a = $\frac{dV}{dt}, V = \frac{dx}{dt}$
- $a = \frac{dV}{dt} = \frac{dV}{dx}\frac{dx}{dt} = \frac{dV}{dx}V$

- Now F = ma yields

$-\frac{dP}{dx}dx\mathrm{dydz} = \rho\mathrm{dxdydz} * \frac{dV}{dx}V \rightarrow dP = -\rho V dv$ (2) Euler's Equation

- Leonhard Euler (1707 – 1783), Swiss mathematician, physicist, graph theory
- Euler Equation relates the rate of change of momentum to force
- Euler Equation is valid for inviscid flow
  - Gravity is also neglected
  - Flow is assumed to be steady
- Bernoulli's Equation
  - Daniel Bernoulli (1700 – 1782), Dutch mathematician → fluid mechanics,
  - probability, statistics
  - Consider the flow over the following airfoil



- To relate P_1 and V_1 at point 1 to P_2 and V_2 at point 2, the Euler's Equation is Integrated:

$$dP = -\rho V dv$$
$$dP + -\rho V dv = 0$$

$$\int_{P1}^{P2} dP + \rho \int_{V1}^{V2} V dv = 0$$

$$P_2 - P_1 + \rho[\frac{{V_2}^2}{2} - \frac{{V_1}^2}{2}] = 0$$

$$P_1 + \rho\frac{{V_1}^2}{2} = P_2 + \rho\frac{{V_2}^2}{2}$$ (3) Bernoulli's Equation

$$P + \rho\frac{V^2}{2} = constant\ along\ streamline$$

Important points about Bernoulli's Equation:

- Only valid for inviscid (frictionless), incompressible flow
- Relate properties between different points along a streamline
- Not valid for compressible flow, must be treated with ρ as a variable
- Euler's and Bernoulli's equations are essentially Newton's Second Law applied to fluid mechanics