2021

Grade 9

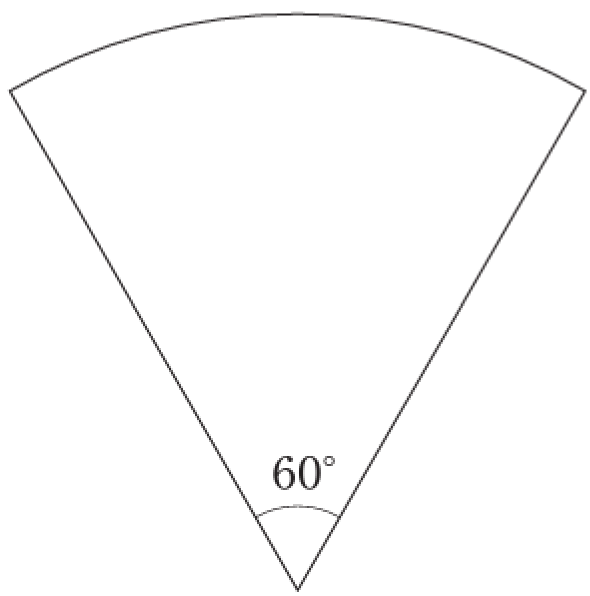
Mathematics

[1] Calculate $(5x + 6y) - (3x - 2y)$.

[2] If you purchase 2 notebooks and a 800-yen pencil case, the total cost is the same as the cost of purchasing 4 notebooks and a 500-yen mechanical pencil.

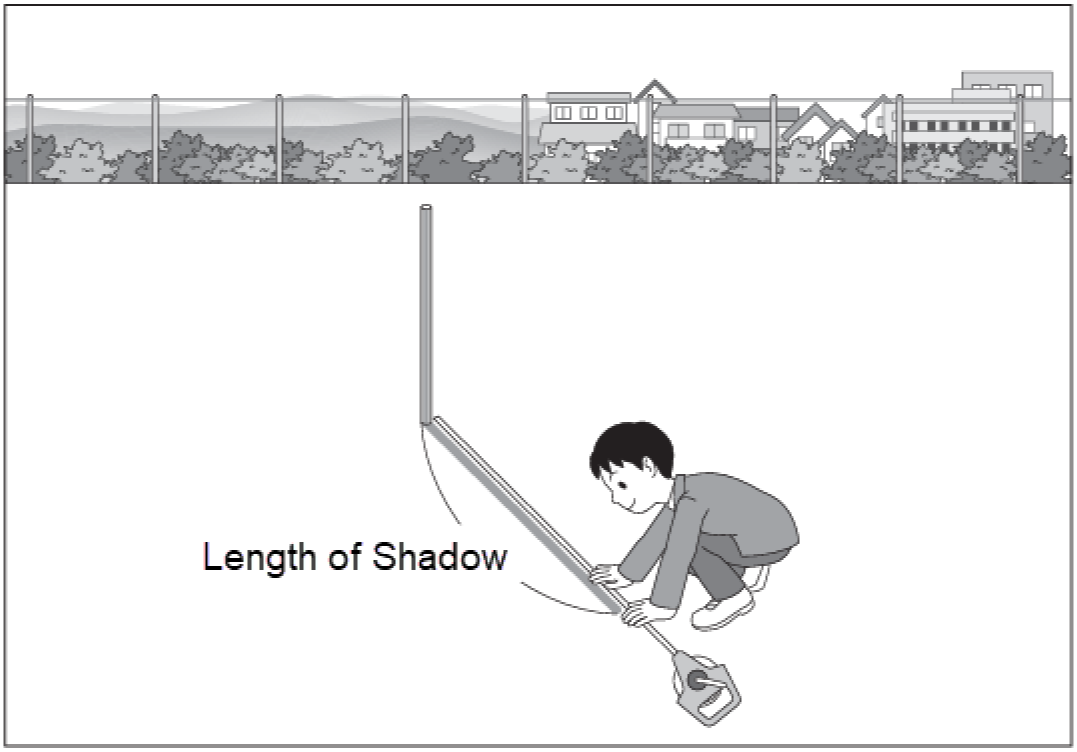
To determine the cost of one notebook, write an equation by letting the cost of one notebook as *x*-yen. You do not have to solve the equation you write.

[3] There is a sector with the central angle of 60° as shown below. How many times as long is the length of the arc of this sector as the circumference of a circle with the equal radius. Select the correct answer from (a) through (e) below.



(a) $\frac{1}{2}$ times (b) $\frac{1}{3}$ times (c) $\frac{1}{4}$ times (d) $\frac{1}{5}$ times (e) $\frac{1}{6}$ times

[4] A 1-meter long wooden stick was placed perpendicular to the ground, and we investigated the lengths of its shadow hourly on a certain day from 8 am till 4 pm.



The table below shows the elapsed time since 8 am and the length of the shadow at that time.

Elapsed time (hour)	0	1	2	3	4	5	6	7	8
Length of shadow (cm)	190	124	96	80	79	96	130	193	350

For the relationship between the elapsed time since 8 am and the lengths of the shadow, we can say that "<u>if we fix the elapsed time, there is a unique corresponding value for the length of the shadow</u>."

If we are to re-phrase the underlined statement as below, write the phrases that should go into [(1)] and [(2)].

[(1)] is a function of [(2)].

[5] The data below show the number of side-to-side steps in 20 seconds for 10 middle school boys. The numbers are arranged in ascending order.

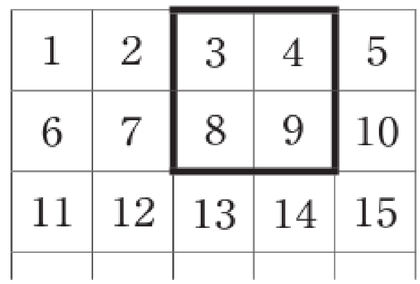
Data

43	46	46	52	53	55	56	56	56	57

Determine the median of the data.

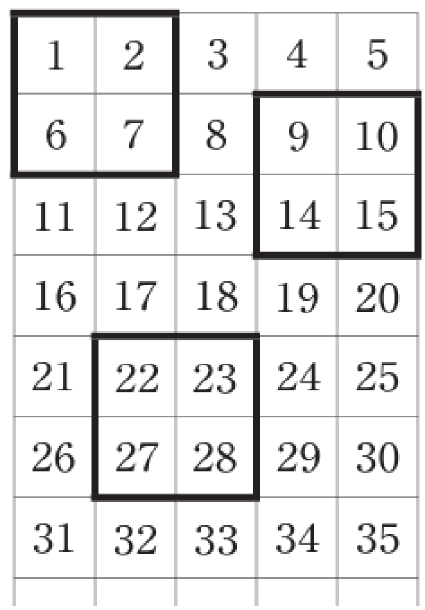
[6] As shown on the right, there is a chart that lists natural numbers in order, 5 numbers in each row. You are grouping 4 numbers in a 2-by-2 cells. For example, if we group 4 numbers as shown in Figure 1, the number on top left is 3, top right 4, bottom left 8 and bottom right 9.

Figure 1



Yuta and Mana made 3 groups of 4 numbers as shown in Figure 2. They are investigating the sum of those 4 numbers in each group.

Figure 2



When the numbers are 1, 2, 6 and 7,

$$1 + 2 + 6 + 7 = 4 \times 4$$

When the numbers are 9, 10, 14 and 15,

$$9 + 10 + 14 + 15 = 4 \times 12$$

When the numbers are 22, 23, 27, and 28,

$$22 + 23 + 27 + 28 = 4 \times 25.$$

Based on these observations, Yuta conjectured that the sum of 4 numbers grouped this way will always be a multiple of 4.

Answer the following questions (1) through (3).

(1) When the 4 numbers grouped together are 12, 13, 17 and 18, we are going to check if their sum will be a multiple of 4 in the following way. Write the expression that will go into the [ ] below.

When the numbers are 12, 13, 17 and 18, $12 + 13 + 17 + 18 = 60 = [\qquad\qquad]$

(2) Two of them are discussing if the sum of 4 numbers grouped in this way will always be a multiple of 4.

Yuta:	When the top left number is 1, the bottom left is 6. When we group 4 numbers this way, the bottom left number is 5 more than the top left number.
Mana:	That's because each row has 5 consecutive natural numbers.
Yuta:	So, if we let the top left number be *n*, we can write the bottom left number as *n* + 5.
Mana:	Let's express the top right and the bottom right numbers in terms of *n* and investigate their sum.

Complete the explanation of why Yuta's conjecture, "the sum of 4 numbers grouped this way will always be a multiple of 4," is always true.

Explanation

If a natural number, *n*, is the top left number of the four numbers grouped in this manner. Then, the top right number can be expressed as *n* + 1, the bottom left number as *n* + 5, and the bottom right number as *n* + 6.

Therefore, the sum of these 4 numbers is

$n + (n + 1) + (n + 5) + (n + 6)$

=

(3) Two of them decided to explore if the sum of 4 numbers grouped together like this will be a multiple of 4 even when the chart is revised to have 6 numbers in each row. So, they created a chart as shown in Figure 3 and considered the sum of 4 numbers grouped together.

Figure 3

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30

When the numbers are 1, 2, 7, and 8, $1 + 2 + 7 + 8 = 18 = 2 \times 9$

When the numbers are 17, 18, 23 and 24, $17 + 18 + 23 + 24 = 82 = 2 \times 41$

From these results, if the chart is like the one shown on Figure 3, the 4 numbers grouped together in this manner will not sum to a multiple of 4. In order to investigate what she could say about the sum of 4 numbers grouped in this way, she expressed the top left number as n, the top right number as $n + 1$, the bottom left number as $n + 6$, and the bottom right number as $n + 7$, and calculated as shown below.

Mana's calculation

$n + (n + 1) + (n + 6) + (n + 7)$

$= n + n + 1 + n + 6 + n + 7$

$= 4n + 14$

$= 2\,(2n + 7)$

n	$n + 1$
$n + 6$	$n + 7$

As you can see from Mana's calculation above, the sum of 4 numbers grouped this way will be 2 (2n + 7), or a multiple of 2. Two of them are discussing about this observation.

Mana:	We can tell that the sum of 4 numbers grouped this way in a chart that has 6 consecutive natural numbers in each row will be the double of $2n + 7$.
Yuta:	I wonder what kind of number is $2n + 7$.

Note that $2n + 7$ in $2(2n + 7)$ can be re-written as $n + (n + 7)$. Thus, we know that the sum of 4 numbers grouped this way will be the sum of two of the numbers at top left, top right, bottom left and bottom right.

The sum of the 4 numbers grouped this way will be the double of the sum of which 2 numbers? Write your answer in the form, "_________ is ____________."

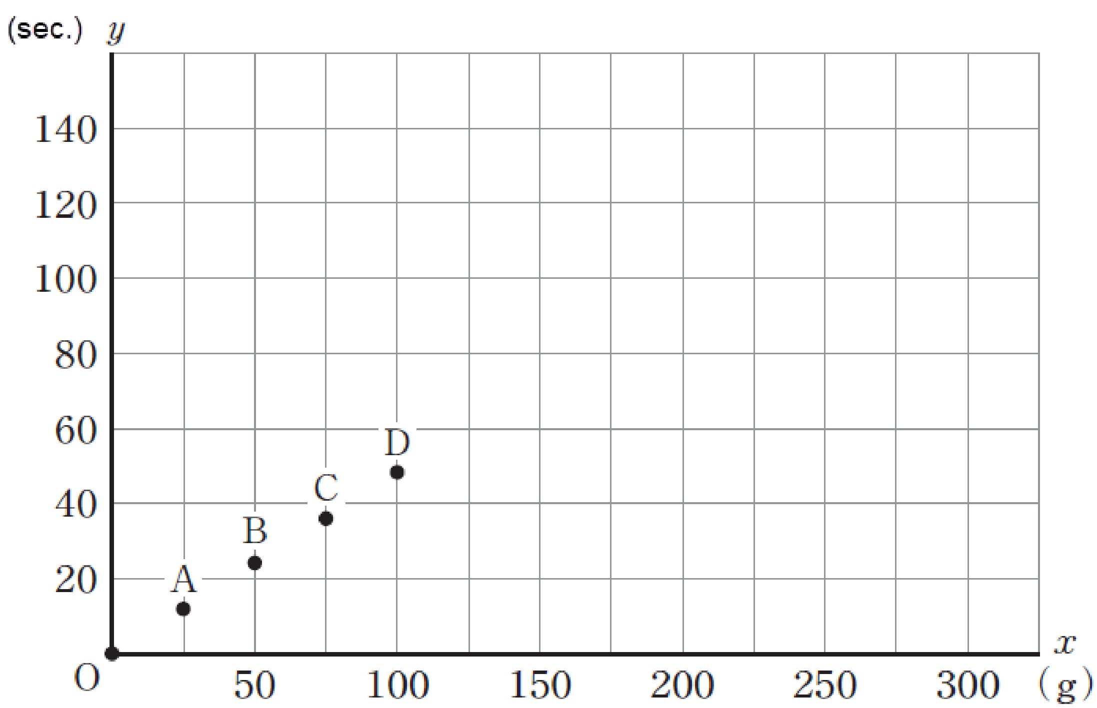
[7] Kento, as the class leader, decided to built a sand timer for 2-minute speeches the students in the class will have to make by putting together 2 plastic drink bottles. He will put sand in that bottles, and a construction paper with a hole will be placed in between the bottles through which sand can fall from the top bottle to the bottom one.

Kento thought that the weight of sand he will put in the timer will determine the amount of time it will take sand to completely fall from the top to the bottom bottle. Thus, he experimented with various weights of sand (x g) and measured the amount of time it took for sand to completely fall (y seconds). He summarized the results of his experiment in the table and the graph shown below.

Results of Kento's experiment

Weight of sand and time it took to fall completely

Weight of sand x (g)	0	25	50	75	100
Time took for sand to fall completely y (sec.)	0	11.9	24.2	36.0	48.3



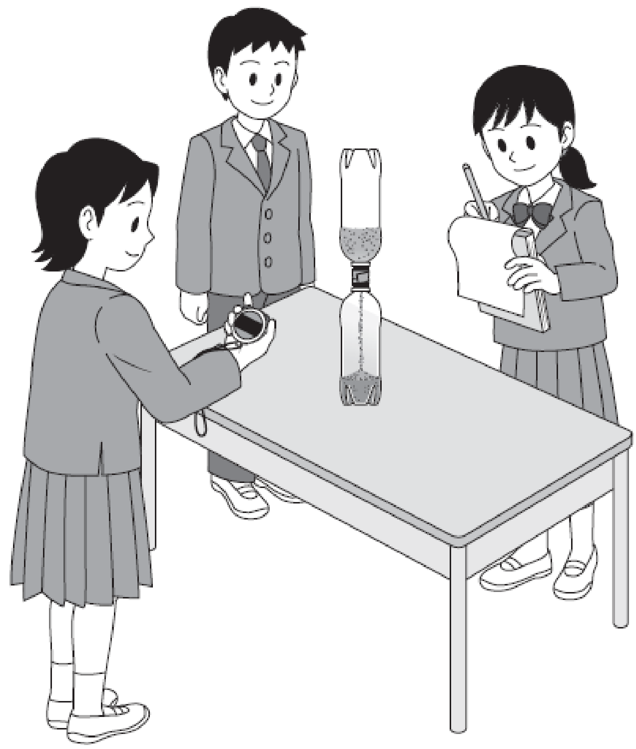
Answer the following questions, (1) and (2).

(1) In the graph shown above, which point shows that 75 g of sand took 36.0 seconds to fall completely? Select one from points A through D.

(2) Kento is trying to figure out the amount of sand needed to measure 2 minutes.

To do so, Kento considered that the points A through D are on a straight line starting from the origin, and he assumed that as the weight of sand increased, the amount of time will continue to fall on the same line.

Explain how Kento can determine the weight of sand needed to measure two minutes. You do not have to determine the actual weight.



[8] Momoka will be going on camping in City A in May. In order to prepare for her trip, she researched weather related conditions in City A as they are closely related to how comfortable it will be to camp there. She found the data on daily high and low temperature, sunshine duration, maximum gust, and amount of rain from last May on Internet. She also determined the daily temperature difference by calculating the difference between the high and low temperature. She summarized her findings in the table below.

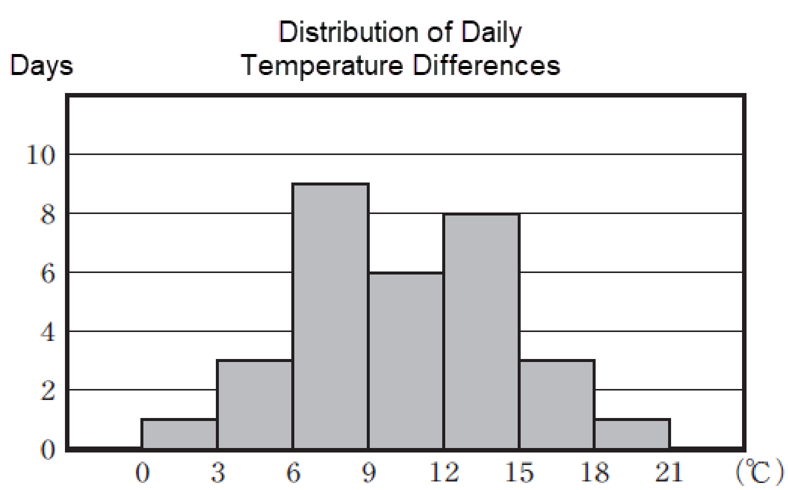
Results

Date	High (°C)	Low (°C)	Temp. Difference (°C)	Sunshine duration (Hrs.)	Maximum Gust (m/sec)	Amount of rain (mm)
1	20.9	6.9	14.0	5.8	7.4	0.0
2	25.9	9.1	16.8	12.0	7.3	0.0
3	27.3	12.8	14.5	10.3	8.2	0.0
4	20.3	11.8	8.5	2.5	9.5	0.0
5	23.5	9.4	14.1	9.9	11.9	0.5
6	13.2	5.5	7.7	0.1	8.7	2.0
|	|	|	|	|	|	|
31	20.9	9.2	11.7	2.2	9.1	0.0

Note: Sunshine duration is the total amount of time during a day when shadow is observed due to sunshine.

Answer the following questions (1) through (3).

(1) As Momoka examined the results of her research, she became a little concerned that the daily temperature differences varied a bit. Thus, she summarized the distribution of daily temperature differences in the histogram as shown. For example, you can see that there were 3 days when the daily temperature difference was greater than or equal to 3°C but less than 6°C.



Find the frequency for the class, greater than or equal to 9°C but less than 12°C.

(2) As she examined the histogram above, she noted that two classes, greater than or equal to 6℃ but less than 9℃ and greater than or equal to 12℃ but less than 15℃, had frequencies higher than other classes, making the data look like having 2 peaks. She re-examined the data she obtained from her research, and she hypothesized that days with longer sunshine durations have a larger temperature difference. Therefore, she created a frequency distribution chart for the days with sunshine duration of less than 6 hours and those days with 6 hours or more, using relative frequency for each class as shown below.

Frequency Distribution of Temperature Difference

Temp. Diff. (℃)	Less than 6 hrs.		6 hrs. or more	
Greater than or equal to ~ Less than	Freq. (days)	Rel. Freq.	Freq. (days)	Rel. Freq.
0 ~ 3	1	0.05	0	0.00
3 ~ 6	3	0.16	0	0.00
6 ~ 9	9	0.47	0	0.00
9 ~ 12	4	0.21	2	0.17
12 ~ 15	2	0.11	6	0.50
15 ~ 18	0	0.00	3	0.25
18 ~ 21	0	0.00	1	0.08
Total	19	1.00	12	1.00

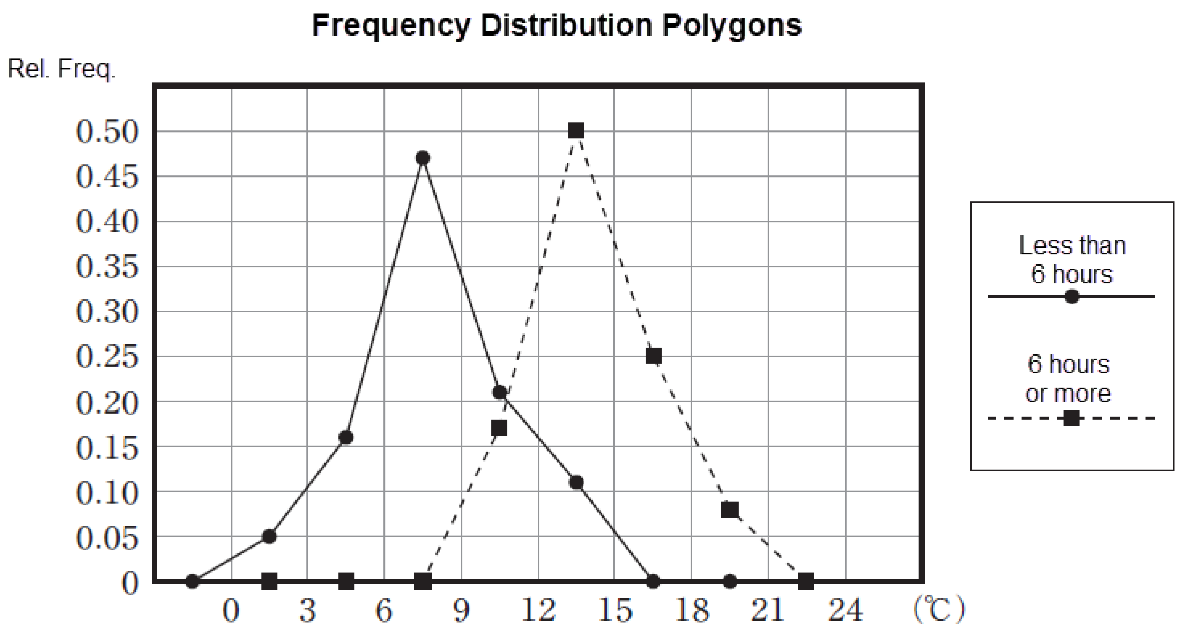
The reason relative frequencies like above is used when comparing distribution of data in 2 data sets is because of the following.

> The reason relative frequencies are used to compare distributions of data in 2 data sets is because [ ] for “less than 6 hours” and “6 hours or more” are different.

Select the appropriate words that should go into the [   ] above from A through D below.

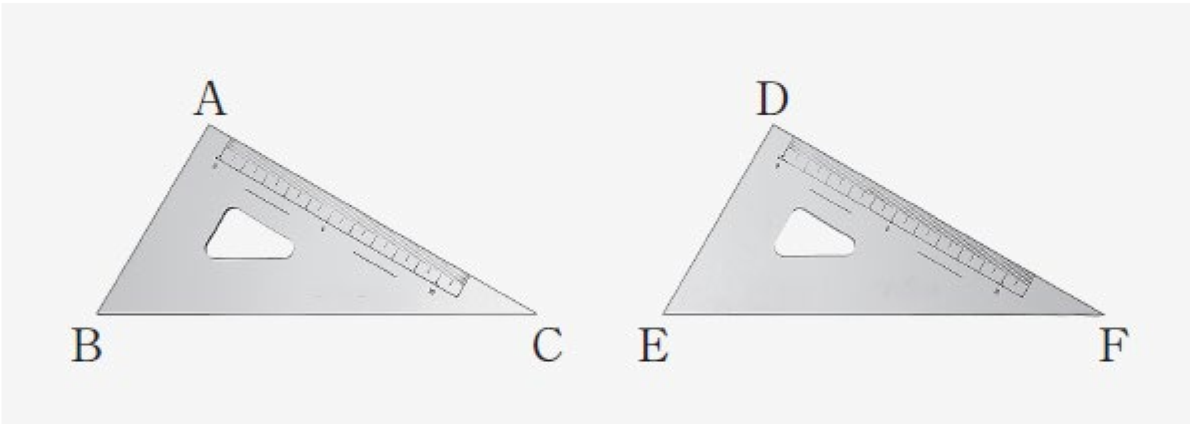
A	Sunshine duration	B	Temperature difference
C	Frequencies among classes	D	Total of frequencies

(3) Momoko created frequency distribution polygons with the relative frequencies on the vertical axis and temperature difference on the horizontal axis, as shown below.



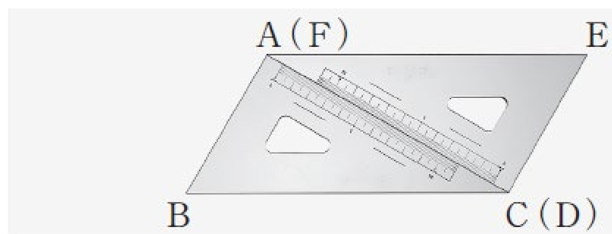
Based on these frequency distribution polygons, we can argue that “daily temperature difference tends to be greater on days with sunshine duration is 6 hours or more than those days with less than 6 hours of sunshine duration.” Explain what such an argument may be justified by using 2 characteristics of frequency distribution polygons in the graph.

[9] There are 2 pieces of $30° - 60° - 90°$ set squares (triangular rulers) and labeled them as $\triangle$ABC and $\triangle$DEF, as shown below. Naoki and Yui are thinking about quadrilaterals they can form by putting together these two set squares.



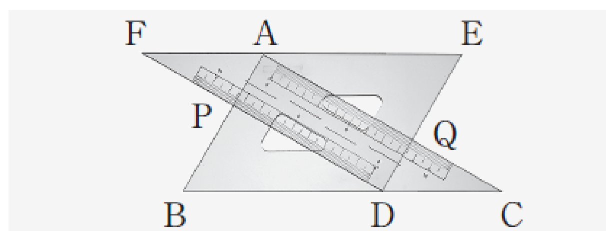
They first created quadrilateral ABCE by putting vertices A and F and vertices C and D together.

Figure 1



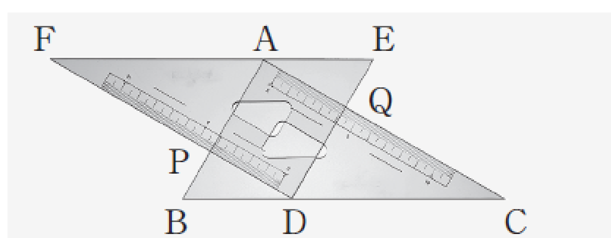
Then, they overlapped $\triangle$ABC and $\triangle$DEF so that vertex D will be on side BC and side EF will be parallel to side BC as shown on Figure 2. If we call the intersection of sides AB and FD as P and the intersection of sides ED and AC as Q, we have quadrilateral APDQ.

Figure 2



Finally, they slid $\triangle$DEF to the left so that vertex D will still be on side BC but sides EF and BC are parallel as shown in Figure 3.

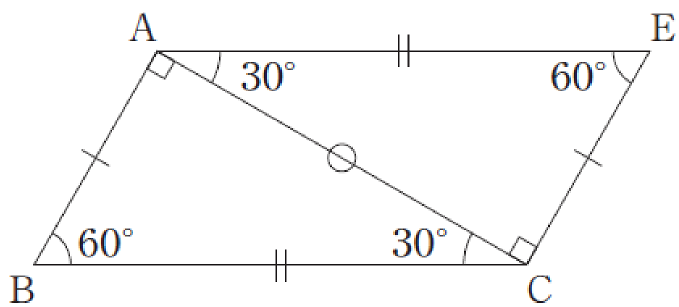
Figure 3



Answer the following questions (1) through (3).

(1) Naoki and Yui conjectured that quadrilateral ABCD in Figure 1 will be a parallelogram. In order to prove their conjecture, they drew Figure 4 below.

Figure 4



In Figure 4, since △ABC and △DEF are congruent, we know that the measures of corresponding sides and angles are equal.

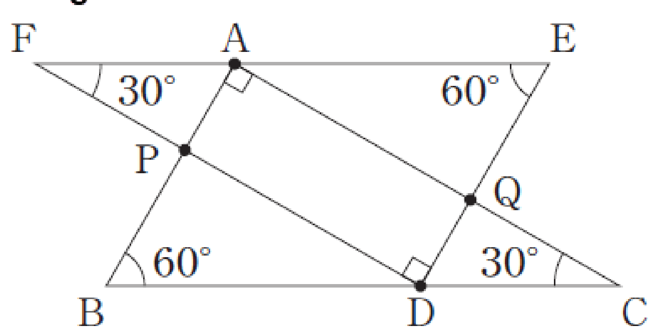
Using this fact, we can prove quadrilateral ABCD is a parallelogram by using the conditions for parallelogram. Using either A or B below, prove that quadrilateral ABCD is a parallelogram. You may use either A or B.

A A quadrilateral in which 2 pairs of opposite sides are both parallel is a parallelogram.

B A quadrilateral in which the measures of 2 pairs of opposite angles are equal is a parallelogram.

(2) Two of them conjectured that quadrilateral APDQ obtained when 2 set squares are overlapped as shown in Figures 2 and 3 is a rectangle. In order to prove their conjecture, they drew Figure 5.

Figure 5



Quadrilaterals with 4 congruent angles will be rectangles. In quadrilateral APDQ, since ∠PAQ = ∠ PDQ = 90°, if we can say ∠ AQD = 90°, then we know quadrilateral APDQ is a rectangle.

So, Naoki came up with the following reasoning to show ∠ AQD = 90°.

Naoki's reasoning

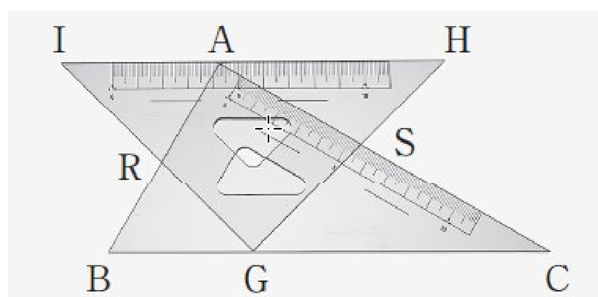
(1)	Since ∠ AFP is an external angle, its measure equals the sum of measures of ∠ AFP and ∠ FAP.
(2)	Of the angles created when line AB intersects lines FE and BC, ∠ FAP and ∠ PBD are alternate interior angles, therefore, ∠ FAP = ∠ PBD = 60°.
(3)	From (1) and (2), ∠ AFP + ∠ FAP = $30° + 60° = 90°$. Thus we can say ∠ AFP = 90°.

In (2) of Naoki's reasoning, the reason we can say that ∠ FAP and ∠ PBD are alternate interior angles is because lines FE and BC are in a particular relationship. Express that relationship using appropriate symbol.

(3) Two of them switched the set square piece that was slid to left (△DEF) with the $45° - 45° - 90°$ set square piece and thought about the quadrilateral formed by the overlapping region.

As shown on Figure 6 on the right, if we label the $45° - 45° - 90°$ set square piece as △GHI and the point of intersection between sides AB and IG as R and the point of intersection between sides HG and AC as S, we can obtain quadrilateral ARGS.

Figure 6



They conjectured that even when △GHI is slid to the left so that vertex G will remain on side BC and sides HI and BC are parallel, quadrilateral ARGS will not be a rectangle. They drew Figures 7 and 8 as shown below.

Figure 7

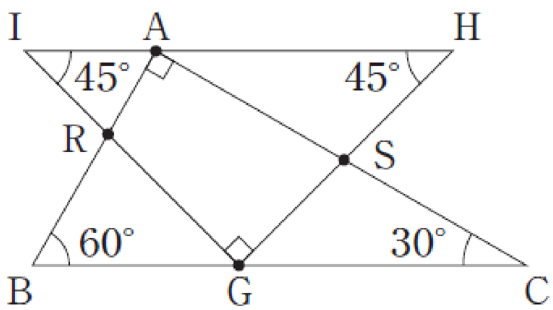
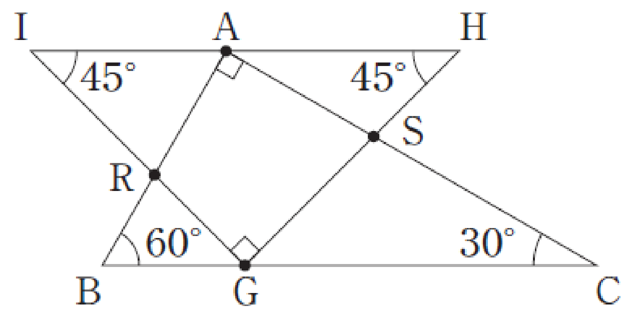


Figure 8



Two of them noticed that quadrilateral ARGS will not be a rectangle, and they are discussing what kind of quadrilateral ARGS may be.

Naoki	When we slide △GHI, the sides of quadrilateral ARGS will get shorter and longer. What about the angles?
Yui	∠ RAS and ∠ RGS are both 90°, so they do not change. What about the measures of ∠ ARG and ∠ ASG?

Even when △GHI is slid, the sum of ∠ ARG and ∠ ASG in quadrilateral ARGS will always be 180°. What else will always be the case for ∠ ARG and ∠ ASG?