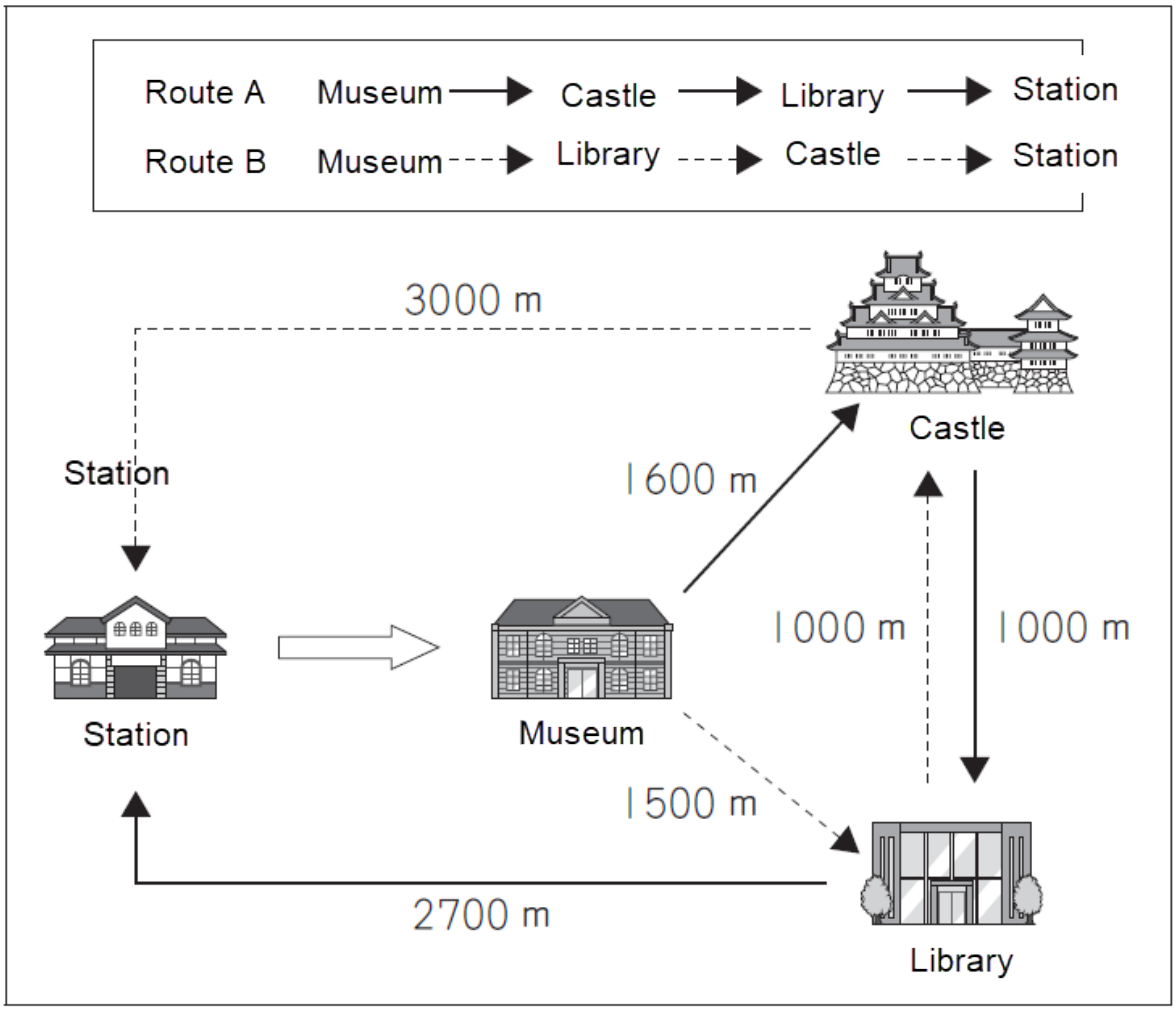
2021
Grade 6
Mathematics

[1] Takeru and his friends are studying local history, and they will be visiting a museum, a castle, and a library.

(1) Takeru and his friends are gathering at a train station, and they will go to the museum first. After the museum, they are thinking about two possible routes as shown below.

Takeru and his friends decided to follow Route A because the total distance for Route B was shorter than that of Route B.

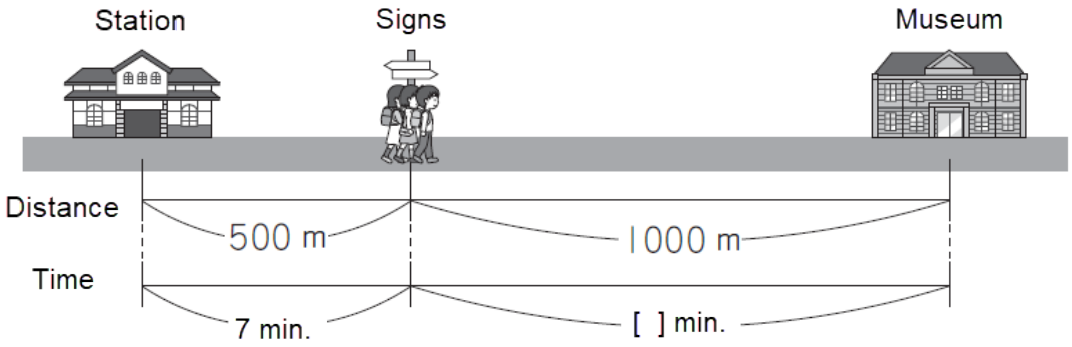
2 Routes and Distances



The total distance for Route A is how many meters shorter than that of Route B?

Write how you determined the answer using equations and words.

(2) After walking for 7 minutes from the station, they found signs saying "700 m to Museum" and "500 m to Station."

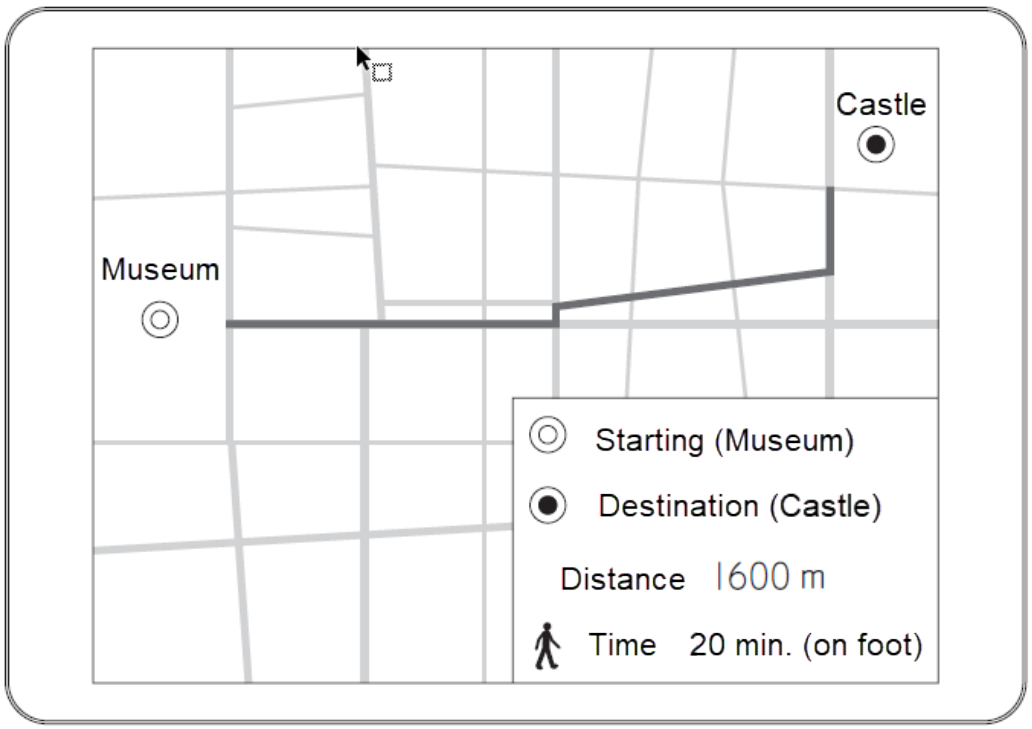


If they continue walking at the same rate of 500 m in 7 minutes, how many minutes will it take for them to walk 1000?

Write your answer.

(3) After the museum, they will be visiting the castle.

Before they started walking toward the castle, they searched the distance and estimated amount of time using Internet.



Takeru

The distance from the museum to the castle is 1600 m and takes 20 minutes.

Honoka

I wonder if the walking speed is about the same as ours.

Shiori

It took us 7 minutes to walk 500 m.

The table below shows the distance and time they found on Internet and the distance and time for their pace.

Distance and Time

	Distance (m)	Time (min.)
(*a*) Internet	1600	20
(*b*) Takeru's group	500	7

To determine which route is faster, the following calculations have been done.

a) Internet $1600 \div 20 = 80$

b) Takeru's group $500 \div 7 = 71.4 \ldots$

What can we say based on the calculations above?

Select one from 1 through 4 below.

1. The distance traveled in one minute are 80 m and about 71 m, therefore, a) is faster.
2. The distance traveled in one minute are 80 m and about 71 m, therefore, b) is faster.
3. The time it takes to travel 1 m are 80 minutes and about 71 minutes, therefor, a) is faster.
4. The time it takes to travel 1 m are 80 minutes and about 71 minutes, therefore, b) is faster.

(4) Takeru's group has arrived at the castle.

They began exploring the castle at 1:35 pm, and they decided to depart from the castle in 50 minutes. What time are they planning to depart from the castle? Write the time.

Time they started exploration.

Time to depart from the castle

(5) Takeru's group decided to take a bus to get back to the train station. Write the expression to calculate the number of minutes it takes for a bus traveling at 540 meters per minute to travel 2700 m. You do not have to actually calculate the answer.

[2] There is a triangle as shown in Figure 1.

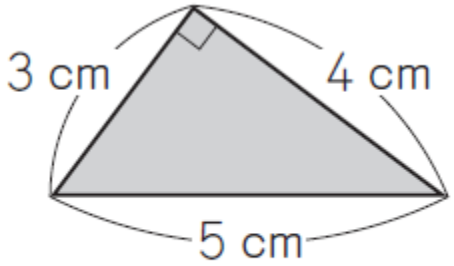
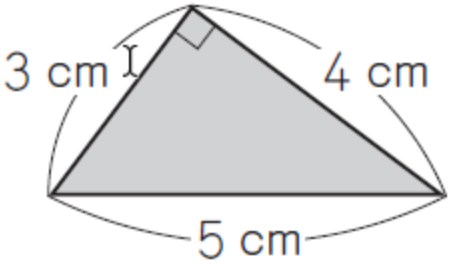
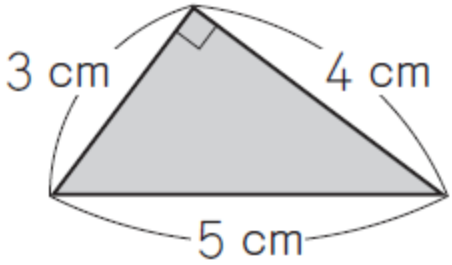


Figure 1

(1) What is the area of the triangle shown in Figure 1? Write both the expression to calculate the area and the answer (in cm^2).

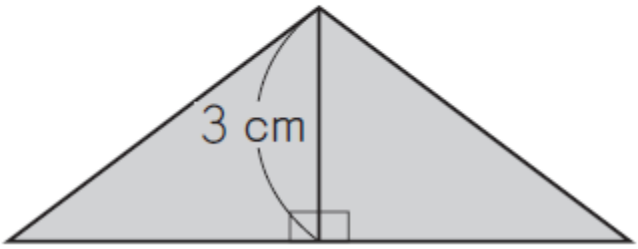
(2) There are 2 copies of the triangle shown in Figure 1.



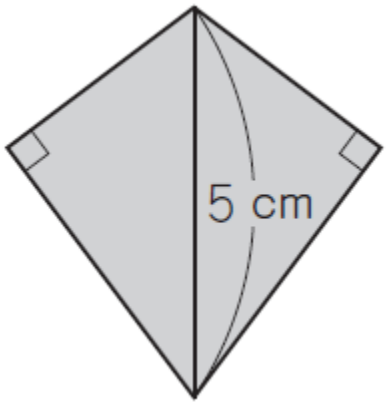


Using these two triangles, we can make figures like (1) and (2) below by putting the two sides that are the same length.

(1)



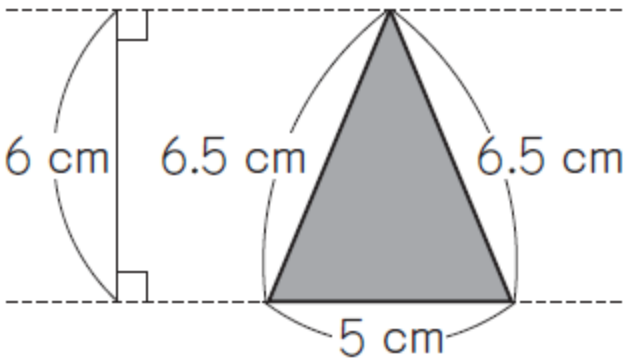
(2)



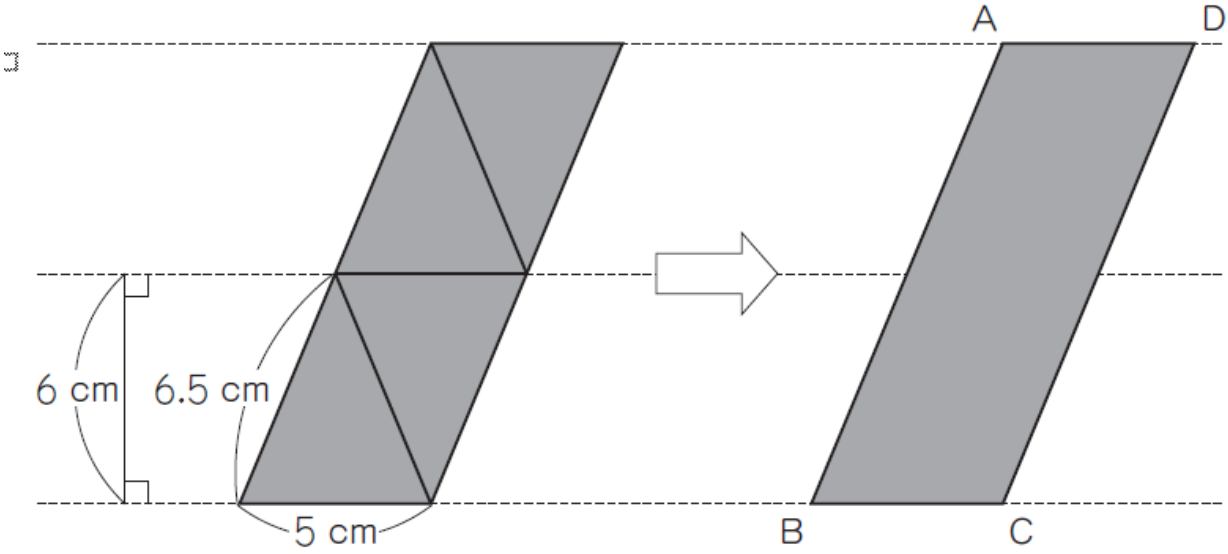
What can we say about the area of figures (1) and (2)? Select one statement from 1 through 4 below.

1. The area of figure (1) is greater.
2. The area of figure (2) is greater.
3. The areas of figures (1) and (2) are equal.
4. We cannot compare the areas of figures (1) and (2) with what we know.

(3) There is an isosceles triangle shown below.



By using 4 copies of the isosceles triangle above by putting the sides of the same length together, the following parallelogram ABCD was constructed.



We are going to calculate the area of the parallelogram using the formula.

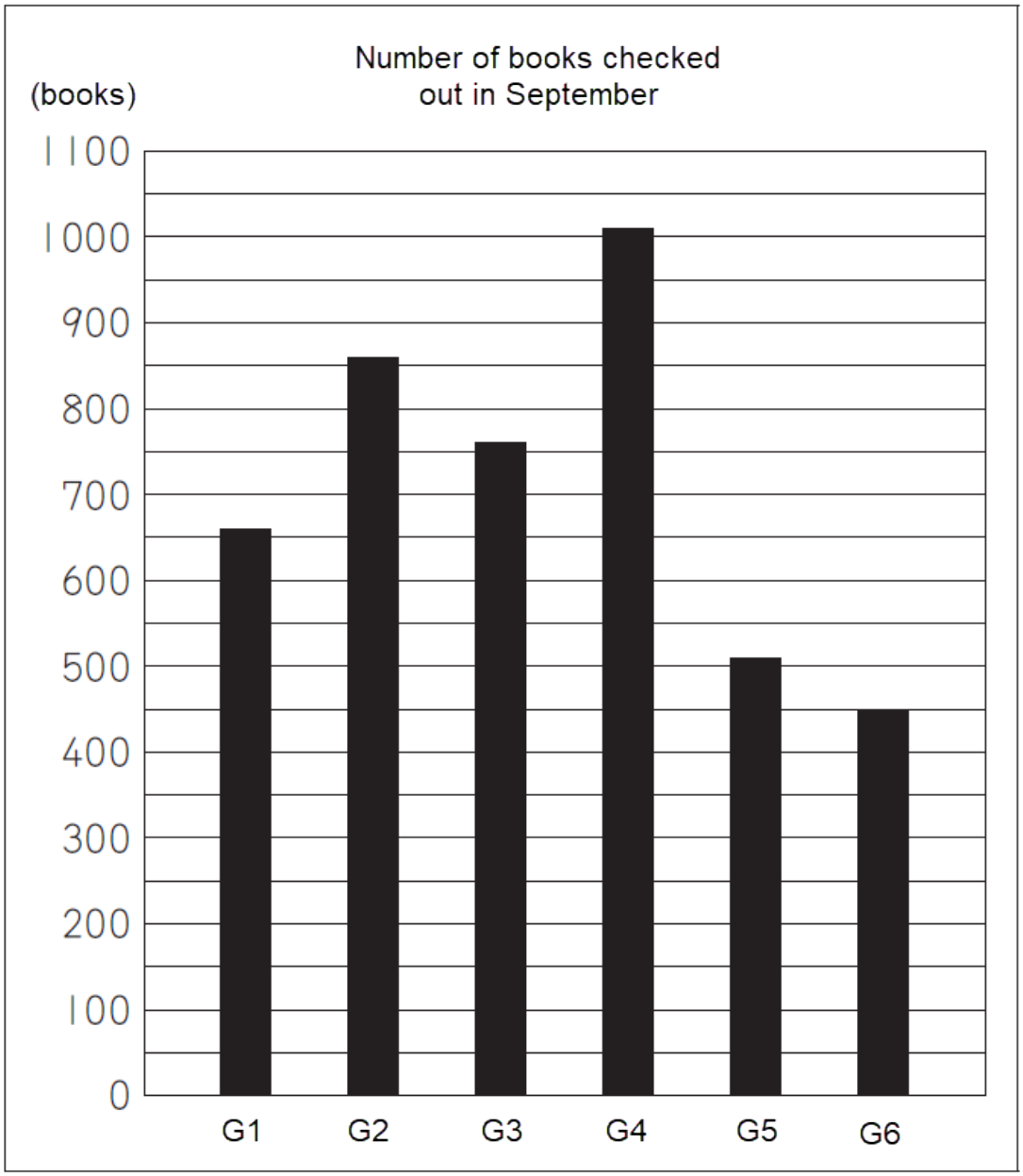
If we consider side BC as the base, explain how to calculate the area of the parallelogram using mathematical expressions and/or words. Make sure you clearly explain how you determined the height of parallelogram ABCD. Write the area of the parallelogram (in cm^2).

[3] Hiyori and her friends on the School Library Committee are discussing the data on the book circulation.

Hiyori

It seems like students in Grades 5 and 6 are not checking out as many books as students in other grades.

So, they created the graph below to show the number of books that have been checked out in September.



(1) Based on the graph above, how many books did students in Grade 6 check out in September? Select the most appropriate answer from (a) through (d) below.

(a) 401 books

(b) 405 books

(c) 410 books

(d) 450 books

(2) What can we say about the number of books checked out in September? Select the most appropriate statement from (a) through (d) below.

(a) Students in Grade 2 checked out the most books.

(b) Students in Grade 2 checked out almost twice as many books as students in Grade 3 did.

(c) Students in Grade 5 checked out only about a half as many books as students in Grade 4 did.

(d) The difference in the number of books checked out by students in Grade 1 and students in Grade 3 is about 200 books.

The committee examined the data from other months and found that students in Grades 5 and 6 checked out fewer books that students in other grades.

Hiyori

I wonder why students in Grades 5 and 6 checked out fewer books than students in other grades?

Sohta

I wonder if there are fewer Grades 5 and 6 students who like to read books.

The Committee conducted a survey with 189 Grades 5 and 6 students.

(3) First, the committee focused on two questions that asked whether students liked to read and how often they check out books from the library.

Question 1 Do you like to read?
Yes -------- 171
No --------- 18

Question 2 Did you check out 5 or more books from the library in September?
Yes -------- 61
No --------- 128

Hiyori

There were 171 people who like to read, but only 61 people have checked out 5 or more books from the library in September.

Sohta

I wonder about how many people who like to read but didn't check out many books from the library.

They decided to organize the results of those two questions in the table below.

Results on Questions 1 and 2 (people)

		Did you check out 5 or more more books from the library in September?		Total
		Yes	No	
Do you like to read?	Yes	①	②	③
	No	④	⑤	⑥
Total		⑦	⑧	189

They found out that 114 people answered "yes" to the question, "Do you like to read?" but "no" to the question, "Did you check out 5 or more books from the library in September?"

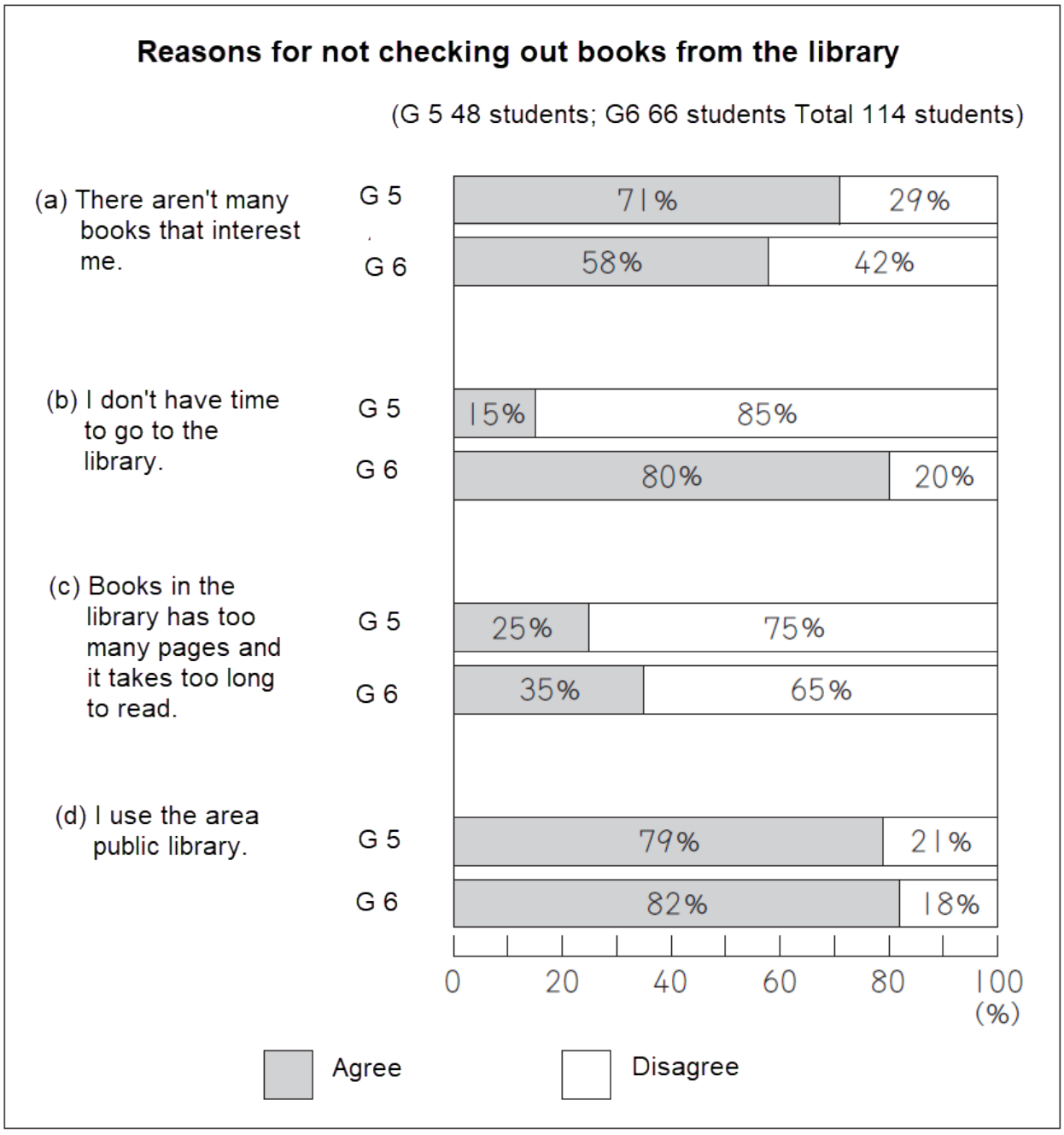
Where should "114" be placed in the table. Select the appropriate number from 1 through 8 above.

(4) Next, the committee focused on those 114 people who like to read but didn't check out many books from the library.

Hiyori

I wonder 5th graders and 6th graders have different reasons for not checking out many books.

So, they separated the responses of grade 5 students from grade 6 students and summarized in the table below.



The committee is discussing what they noticed from the graphs above.

Sota and Ayano focused on the rate of those who answered “agree” on items (a) through (d) in the summary above.

Sota

Fir some items, the rates are about the same for 5th graders and 6th graders.

Ayano

Fir other items, the rates are very different across grades.

For which item is the difference of rates of students who answered “agree” the greatest between Grades 5 and 6? What are the rates of students who answered “agree” for that item in Grade 5? Grade 6?

Write the item and the rates using words and numbers.

(5) The committee members are discussing the results of the survey.

Hiyori

We learned that some people feel that there aren’t many books that are of their interest in the library. Also, there are those who use local public libraries, too.

Sota

In order to get more students check out books from the library, how about asking the library to get more books that 5th and 6th graders will be interested in reading?

Therefore, they decided to investigate what books 5th and 6th graders are interested in reading, as well as what books they have been reading.

What kinds of data should the committee collect?

Select two appropriate items from 1 through 5 below.

1. Days of the week that students are using the school library.
2. Titles of the books they would like the school library to get.
3. Time of the day when they often use the school library.
4. Days of the week that they visit local public library.
5. Titles of books they have recently checked out from a local public library.

[4] Koharu and her friends are looking back on different division problems they have studied so far.

(1) There are 23 balls. We are going to put 6 balls in a box.

In order to figure out how many boxes are needed to put all of the balls away, we did the following calculation.

$$23 \div 6 = 3\ rem.\ 5$$

At least how many boxes do we need to put all of the balls away?

(2) We are going to share 4 L of juice equally among 8 people.

How much juice (in L) will each person get? Write the expression for the calculation necessary and the answer.

(3) There is a 14-m tape and a 20-m tape.

Koharu and her friends are discussing how many times as long is the 14 meters as 20 meters.

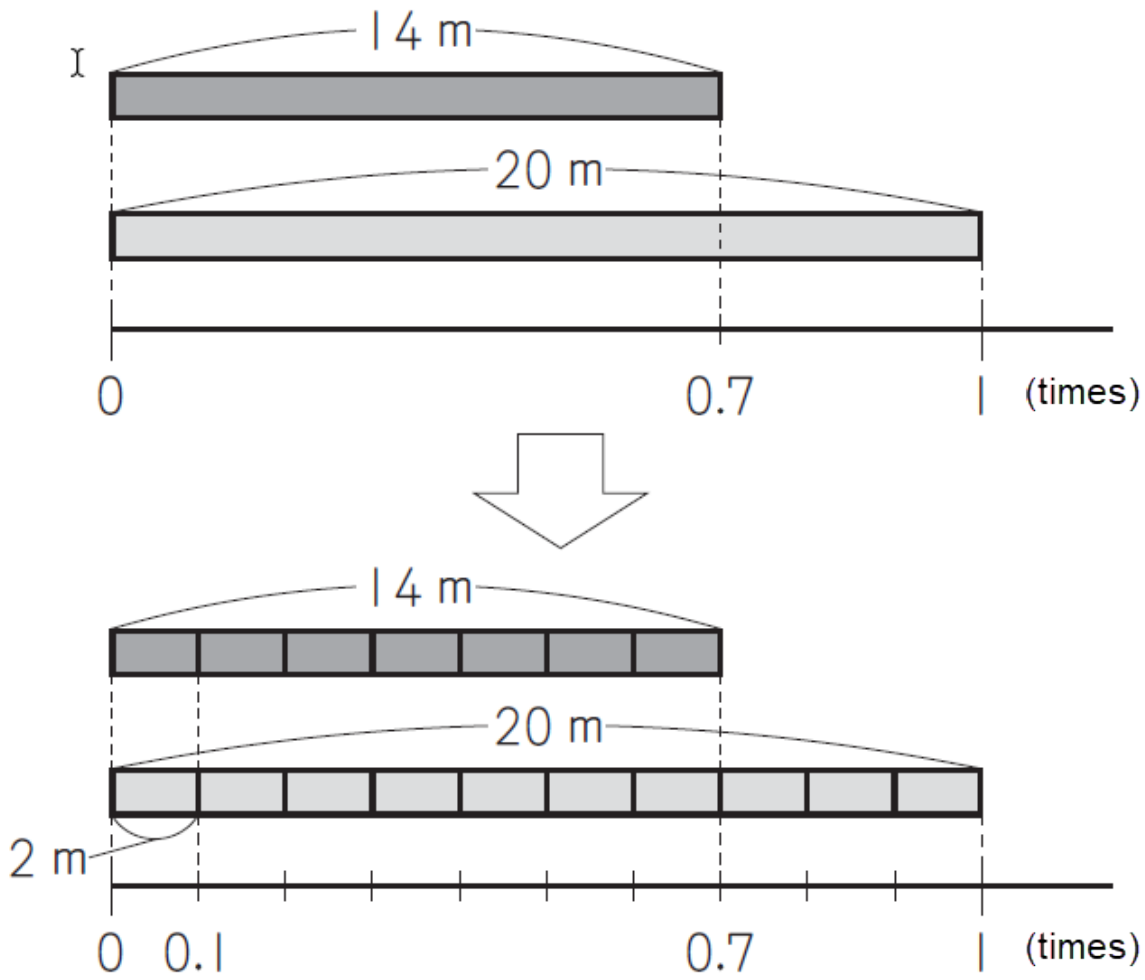
Koharu

$14 \div 20 =$ 0.7. So, it is 0.7 times as long. 0.7 times as long means if we consider 20 meters as 1, 14 meters will correspond to 0.7.

Rento

Is it really true that if we consider 20 meters as 1, 14 meters will correspond to 0.7?

When Yuhma heard Rento's comment, he tried to explain the reason why 14 meters will correspond to 0.7 when 20 meters is considered as 1 based on the idea that each of 10 equally divided parts of 20 meters will correspond to 0.1. He drew the diagram below to help him explain.



Yuhma's explanation

Yuhma

If we consider 20 meters as 1, 2 meters will correspond to 0.1.

14 meters is 7 of 2 meters. Therefore, if we consider 20 meters as 1, 14 meters will correspond to 0.7.

There are a 12-m tape and a 30-m tape, too.

Takumi and his friends are thinking about how many times as long is 12 meters as 30 meters.

Takumi

$12 \div 30 = 0.4$. So, it is 0.4 times as long.

Sakura

Is it really true that if we consider 30 meters as 1, 12 meters will correspond to 0.4?

Explain the reason why 12 meters will correspond to 0.4 when we consider 20 meters as 1 using "Yuhma's explanation" by making explicit what length will correspond to 0.1. Use words and numbers in your explanation.

* If necessary, you may use the diagram below.