[1] Answer the following questions (1) through (7).

(1) Calculate the following.

(i) $-2 \times 3 + 2$

(ii) $6\left(\frac{2}{3}a - \frac{3}{2}b\right) - (a - 3b)$

(iii) $(2\sqrt{3} - 1)^2$

(2) There is a rectangle whose length is 3cm longer than twice its width. Answer the following 2 questions.

(i) If we let the length of width as x cm, express the area of the rectangle in terms of x.

(ii) If the area of the rectangle is 7 cm^2, find the length of width.

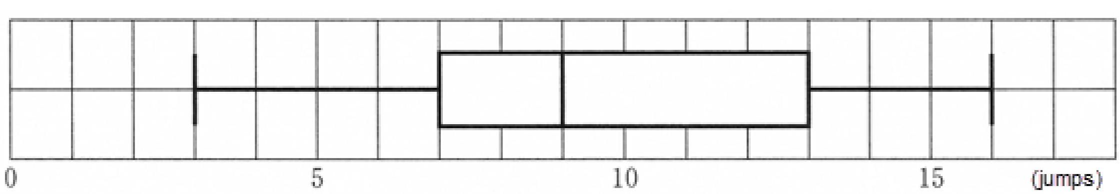
(3) At middle school A, they have a long jump competition in which classes who has the longest consecutive successful jumps will win. The table below shows the records of the consecutive successful jumps a 7th grade class recorded when they practiced.

Answer the following 2 questions.

Attempt #	1	2	3	4	5	6	7	8
# of consecutive jumps	3	11	7	12	14	7	9	16

(i) Find the median of the data.

(ii) The number of consecutive jumps in their 9th attempt was a. The graph below is the box-and-whisker plot of the data for the 9 attempts. List all possible values of a, the number of consecutive jumps in the 9th attempt.



(4) Answer the following 2 questions.

(i) How many prime numbers less than 20 are there?

(ii) We are going to roll two dice of different sizes. Let *a* be the roll on the large die and *b* be the roll on the smaller die. What is the probability that $2a + b$ will be a prime number. Assume that the dices are fair dice so that the likelihoods of rolling 1 through 6 are equally likely.

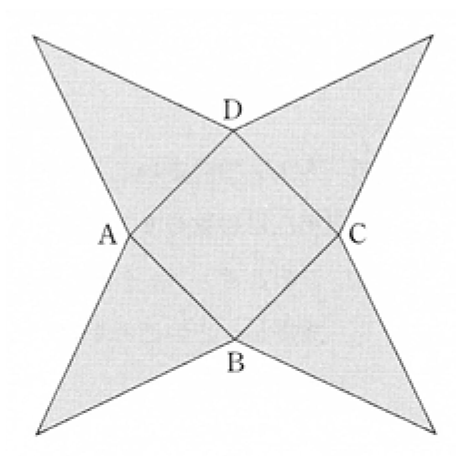
(5) If the solution for the following system of equations is $x = 2$ and $y = -1$, find the values of a and b.

$$\begin{cases} -ax + 3y = 2 \\ 2bx + ay = -1 \end{cases}$$

(6) Answer the following 2 questions.

(i) If we let *S* be the area of the base of a pyramid or a cone and *h* be its height, the volume may be calculated using the formula $V = \frac{1}{3} Sh$. Solve this equation for *h*.

(ii) The figure below is a net of a square pyramid. The length of diagonal AC is 4cm. The volume of this pyramid was calculated to be $\frac{32}{3}$ cm^3. Find the height of this pyramid.



(7) As shown below, there are 3 points, A, B, and C. Find the point P which satisfies the following conditions by construction. Label the point with "P."

You may not use the corners of set squares to draw lines. Leave all lines used in your construction.

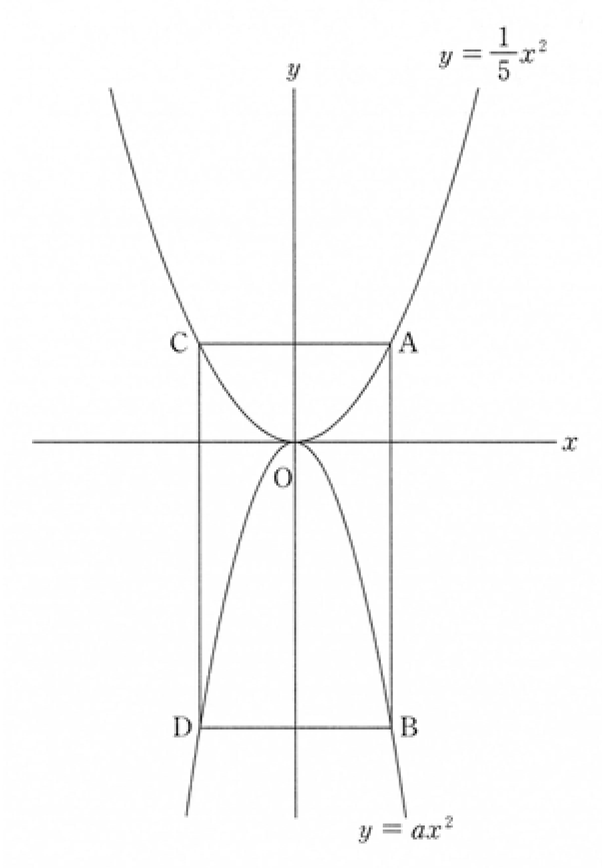
Conditions

- Point P is on the line connecting the midpoint of AC and point B.
- Lines AP and BP are perpendicular.

[2] As shown in the figure below, there is a graph of function, $y = \frac{1}{5}x^2$, and point A on the graph. The point of intersection between a line passing through point A and parallel to the *y*-axis and the graph of function, $y = ax^2$ is labeled B. The *x*-coordinate of point A is 5 and the *y*-coordinate of point B is –15. Points C and D are the reflection images of points A and B along the *y*-axis, respectively. Quadrilateral ABCD is a rectangle.

Answer the following questions (1) through (3).

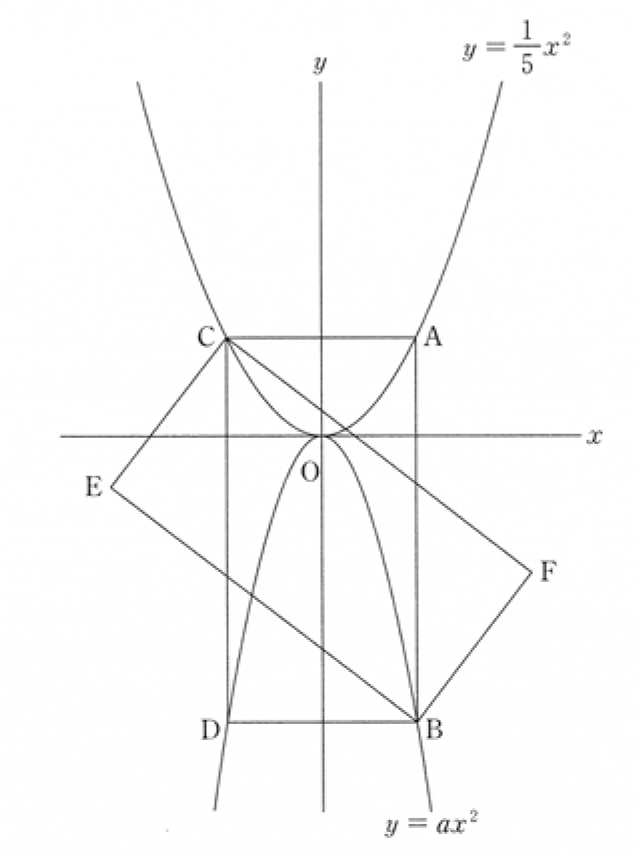
Note that $a < 0$.



(1) Find the value of *a*.

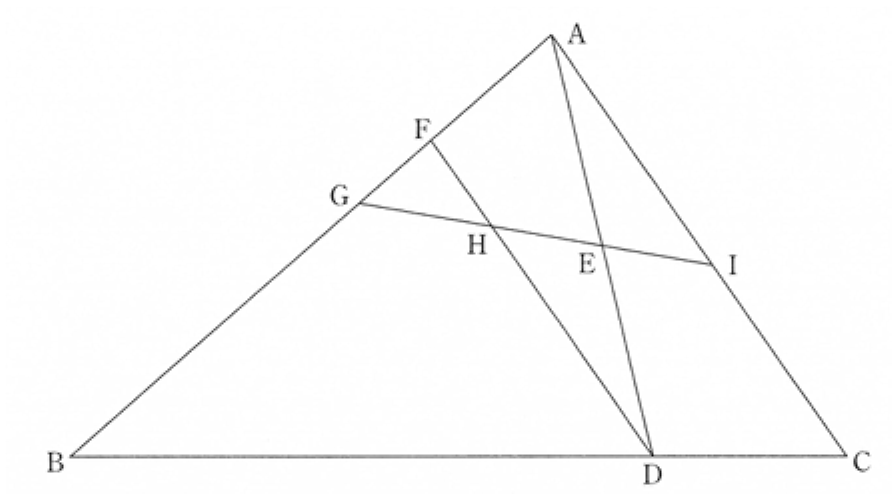
(2) Determine the equation of the line that passes through points B and C.

(3) As shown below, we drew a rectangle CEBF which is congruent to rectangle ABCD. Find the equation of the line that passes through points E and F.



[3] There is ΔABC as shown in the figure below. Let point D be on side BC such that BD : DC = 3 : 1. Let E be the midpoint of segment AD. The point of intersection between the line passing through D and parallel to side AC and side AB will be labeled as point F. Point G is on segment BF but different from points B or F. Let points H and I be the points of intersection between line GE and segments DF and side AC, respectively.

Answer the following questions (1) through (3).



(1) We are going to prove AI = DH in the box below. From A through D below, select the appropriate choice of blanks (a) and (b). Write the appropriate word/phrase that should go into blank (c).

To prove AI = DH, we need to show [(a)] and [(b)] are [(c)].

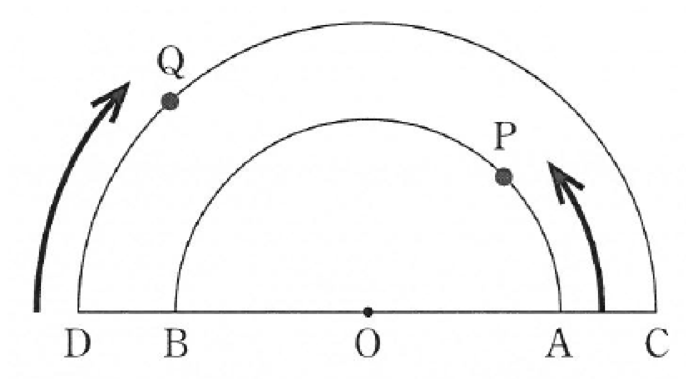
Choices

A ΔAEI	B ΔABD	C ΔAFD	D ΔDEH

(2) Prove AI = DH following the idea in question (1).

(3) If GE // BC, find the ratio of the areas of ΔAEI and quadrilateral BDHG, using the smallest positive whole numbers.

[4] As you can see on the right, there are two semicircles centered at point O with segments AB and CD as their diameters.



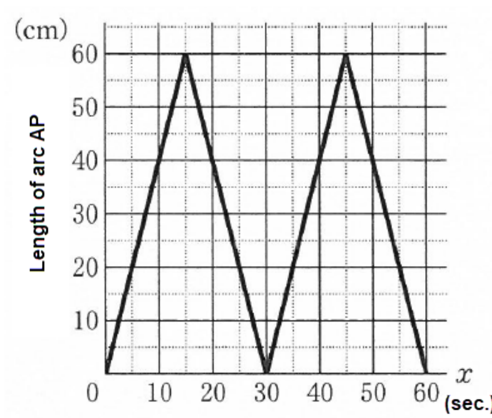
Point P starts from point A and point Q starts from point D at the same time. Point P travels along the arc AB at a constant speed starting from point A and keeps traveling A → B → A → B → A → … Likewise, point Q travels along the arc CD at a constant speed and keeps traveling D → C → D → C → D → … The length of arc AB is 60 cm and the length of arc CD is 90 cm. Also, the speed of point P is 4 cm per second and the speed for point Q is 9 cm per second.

Read the following conversation and answer questions (1) through (5).

Conversation

Teacher: Let's think about the situation where 3 points, O, P and Q are lined up in that order. Let's first think about the locations of points P and Q x seconds after they left points A and D at the same time.

Student X: I'm thinking about the movement of point P. Since the length of arc AB is 60 cm and the speed of point P is 4 cm per second, we know that it will be 15 seconds after point P left point A when P reaches point B for the first time. The relationship of the length of arc AP and x can be graphed like how it is shown below.

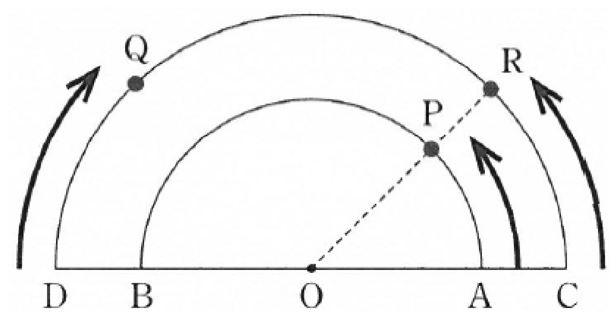


Student Y: I'm thinking about the movement of point Q. Since the length of arc CD is 90 cm and the speed of point Q is 9 cm per second, it will be [(a)] second after point Q left point D when Q reaches point C for the first time. I wonder we can observe something about the way the length of arc DQ changes if we represent it on a graph.

Student X: Even if we can figure out the way the lengths of arcs AP and DQ change, it is still difficult to figure out when these 3 points will line up as P and Q travel on different semicircles.

Conversation (cont.)

Teacher: Suppose point R is the point of intersection between line OP and arc CD, as you can see in the diagram below. Since the speed of point P on arc AB is 4 cm per second, the speed of point R on arc CD will be [(b)] per second.



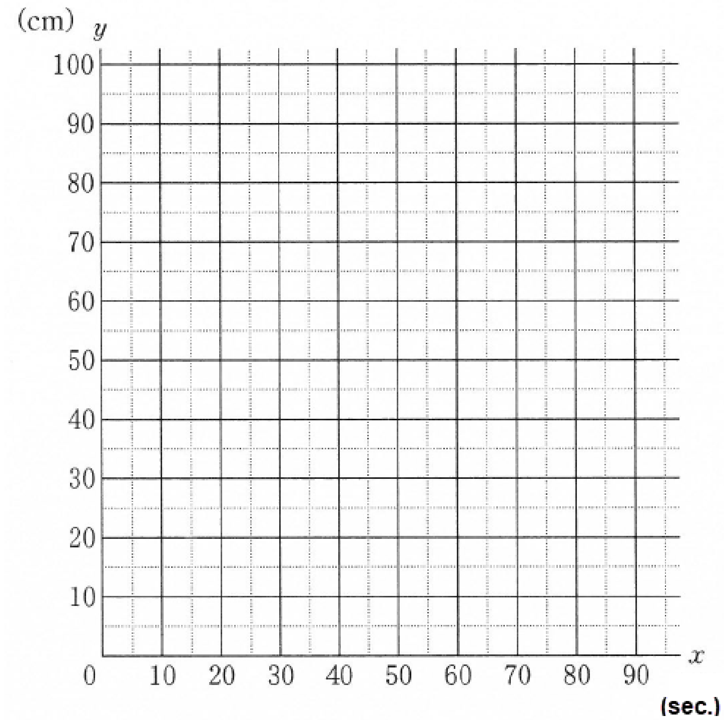
Student Y: Now we can think about 2 points Q and R on the same arc, CD, so that makes it simpler to think about. When 3 points O, P, and Q are on a line, we can say that (length of arc CR) + (length of DQ) = 90 cm.

Student X: Since (length of arc CR) = 90 – (length of arc DQ) = (length of arc CQ). So, we can think of the situation (length of arc CQ) = (length of arc CR), too. So, let's first investigate how the length of arc CQ changes. If we say the length of arc CQ x seconds after Q left point D as y cm, the relationship between x and y before point Q reaches point C for the first time can be expressed as y = 90 – 9x.

(1) Find the most appropriate numbers that should go into blanks (a) and (b) in the conversation above from A through F below.

A 4 B 6 C 8 D 10 E 12 F 14

(2) Let the length of arc CQ x seconds after point Q left point D be y cm. Graph the relationship between x and y when $0 \leq x \leq 30$.



(3) Determine how many seconds after points P and Q left points A and D, respectively, at the same time, do the 3 points O, P and Q line up for the first time?

(4) Sometime after the two points left points A and D originally, point P reached point A and point Q reached point D at the same time for the first time. Determine how many times points O, P and Q lined up until that time.

(5) Determine the size of angle POQ 144 seconds after point P left A and point Q left D.