# RATIO, RATE AND PROPORTIONAL RELATIONSHIPS IN JAPANESE CURRICULUM MATERIALS

Tad Watanabe

Kennesaw State University, USA

*Ratio, rate and proportional relationships are arguably the most important topics in middle grades mathematics curriculum before algebra. However, many teachers find these topics challenging to teach while students find them difficult to learn. In this study, Japanese curriculum materials' treatments of these topics were investigated. Both vertical and horizontal analyses (Charalambous et al., 2010) were conducted, examining when and what specific topics are discussed and how they are treated in the Japanese curriculum materials.*

## INTRODUCTION

Ratio, rate and proportional relationships are arguably the most important topics in middle grades mathematics curriculum before algebra. However, in the United States, and perhaps in other countries, both teachers and students find these topics challenging. Japanese students have consistently performed near the top of the world in many international studies. One important factor in students' opportunities to learn is the textbooks that are used. Kilpatrick, Swafford, and Findell (2001) pointed out, "what is actually taught in classrooms is strongly influenced by the available textbooks" (p. 36). Therefore, understanding how Japanese textbooks treat these topics may be of interest to mathematics educators outside of Japan. The goal of this study is to examine how one of the most widely used textbook series in Japan introduces and develops these topics.

## RATIO, RATE AND PROPORTIONAL RELATIONSHIP

In Japanese, the term *hi* corresponds to the English term ratio, and *hirei* and *hirei no kankei* corresponds to proportion and proportional relationship, respectively. However, there is no Japanese word that directly corresponds to the term rate. The Japanese word, *wariai*, can be translated into ratio, rate or even proportion, depending on the context. According to Tabata (2010), *wariai* was an everyday Japanese term that was adopted into school mathematics after World War II, in part so that it could refer to related yet distinct ideas about ratio. However, the ambiguity of the term *wariai* may actually be due to the ambiguity of the terms ratio and rate in English. There appear to be different interpretations of ratio and rate. Some consider a ratio as a multiplicative comparison of two quantities from the same measure field while a rate is a comparison of two quantities from distinct measure fields. Thus, ratio and rate are mutually exclusive. However, others consider a rate as a special ratio where two quantities being compared come from different measure spaces while a ratio is a comparison of any two quantities. Still others consider a ratio to be a comparison of two non-varying quantities while a

In Rezat, S., Fan, L., Hattermann, M., Schumacher, J., & Wuschke, H. (Eds.). (2019). *Proceedings of the Third International Conference on Mathematics Textbook Research and Development* (pp. 353–358). Paderborn: Universitätsbibliothek Paderborn.

rate represents a proportional relationship between co-varying quantities. For the purpose of this research, we considered the union of these different interpretations of ratio and rate as the Japanese curriculum materials were analysed.

## RESEARCH QUESTIONS

This content analysis study examined the following two research questions.

RQ 1: What specific topics related to ratio, rate and proportional relationships are discussed in the Japanese curriculum and in which grade, or grades?

RQ 2: How are the ideas related to ratio, rate and proportional relationships introduced and developed, both within and across grades, in the Japanese curriculum materials?

## METHODOLOGY

### Curriculum materials

There were two different data sources for the current study. One data source consists of documents published by the Japanese Ministry of Education, Culture, Sports, Science and Technology (MOE). The MOE publishes the National Course of Study (COS), which specifies what topics are to be taught in what grade level. The MOE also publishes a second document, often called *Teaching Guide*, that explains and elaborates the specific standards in the COS. The other data source was the textbook series published by Tokyo Shoseki, one of the most widely used series in the elementary and lower secondary schools. This particular series has been translated into English (Fujii & Iitaka, 2012). Thus, even though the analysis was conducted on the original Japanese version, the English version will be used whenever aspects of this textbook series are discussed.

### Data analysis

This study employs both vertical and horizontal analysis (Charalambous et al., 2010) to analyze the content of the curriculum materials. Vertical analysis, also referred to as macro analysis by others (e.g., Li, Chen, & An, 2009), examines curriculum materials to identify what mathematics is taught at what grade levels. Thus, it was the primary method to answer the first research question. In contrast, horizontal analysis, also referred to as micro analysis by others (e.g., Li, Chen, & An, 2009), focuses the analysis on a particular mathematics topic and how it is treated in the curriculum materials. In other words, horizontal analysis may identify the progression of ideas through the curriculum materials.

## RESULTS

The results of the content analysis will be presented according to the two research questions.

### RQ 1: Which ideas are discussed in what grade level(s)?

Table 1 summarizes the topics related to ratio, rate and proportional relationship discussed in the Japanese COS. Ideas related to ratio, rate and proportional

relationships are found primarily in two domains, Measurement and Quantitative Relationships. The Quantitative Relationships domain includes three sub-domains--ideas related to functions, expressions and equations, and data handling. The topics of two co-varying quantities, ratio, and proportional relationships are found in the Function sub-domain. Per unit quantity and speed are found in the Measurement domain.

| Grades | Topics |
|---|---|
| 4 | Relationships of two co-varying quantities |
| 5 | Per unit quantity (comparison of two quantities from different measure spaces) |
| | Percentage |
| | Simple proportional relationships |
| 6 | Speed |
| | Ratio |
| | Direct and inverse proportional relationships |
| 7 | Direct and inverse proportional relationships |

Table 1. Topics related to ratio, rate and proportional relationships in COS.

Table 2 lists the textbook series units and their mathematical content related to the topics identified in the COS. In the Grade 4 unit, students explore a variety of co-varying quantities, including proportional relationships. However, no explicit mention of proportional relationships occurs in the unit. The COS specifies that simple proportional relationships are to be discussed in Grade 5, but no unit focuses on that idea. Rather, the idea of proportional relationships is introduced in the unit on volume through the following problem:

> As shown on the right (accompanying figure is omitted), we are going to change the height of a cuboid from 1 cm to 2 cm, 3 cm, … without changing its length or width. Investigate how the volume changes. (Fujii & Iitaka, 2012, Grade 5, p. A20)

Students are expected to organize the results in a table showing various heights and corresponding volumes. Upon conclusion of this investigation, the textbook provides the definition of proportional relationship as follows:

> Suppose there are two quantities, □ and ○. If ○ becomes 2, 3, … times as much while □ becomes 2, 3, … times as much, we say that "○ is **proportional** to □." (Fujii & Iitaka, 2012, Grade 5, p. A20, emphasis original)

This one-page investigation is the only discussion of proportional relationships in Grade 5. However, this is consistent with how *Teaching Guide* defines a "simple" proportional relationship. *Teaching Guide* specifically states that "a simple case means

students become aware that sometimes when one quantity becomes 2, 3, 4, … times as much, the other quantity will also become 2, 3, 4, … times as much" (translated by Author).

| Grade | Title of unit | Mathematical content |
|---|---|---|
| 4 | How do quantities change? | Expressing the relationship of two co-varying quantities in equations using symbols. |
| 5 | Let's think about how to compare (1) | Arithmetic mean |
| | | Per unit quantity |
| | Let's think about how to compare (2) | Percentage |
| 6 | Let's think about how to express proportions | Ratio |
| | Let's think about how to express speed | Speed |
| | Let's investigate proportional relationships | Direct proportional relationship |
| | | Inverse proportional relationship |
| 7 | Direct and Inverse proportions | Direct proportional relationship |
| | | Inverse proportional relationship |

Table 2. Titles of units in the textbooks (Fujii & Iitaka, 2012) where topics related to ratio, rate and proportional relationships are discussed. The titles are listed in the order they appear in the textbooks.

**RQ 2: How are the ideas introduced and developed?**

Because of the space limitation, we will only share the results concerning proportional relationships. As noted above, proportional relationships are introduced in Grade 5 and further developed in Grades 6 and 7. Thus, we tried to answer the following questions: What are the emphases in each grade, and what new ideas are being introduced?

The idea of proportionality is introduced in Grade 5 as students investigate the relationship between the height and the volume of rectangular prisms with the same base. Students are asked when the height becomes 2, 3, … times as much, how the volume changes, and the definition of a proportional relationship is given as shown above. While investigating two co-varying quantities in Grade 4, the textbook asked several times how one quantity changes as the other quantity increases by 1 unit. Thus,

investigating how a quantity changes when the other quantity becomes so many times as much is a new perspective introduced in Grade 5.

In Grade 6, students are given a table showing the number of minutes and the depth of water as water is being poured into a fish tank. The textbook explicitly states that the depth of water is proportional to the number of minutes. Students are then asked to find the quotients when the depth of water is divided by the amount of time to discover that the quotients of corresponding quantities are constant. Students then conclude that if $y$ is proportional to $x$, their relationship can be expressed in an equation, $y$ = (fixed number) × $x$. Later in the unit, this relationship is contrasted with inversely proportional relationships where the products of the two corresponding quantities are constant and their relationship can be expressed in an equation, $y$ = (fixed number) ÷ $x$.

In Grade 5, students only considered what happens to the corresponding quantities when the other quantities become 2, 3, 4, … times as much; that is, only whole number multiples. In Grade 6, students discover that the same relationship holds even if the scale factors are positive decimal numbers or fractions. Students also graph proportional relationships and are expected to discover that graphs of proportional relationships will be straight lines that start on the bottom left corner of the graph paper.

In Grade 7, there seem to be two foci in the units of direct and inverse proportional relationships. The first emphasis is understanding the idea of functions, which is the topic of the first sub-unit. Students examine proportional relationships as a type of functional relationship. As students learn about functions, they are introduced to the idea of domain and range. The second emphasis in Grade 7 is to extend the range of the domain, the range and the constant of proportionality – these formal terms are introduced in Grade 7 – to the entire set of rational numbers. Students will then be able to graph proportional relationships on $x$-$y$ coordinates, showing all four quadrants. They learn that the graph of a proportional relationship is a straight line through the origin.

## DISCUSSION

The analysis shows that the Japanese curriculum treats ideas related to ratio, rate, and proportional relationships carefully and systematically. Moreover, beginning in elementary school, *Teaching Guide* clearly locates the study of proportional relationships within a broader study of functions. Although the idea of proportional relationships is discussed in four grade levels, there are new and mathematically more sophisticated expectations in every grade level.

The Japanese curriculum clearly distinguishes the multiplicative comparison of two quantities from different measure spaces from that of two quantities from the same measure space. Both types of comparisons are discussed in Grade 5, but the idea of per-unit quantity—that is, comparison of two quantities from different measure spaces—occurred in Grade 6 in the previous revision of the COS. Thus, the order of

introduction of comparison types still seems to be an open question among Japanese mathematics educators.

## LIMITATIONS AND FUTURE RESEARCH

One limitation of the current study is that only one textbook series was analysed. Although the grade level placement of a specific topic is defined by the COS, the way a particular series sequences topics within a grade level is left up to each publisher. Examining other series would surely deepen our understanding of the way the Japanese curriculum treats these important ideas. Also, *Teaching Guide* discusses the relationship between proportional relationships and operations of multiplication and division with decimal numbers and fractions. Those topics were not included in this study's analysis. Expanding the scope of analysis may give us additional insights about the treatment of proportional relationships in Japanese curriculum materials.

## References

Charalambous, Y. C., Delaney, S., Hsu, H., & Mesa, V. (2010). A comparative analysis of the addition and subtraction of fractions in textbooks from three countries. *Mathematical Thinking and Learning*, *12*, 117 – 151.

Fujii, T., & Iitaka, S. (2012). *Mathematics International*. Tokyo: Tokyo Shoseki.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Li, Y., Chen, X., & An, S. (2009). Conceptualizing and organizing content for teaching and learning in selected Chinese, Japanese and US mathematics textbooks: The case of fraction division. *ZDM Mathematics Education*, *41*, 809–826.

Tabata, T. (2010) Wariai, hi, hirei [Wariai, ratio and proportional relationships.] *Arithmetic Education, Journal of Japan Society of Mathematical Education*, *59*(6), 60–61.