

**Junior High School Teaching Guide for the Japanese Course of Study:**

**Mathematics**

**(Grade 7-9)**

中学校学習指導要領解説

数学編

日英対訳

2010年1月

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「青年海外協カ隊必携としての日本の教育情報の整備と活用」

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# Preface

For a number of years, many Japanese educators have been engaging in international cooperation with their educational experiences. English translation of ‘*The Teaching Guide of the Course of Study for Junior High School Mathematics*’(2008) as a guidebook on Japanese curriculum standards<sup>2</sup> is contained in this material along with original Japanese sentences. This was made for those who participate in international education cooperation abroad with Japanese educational background. As well as the English translation of the Elementary School Mathematics, the translation work was conducted as a part of the project “Preparing the Educational Information of Japan for Japan Overseas Cooperation Volunteers (JOCV)” (Representative: Mariko Sato / CRICED, University of Tsukuba) of International Cooperation Initiative by Ministry of Education, Culture, Sports, Science and Technology (MEXT). The following is the background and how to use of this book.

## 1) Revision of the Curriculum Standard in Japan<sup>3</sup>

In Japan, curriculum standards are determined by the Course of Study and School Education Law (Act). School Education Law defines the number of classes and subjects.

Japanese curriculum standards are designed within the frameworks designated by superior laws and councils and course of study is set by the committee for each subject established under Curriculum Council. The course of study is developed based on the information given from the academic achievement survey conducted by MEXT, lesson studies at laboratory schools, international comparative studies of students’ achievement, and researches on the results of educational studies by academic societies.

Fundamental Law of Education was amended in December 22, 2006, under the following objective of the law.

“We, the citizens of Japan, desire to further develop the democratic and cultural state we have built through our untiring efforts, and contribute to the peace of the world and the improvement of the welfare of humanity. To realize these ideals, we shall esteem individual dignity, and endeavor to bring up people who long for truth and justice, honor the public spirit, and are rich in humanity and creativity, while promoting an education which transmits tradition and aims at the creation of a new culture. We hereby enact this Act, in accordance with the spirit of the Constitution of Japan, in order to establish the foundations of education and promote an education that opens the way to our country’s future.”

Under the amended Fundamental Law of Education, School Education Law (Act) was revised on June 27, 2007, and the goal of the school education was changed as follows.

“Particular attention must be paid to achieve basic knowledge and skills, to cultivate thinking, decision making and expressing ability to solve problems by using those knowledge and skills, and to nurture the attitude to willingly pursue leaning in order to lay foundation for lifelong learning.”

In accordance with this revision of School Education Law, the new course of study for compulsory education was announced on March 20, 2008. Shift to the revised version started in April, 2009 and it will be implemented completely from April, 2011.

While the Course of Study is regarded as a legal document, MEXT has published guides for the course of study, though they are not legally binding, to explain the gist of it. ‘The Teaching Guide of the Course of Study for Junior high School Mathematics’ is one of those guidebooks.

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<sup>2</sup>‘Course of Study’ is an English translation for ‘Gakushu Shidou Youryo’ adopted in the Occupation after World War II. Today it could be regarded as Curriculum Standards.

<sup>3</sup>The revision process of Japanese curriculum standards and school curriculum is well described following articles: Isoda, M., Stephen, M. Ohyara, Y., Miyakawa, T. (2007). *Japanese Lesson Study in Mathematics*. NJ: World Scientific.

## 2) Control Methods of Curriculum Implementation on National Level by the Course of Study

Generally, curriculum has usually three meanings: Intended curriculum, Implemented curriculum and Achieved curriculum. On the other hand, in Japan, when we say curriculum in Japanese, it usually means intended curriculum, i.e. curriculum standards, because curriculum implementation is controlled by the standards. In Japanese Education System, intended curriculum is given by the national curriculum standards and each school defines their original curriculum based on the national standards. At the same time, in the textbook certification system, teacher must use textbooks on the standards. Each textbook receives the certification if 90 schools. On this context, curriculum implementation is supported by the textbook certification system. Children's achievement is evaluated according to the evaluation standards which are given by the national curriculum standards and national achievement test is also based on the national curriculum standards. In national law, at the end of each school year, children's achievement should be described in the special format given by the law based on the standards. Until assessment, curriculum implementation is also controlled by the standards in the case of Japan.

## 3) How to use this book

This book shows the composition of Guidebook for the curriculum standards in Japan and the elementary mathematics curriculum in Japan. There are roughly two ways of using this book.

The first is to use the book as a reference book to learn verbal expressions in English-speaking countries by listing parallel translation in Japanese and in English. Half of teachers sent as Japan Overseas Cooperation Volunteers (JOCV) are mathematics and science teachers, and many of them directly work on improvement of mathematics education at secondary school. They are expected for the first time in their life to work and develop activities in English. This book is useful to help those teachers to find expression when they talk about education in English. Even if they are posted in non-English speaking area, knowing English expression could be a clue to find right expression in the native tongue of the area.

Another usage is as an evidence document. The Course of Study as an evidence document is not for everyday reading even in Japan. There are a number of teachers who believe they can teach without reading curriculum standards all around the world. In Japan, on the other hand, textbooks are developed based on the curriculum standards of education. Therefore, when it comes to discussing why a theme is taught in a classroom, typical documents for explaining the reason as evidence are guidebooks for the Course of Study like this book.

At present, thousands of educators are trained within and outside of Japan by teacher trainers with background in Japanese education. On the training, it is required to explain situation in Japan precisely. When holding teacher training outside of Japan, when training trainees and exchange students in Japan, and when conducting cross-national research on education, teacher trainers often use a phrase "in Japanese cases". In some cases, personal experiences which would not gain support in Japan were transmitted to abroad as "Japanese case".

For example, JICA recognizes following characteristics in Japanese Education: "child-centered approach", "well-sequenced curriculum", and "Improvement through lesson studies". To discuss questions like "How 'child-centered approach' was described in Japanese education curriculum documents?" and "What type of education curriculum is described as 'well-sequenced curriculum'?", this translation of The Teaching Guide of the Course of Study for Junior High School Mathematics will become the foundation.

I would like to express our sincere appreciation to the team of translators without whom this project would not have been completed in such a timely fashion: Akihiko Takahashi, Ph.D., DePaul University, Tad Watanabe, Ph.D., Kennesaw State University, Thomas McDougal, DePaul University, Regan Tieff, DePaul University.

At the same time, I would like to deeply acknowledge Shigeo Yoshikawa, Inspector, Ministry of Education, Culture, Sports and Science (MEXT), Japan who gave the editor necessary information, guidance and clear suggestion for translation based on the edition of the ministry of education, Japan, Nagata Junichiro, Investigator, MEXT and Mari Usami & Masaru Sanuki, CRICED, University of Tsukuba who support this translation through the back translation to Japanese from English translation and compared it with the original Japanese document.

Without their contributions, suggestions and supports, I could not complete this work.

24, January, 2010.

Masami Isoda

Editor of the English translation of

*The Guide of the Course of Study for Junior High School MATHEMATICS*

with the translation on the opposite page



## Contents

Chapter 1. General Overview	1
1. Background of this Revision	1
2 Aims of reforms in mathematics	3
3. Main Points of the Revision for Mathematics	6
Chapter 2. Objectives and Content of Mathematics	15
Section 1: Objectives	15
1. Objectives of Mathematics	15
2. Objectives for each grade	19
Section 2: Content	25
1. Approaches to Content Organization	25
2. Overview of the contents in Each Domain	33
A. Numbers and Algebraic Expressions	33
B. Geometrical Figures	38
C. Functions	43
D. Making Use of Data	48
Section 3: Contents of Each Grade	54
1. Grade 1 of Lower Secondary School	54
2. Grade 2	82
3. Grade 3	100
Chapter 3 Syllabus Design and Handling the Content	124
1. Consideration Points for Designing a Syllabus	124
2. Consideration Points for Handling the Content in Chapter2	126

# Chapter 1. General Preview

## 1. Background of this Revision

The 21st century is the so-called, “era of the knowledge-based society” where new knowledge, information, and technology have become dramatically important in various aspects of life, such as politics, the economy, and culture.

This process of moving into a knowledge-based society and globalization will intensify international competition for ideas, knowledge, human resources, and at the same time will emphasize the importance of the coexistence and global cooperation among different cultures and civilizations of the world.

In this atmosphere, it will become more and more important to nurture a “zest for living” that emphasizes scholastic ability, a rich heart and mind, and the harmony of a healthy body. On the other hand, various studies such as the Programme for International Student Assessment (PISA) study done by the Organization for Economic Co-operation and Development (OECD) indicate the following about students in Japan:

- ① a difficulty in reading comprehension and in writing problems that require thinking, decision-making, and expressing;
- ② a problem in widening the distribution of scores in reading comprehension, which indicates problems at home where not enough time is spent studying, a lack of motivation to learn, as well as a poor study and living environment;
- ③ a lack of confidence in their abilities; feelings of insecurity about the future; and lowered physical strength

Consequently, to enrich the education for students who will live in the 21st century, in February 2005 the Ministry of Education, Culture, Sports, Science and Technology-Japan (MEXT) requested the Central Education Council reform the standards for the curriculum, and also to improve the quality and skill of teachers and the infrastructure of education. In April of the same year, the discussion on these subjects commenced. During this period, the Fundamental Law of Education and the School Education Law were amended, the balance between knowledge, morality, and health was reaffirmed (the Fundamental Law of Education, Article 2,1), as was the importance of fundamental and basic knowledge and skills, of the ability to think, to make decisions, to express, and of the motivation for learning (School Education Law, Article 30,2). For school education, it is now stated in the law that it is necessary to promote these objectives in a harmonious way. Based on the amendment of the laws, these fundamental issues in education were discussed in the Central Education Council. The discussion lasted 2 years and 10 months, and in January 2008 the report, *The Reform of the Curriculum for Kindergarten, Elementary Schools, Lower Secondary Schools, Upper Secondary Schools and Special-Needs Schools*, was submitted.

Based on the problems our students are facing, this report gives the following points that were considered as the basis for reform as well as directions for each school level and subject:

- ① Reform of the curriculum based on the amendment of the Fundamental Law of Education
- ② Establishing a shared concept of the “zest for life”
- ③ Acquiring fundamental and basic knowledge and skills
- ④ Fostering the ability to think, to make decisions, and to express
- ⑤ Allocating the necessary hours of classes to acquire solid academic ability
- ⑥ Motivating students to learn and helping them develop sound study habits
- ⑦ Enriching teaching for fostering rich minds and good physical health.

Specifically regarding ①, in order to develop Japanese people who are courageous and rich in spirit who can lead the world of the 21st century, the Fundamental Law of Education was amended for the first time in approximately 60 years, and new ideals for future education were established. In the School Education Law amended by the Fundamental Law of Education, the new objectives of compulsory education were ordained, and reform of the curriculum is required to be fully grounded in the revision of the objectives of each school level.

In ③, fundamental and basic skills such as reading, writing, and calculation are to be completely acquired, so that they can form the foundation for future learning as appropriate to each developmental stage. For example, in grades 1 through 4, the emphasis is given on understanding obtained through experience and repeated practice.

Building on this foundation, in order to foster the ability to think, make decisions, and to express as described in ④; learning activities, such as experiments, observations, and writing reports and essays, in which knowledge and skills are utilized are enhanced according to each developmental stage. In addition, in order to develop language ability to support these activities, it was pointed out that in each subject it is necessary to acquire basic Japanese language skills — reading out loud, reading silently, reading and writing kanji characters — followed by learning activities such as recording, summarizing, describing, and writing essays.

In order to foster a rich mind and good physical health as described in ⑦, in addition to enriching character education and physical education classes, by emphasizing language skills in Japanese language class and other classes and by nurturing learning through experience, it was suggested that it is necessary to let students gain confidence in life by interacting with others, with society, nature, and the environment.

Based on the report of 3/28/2008, the School Education Law Enforcement Regulation was amended, and educational guidelines for kindergarten and elementary and lower secondary schools were made public. Educational guidelines for elementary schools will be implemented ahead of schedule for courses such as mathematics and science from 4/1/2009, and will be fully implemented by 4/1/2011.



## 2. Aims of reforms in mathematics

As noted in section 1, the Central Education Council, in its January 2008 report, “The Reform of the Curriculum for Kindergarten, Elementary Schools, Lower Secondary Schools, Upper Secondary Schools and Special-Needs Schools”, indicated the basic ideas for revising the Course of Study, the framework of curriculum, and the main points of reform in educational contents; the report also summarized the main points of reform for each subject. The revision of the mathematics curriculum is based on those recommendations.

### A. Main reform policies

The Central Education Council report indicates the main reform policies for elementary and lower and upper secondary school mathematics as follows:

- (a) In mathematics, considering its challenges, mathematical activities must be further enriched so that students will acquire fundamental and basic knowledge and skills, develop the ability to think and express mathematically, and increase their motivation to learn throughout elementary, lower secondary, and upper secondary school, according to their developmental stage.
- (b) The fundamental and basic knowledge and skills of numbers, quantities, and geometrical figures are the foundation for daily living and learning. With the advancement of scientific technology, it is actively being discussed more than ever that mathematics and science education should meet international competitive levels. Consequently, to firmly establish the fundamental and basic knowledge and skills of numbers, quantities, and geometrical figures in students, while retaining the importance of a systematic nature of mathematics, the curriculum may adopt repeated learning (spiral) according to developmental stages and grade levels of students by overlapping some of the content across grades.
- (c) Mathematical thinking and expression play an important role in rational and logical thinking as well as in intellectual communication. For this reason, instructional content and activities that foster mathematical thinking and expression should be clearly indicated. We will enrich the kind of teaching where students are taught to think systematically, in logical steps, by reasoning, and to understand the connections among words, numbers, algebraic expressions, figures, tables, and graphs. This kind of teaching will also allow students to learn appropriate usage, problem solving, how to explain one’s ideas clearly, and how to express and communicate one’s ideas to others.
- (d) It is important that we motivate students to learn mathematics, and students should experience the meaning of learning and utilize what was learned. For this reason, the following objectives are emphasized: Help students understand numbers, quantities, and geometrical figures through learning activities which serve as a basis for understanding their meanings.
  - Help students feel progress in learning, such as a depth and broadening of understanding through repeated learning (spiral) that is designed according to the developmental stage and grade level of each student.
  - Help students apply what has been learned to activities in daily life, to the study of other subjects, and to learning more advanced mathematics.
  - Help students feel progress in learning, such as a depth and broadening of understanding through repeated
- (e) Mathematical activities play an important role in helping students acquire fundamental and basic knowledge and skills, in increasing students’ ability to think and express mathematically, and in enabling students to feel a joy and purpose in learning mathematics. To enrich the teaching of mathematics through mathematical activities — with experiential activities and an emphasis on language — concrete examples of mathematical activities should be provided in the curriculum for elementary and lower secondary school; and in upper secondary school, project -based learning is introduced in the required subjects and in more popular elective courses.

In (a), the basic policies for the reform for elementary and lower and upper secondary schools are shown. These policies show how mathematics curriculum will respond to the content of School Education Law, Article 30, 2 (Article 49 for lower secondary schools and Article 62 for upper secondary schools), “To develop an attitude of actively engage in learning activities as well as the ability necessary for problem solving such as the ability to think, to make decisions, and to express, while mastering the basic knowledge and skills”.

In (b) through (e), specific responses are discussed. Incidentally, “the challenges” mentioned in (a) refers to the challenges identified by the surveys of the curriculum implementation and international achievement studies; for example, “as for the mastery of basic calculation skills, although the achievement levels have not declined, understanding of the meaning of calculation was a challenge”, and “it is apparent that students weren’t able to fully utilize learned knowledge and skills in their everyday situations and their further study”.

Item (b) states the importance of the mastery of basic knowledge and skills about numbers, quantities, and geometric figures for the advancement of scientific technology and international competitiveness and sets the direction for reform.

Item (c) states the importance of the ability to think, make decisions and express ideas in mathematics in order to promote logical thinking and intellectual communication and it sets the direction for reform from those perspectives.

Item (e) emphasizes understanding with experiences, sensing the broadening and deepening of one’s own learning, using what has been learned, and it states the necessity of increasing the motivation to learn and experiencing the values and usefulness of learning.

Item (e) states that concrete examples of mathematical activities must be indicated in the grade level content of the curriculum so that mathematics teaching is further enhanced by the use of mathematical, experiential, and linguistic activities.

## B. Specific points for reform

The Central Education Council report indicates the following specific points for reform.

In lower secondary school, the following points of reform are emphasized so that students will engage actively in mathematical activities, master fundamental and basic knowledge and skills, develop the ability to think mathematically, know the value of mathematics, and deepen their understanding of relationships between mathematics and scientific technology:

- (a) The content domains will be changed from the current three domains, “Numbers and Algebraic Expressions”, “Geometrical Figures”, and “Quantitative Relations”: to a new domain related to probability and statistics called “Making Use of Data”, while the domain, “Quantitative Relations” will be changed to “Functions”. There will be 4 content domains, “Numbers and Algebraic Expressions”, “Geometrical Figures”, “Functions”, and “Making Use of Data”.
- (b) In order to respond to students’ struggles, a sufficient amount of time will be allocated so that careful and detailed instruction can be provided. Moreover, an emphasis is placed on establishing opportunities to review what has been learned as new content is introduced.
- (c) In order to further emphasize mathematical activities, a section will be created to discuss mathematical activities within the discussion of the content of each grade. Special attention must be given to the articulation between elementary and lower secondary school.

For example, concrete examples of mathematical activities such as those that produce mathematics, utilize mathematics, communicate mathematically, and that experience mathematics directly.

Moreover, project study discussed in the current curriculum is positioned as an opportunity to realize mathematical activities; instruction should provide such opportunity as solving problems by integrating what students have learned so far.

- (d) In the domain, “Numbers and Algebraic Expressions”, the following will be emphasized: to deepen students’ understanding of the need for and the value of thinking generally using literal variables, to represent quantities in the surrounding and their relationships using numbers and algebraic expressions with letters, to process algebraic expressions efficiently by using appropriate procedures, and to explain the meaning of algebraic expressions by interpreting them.

For example, ideas such as expressing the size relationships of quantities using inequalities, proportions, the concepts and the terms of rational and irrational numbers, and the quadratic formula will be taught.

- (e) In the domain, “Geometrical Figures”, the following will be emphasized: to base empirical understanding on experiences, to clarify the properties and relationships of objects in the surroundings by considering them as geometrical figures, to logically explain properties of geometrical figures by clearly stating the reasons, and to identify new relationships from such explanations.

For example, ideas such as geometric transformations, projections, the surface area and volume of spheres, and the ratios of area and volume of geometrical figures will be taught.

- (f) In the domain, “Functions”, the following will be emphasized: to consider phenomena in the surroundings as functions and to represent the way they change and correspond by using tables, algebraic expressions and graphs, to identify new relationships and features from tables, algebraic expressions and graphs, and to interpret what such relationships and features may mean in actual situations.

For example, in order to understand direct and indirect proportion relationships based on the concept of functions, the concept and the term, “function” will be taught starting in the first year. Moreover, other phenomena and functions will be taught.

- (g) In the domain, “Making Use of Data”, the emphasis is on capturing trends and features of populations based on data and making decisions based on such analysis.

For example, in addition to probability, which has been taught in lower secondary school, ideas such as identifying trends of a population by using histograms and representative values and by examining samples from a population.

### **3. Main points of revision for mathematics**

#### **A. Improving the goals of lower secondary school mathematics**

The goal of mathematics instruction in lower secondary school is not just enabling students to solve given problems. As stated above, we need to seek a balance among helping students master fundamental and basic knowledge and skills; nurturing the abilities necessary for problem solving such as the ability to think, to make decisions, and to express; and developing the attitude to actively engage in the study of mathematics. To achieve this balance, the following three points of improvement were identified.

##### **① To experience the joy of mathematical activities and the values of mathematics**

In order for students to actively engage in the study of mathematics, they must experience the joy of mathematical activities and the values of mathematics. To do so, it is important that instruction is done through the use of mathematical activities. By including the statements, “through mathematical activities” and “experiencing the joy of mathematical activities and the values of mathematics”, this point was made explicit.

As for “the joy of mathematical activities”, as it has been the case, we must pay attention not only to simply enjoying activities but also to the qualitative aspect, that is how intellectual growth can be brought to students.

The primary purpose of experiencing “the values of mathematics” is to motivate students to actively engage in the study of mathematics. “The values of mathematics” include, for example, the values of mathematical representations and procedures such as “if we express relationships of quantities in equations, we can solve them by transforming them procedurally”.

Other values of mathematics include values of fundamental concepts and properties/laws about quantities and geometrical figures, and values of mathematical ways of observing and thinking. Furthermore, the fact that mathematics may be useful in daily life and the knowledge that mathematics and scientific technologies have developed as they support each other are also “values of mathematics”.

##### **② To increase the ability to examine and represent phenomena mathematically**

To examine phenomena mathematically involves both phenomena in daily life society and phenomena in the world of mathematics. It is necessary to increase the ability to examine phenomena mathematically by each of their characteristics.

The understanding of the processes and the results of mathematical examination of phenomena may be deepened by representing them. By including the new phrase, “to increase the ability to represent”, the importance of representing the properties of quantities and geometrical figures appropriately, giving logical explanations with clear rationale, communicating one’s own ideas and reasoning to share and improve qualitatively is made clear.

##### **③ To nurture the attitude to think and make decisions by using mathematics**

Nurturing the attitude to use mathematics leads to the active engagement in the study of mathematics. By including the new phrase, “the attitude to think and make decisions using mathematics”, the importance of clarifying the purpose of the use of mathematics, providing opportunities for students to think and make decisions using mathematics, and helping them understand the usefulness of mathematics experientially is made clear.

## **B. Improvement of the content of lower secondary school mathematics**

In lower secondary school mathematics, to address the recommendations made by the Central Education Committee, changes have been made to not only the content but also in the way they are stated. Those changes are explained below.

### **① About the structure of the content domains and mathematical activities**

The content domains have been increased from three to the following four: “Numbers and Algebraic Expressions”, “Geometrical Figures”, “Functions”, and “Making Use of Data”. In addition, mathematical activities are discussed in the discussion of content section in each grade. Mathematical activities discussed will include activities used to identify and extend properties of numbers and geometrical figures based on what have been learned, activities to use mathematics in everyday life and in society, and activities to explain one’s own ideas logically by utilizing mathematical expressions and by clearly expressing the rationale. Moreover, some points of considerations for instruction of mathematical activities are indicated in “III. THE CONSTRUCTION OF TEACHING PLANS AND HANDLING THE CONTENT”.

The summary of content of lower secondary school mathematics for each grade and for each content domain is shown in the table 1 (pp. 10, 11 & 12). Table 2 (13 & 14) shows the content of elementary school mathematics.

### **② About specific content**

In order to master the foundational and basic knowledge and skills and to develop the ability to think, make decisions, and express, students should use what they learned in elementary school. For this reason, also by considering the international competitiveness at the compulsory education level, the grade level placements of some content have been changed.

The table below shows the materials that have been shifted across the levels of elementary, lower and upper secondary school, the materials that have been shifted across grades within lower secondary school, and the materials that have been newly included in lower secondary school.

## Shifting of content materials in lower secondary school mathematics

	<ul style="list-style-type: none"> <li>● Sets of numbers and possibilities of four arithmetic operations (from Mathematics I for Upper Secondary School)</li> <li>● Expressing the size relationships using inequalities (from Mathematics I for Upper Secondary School, partially)</li> <li>◎ Solving simple proportions</li> <li>◎ Translation, reflection, and rotation</li> <li>◎ Projection</li> <li>● Surface area and volume of spheres (from Mathematics I for Upper Secondary School)</li> <li>○ Meaning of functional relationships (from Grade 2 of Lower Secondary School)</li> <li>● Distribution of data and representative values (from “Fundamental Mathematics” and “Mathematics B” for Upper Secondary School)</li> <li>◆ Symmetry of geometrical figures (line symmetry and point symmetry) (to Grade 6 of Elementary School)</li> <li>◆ Volume of prisms and cylinders (to Grade 6 of Elementary School)</li> </ul>
	<ul style="list-style-type: none"> <li>○ The relationship between inscribed angles and central angles (to Grade 3 of Lower Secondary School)</li> <li>◆ To organize all possible events systematically (to Grade 6 of Elementary School)</li> </ul>
	<ul style="list-style-type: none"> <li>● Rational and irrational numbers (from Mathematics I for Upper Secondary School)</li> <li>● Quadratic formula (from Mathematics I for Upper Secondary School)</li> <li>● Ratios of area and volume of similar figures (from Mathematics I for Upper Secondary School)The relationship between inscribed angles and central angles (From Mathematics A for Upper Secondary School, partially)</li> <li>● Various phenomena and functions (from Mathematics I for Upper Secondary School)</li> <li>● Sampling (from “Fundamental Mathematics” and Mathematics C for Upper Secondary School)</li> </ul>

Note: ●...Content that is being moved from upper secondary school to lower secondary school

○...Content that is being moved across grades within lower secondary school

◎...New content to be taught in lower secondary school

◆...Content that is being moved from lower secondary school to elementary school

Those materials, such as algebraic expressions with letters, enlarged and reduced drawing, and inverse proportional relationships, that will be discussed both in elementary and lower secondary school as a part of a spiral curriculum, are not listed here. They are discussed in Chapter 2.

### ③ About the discussion of content

instruction, improvements were made, for example, by re-evaluating the grouping of topics, so that the balance between the mastery of fundamental and basic knowledge and skills and the development of the abilities to think, make decisions, and to express is realized. Furthermore, in order to clarify the intention of gradually raising students’ ability through coherent instruction in each content domain across grades, the expressions, “to cultivate → to nurture → to extend”, are used. As for the content to be learned, the phrases, “students should know...” and “students should understand...” are used. This revision emphasizes the importance of students’ use of previously learned knowledge and skills as they think and make decisions. The word “use” is being employed in all situations where appropriate, but in everyday situations and social phenomena, the word “utilize” is used to distinguish those situations to clarify the purpose of instruction. Moreover, if it is necessary to clarify ways of use, the phrase, “to ... by using” is used.

## **C. Improvements related to “THE CONSTRUCTION OF TEACHING PLANS AND HANDLING THE CONTENT”**

### **① Establishing opportunities to re-learn**

When it may be effective to deepen or broaden students’ understanding by intentionally treating those ideas that have already been taught previously as a new content is being introduced, opportunities to re-learn those ideas are to be established.

### **② Further enrichment of mathematical activities**

As mathematical activities are included in the discussion of the content of each grade level, the following types of opportunities are established as points of consideration for instruction.

- To enjoy mathematical activities and to experience the purpose and the necessity of studying mathematics
- To engage in mathematical activities with good perspectives and to reflect on them
- To share the results of mathematical activities

### **③ The role of project study**

Project study is to be emphasized as before. In this revision, project study is considered as the opportunity to solve problems identified by, for example, integrating the content of each content domain. Furthermore, project study is to be included appropriately in teaching plans as they promote students’ engagement in mathematical activities.

	A. Numbers and Algebraic expressions	B. Geometrical figures	C. Functions	D. Data Handling	Mathematical activities
Grade 7	<p><b>Positive numbers, Negative numbers</b>  a. necessity and meaning of positive and negative numbers [“Set of numbers and four fundamental operations”]  b. Meaning of four basic operations with positive and negative numbers  c. Four basic operations with positive and negative numbers  d. Using positive and negative numbers</p> <p><b>Algebraic expressions using letters</b>  a. Necessity and meaning of using letters  b. how to express multiplication and division  c. Addition and subtraction with linear expressions  d. representing with algebraic expressions with letters [“representations with inequality”]</p> <p><b>Linear equations with one unknown</b>  a. Necessity and meaning of equations and the meaning of their solutions  b. Properties of equations and how to solve them  c. Solving and using linear equations [“Proportional expressions”]</p>	<p><b>Plane figures</b>  a. Fundamental methods for constructing figures and their application  b.[ moving basic figures]</p> <p><b>Space figures</b>  a. Positional relationship between straight lines and planes  b. structure of space figures and their representation on a plane [“ projection drawing” ]  c. measurements in basic figures [“ surface area and volume of sphere” ]</p>	<p><b>Direct proportion, Inverse proportion</b>  a. Meaning of functional relationships (moved from the eighth grade)  b. Meaning of direct proportion and inverse proportion  c. Meaning of coordinates  d. tables, algebraic expressions, and graphs of direct proportion and inverse proportion  e. applying direct proportion and inverse proportion</p>	<p><b>Dispersion of data and representative values of data</b> (approximations and errors; using the notation <math>a \times 10^n</math>)  a.[ Necessity and meaning of histograms and representative values]  b.[ applying histograms and representative values]</p>	<p>In learning content within and across domains, opportunities to do mathematical activities such as shown below should be implemented.  a. Activities for finding out the properties of numbers and geometrical figures based on previously learned mathematics.  b. Activities for making use of mathematics in daily life  c. Activities for explaining and communicating each other in one’s own way by using mathematical representations</p>



	A. Numbers and Algebraic expressions	B. Geometrical figures	C. Functions	D. Data Handling	Mathematical activities
Grade 8	<p><b>Calculation of the four basic operations with expressions using letters</b></p> <p>a. Calculation of addition and subtraction with simple polynomials, as well as multiplication and division with monomials</p> <p>b. representing and interpreting algebraic expressions using letters.</p> <p>c. Transforming algebraic expressions according to the purpose</p> <p><b>Simultaneous linear equations with two unknowns</b></p> <p>a. Necessity and meaning of linear equations with two unknowns and the meaning their solutions</p> <p>b. Meaning of simultaneous linear equations with two unknowns and the meaning of their solutions</p> <p>c. solving simultaneous equations and applying them</p>	<p><b>Basic plane figures and properties of parallel lines</b></p> <p>a. properties of parallel lines and angles</p> <p>b. properties of angles of polygons</p> <p><b>Congruence of plane figures</b></p> <p>a. Congruence of plane figures, conditions for congruence of triangles</p> <p>b. Necessity, meaning and methods of proof</p> <p>c. basic properties of triangles and parallelograms</p>	<p><b>Linear functions</b></p> <p>a. phenomena and linear functions</p> <p>b. tables, algebraic expressions and graphs of linear functions</p> <p>c. linear equations with two unknowns and functions</p> <p>d. Using linear functions</p>	<p><b>Probability</b></p> <p>a Necessity and meaning of probability and finding the probability</p> <p>b. Using probabilities</p>	<p>In learning content within and across domains, opportunities to do mathematical activities such as shown below should be implemented.</p> <p>a. Activities for finding out and developing the properties of numbers and geometrical figures based on previously learned mathematics.</p> <p>b. Activities for making use of mathematics in daily life and society</p> <p>c. Activities for explaining and communicating each other in an evidenced, coherent and logical manner by using mathematical representations</p>

	A. Numbers and Algebraic expressions	B. Geometrical figures	C. Functions	D. Data Handling	Mathematical activities
Grade 9	<p><b>Square root</b> a. Necessity and meaning of square roots [“rational numbers, irrational numbers”] b. calculations of expressions with square roots c. using square roots</p> <p><b>Expanding and factoring algebraic expressions</b> a. multiplication and division of polynomials and monomials b. expanding and factoring of simple expressions c. representing and explaining through expressions using letters</p> <p><b>Quadratic equations</b> a. Necessity and meaning of quadratic equations and the meaning of their solutions b. solving quadratic equations by factoring and completing squares c.[ Solving quadratic equations by using the quadratic formula] d. Using quadratic equations</p>	<p><b>Similarity of plane figures</b> a. Similarity of plane figures and conditions for similar triangles b. Basic properties of geometrical figures c. Parallel lines and ratios of line segments. d.[ Relationships between the scale factor, the ratio of areas and the ratio of volumes of similar geometric figures] e. Using the properties of similar geometrical figures</p> <p>Inscribed angle and central angle a. Relationship between the inscribed angle and the central angles, and its proof (moved from eighth grade) [“converse of the Central Angle Theorem”] b Using the relationship between an inscribed angle and a central angle (moved from the seventh grade)</p> <p><b>Pythagorean Theorem</b> a. Pythagorean Theorem and its proof b. Using the Pythagorean Theorem</p>	<p><b>Function <math>y = ax^2</math></b> a. Phenomena and the function <math>y = ax^2</math> b. Tables, algebraic expressions, and graphs of the function <math>y = ax^2</math> c. Using the function <math>y = ax^2</math> d. [Various concrete phenomena and functional relationships]</p>	<p><b>Sample survey</b> [a. Necessity and meaning of a sample survey] [b. Carrying our sample surveys.]</p>	

	<b>A. Numbers and Calculations</b>	<b>B. Quantities and Measurements</b>	<b>C. Geometrical figures</b>	<b>D. Mathematical Relations</b>	<b>Mathematical activities</b>
Grade 1	<b>Meaning and Representation of integers</b> · 2-digit numbers and simple 3-digit numbers <b>Addition and subtraction of integers</b> Addition and subtraction of simple 2-digit numbers	<b>Comparing sizes of quantities</b> · comparing length, comparing [area and volume] <b>Reading clock times</b> (moved from grade 2)	<b>Geometrical figures</b> · observing and composing the shapes of familiar objects (plane figures), solid figures)	<b>Representation with algebraic expressions</b> · representing situations where addition and subtraction are used by using algebraic expressions (moved from “A. Numbers and Calculations”) <b>Representing the number of objects using pictures or figures</b>	a. Activities to count concrete objects b. Activities to express the meaning and ways of calculation c. Activities to compare quantities d. Activities to find and compose various shapes e. Activities to express situations with algebraic expressions
Grade 2	<b>Representation of numbers such as integers</b> · 3-digit numbers, 4-digit numbers, 10000, simple fractions (1/2, 1/4 etc.), etc. Addition and subtraction of integers · Addition and subtraction of 2-digit numbers and addition and subtraction of simple 3-digit numbers <b>Multiplication of integers</b> · multiplication table (ku-ku) [up to $9 \times 9$ ] <u>· multiplication of simple 2-digit numbers</u>	<b>Units and measurements of quantities</b> · units of length (mm, cm, m) · units of volume, (ml, dl, l) (moved from grade 3)  <u><b>Units of time: (days, hours, minutes)</b> (moved from grade 3)</u>	<b>Geometrical figures</b> · triangles, quadrilateral · squares, rectangles, right triangles (moved from grade 3) · shape of a box (moved from grade 3)	<b>Representation with algebraic expressions</b> · relationship between addition and subtraction (moved from “A. Numbers and Calculations”) · representing situations where multiplication is used by using algebraic expressions (moved from “A. Numbers and Calculations”) <b>Simple tables and graphs</b> (moved from “A. Numbers and Calculations”)	a. Activities to find situations where integers are used b. Activities to find rules from multiplication tables. c. Activities to estimate the sizes of quantities d. Activities to draw and construct geometrical figures. e. Activities to express relationships by using figures, diagrams and algebraic expressions
Grade 3	<b>Representation of integers</b> · the unit of ten-thousands (man in Japanese), 100 million (oku in Japanese) Addition and subtraction of integers · addition and subtraction of 3-digit numbers and [4-digit numbers] Multiplication of integers · multiplication of 2-digit numbers and 3-digit numbers, (for example, [3-digit number $\times$ 2-digit number]) Division of integers · division in simple cases when the divisors are 1-digit numbers ( the quotients are 1-digit or 2-digit numbers) Decimal numbers (moved from grade 4) · The meaning and the representation of decimal numbers, addition and subtraction of decimal numbers (the tenths place) Fractions (moved from grade 4 and 5) · The meaning and the representation of fractions, simple addition and subtraction of fractions Soroban (Japanese Abacus) · The representations of numbers on soroban addition and subtraction	<b>Various units and measurements</b> · Units of length (km) and weight (g, kg, [ton]) Measurement using instruments Units of time(seconds), calculations with time	<b>Geometrical figures</b> · isosceles triangle, equilateral triangle (moved from grade 4) · angle (moved from ) · circle, sphere (moved from grade 4)	<b>Representation with algebraic expressions</b> · representing situations where divisions are used, by using algebraic expressions (moved from “A. Numbers and Calculations”) · [making connections between algebraic expressions and diagrams; algebraic expressions that use □] Tables and bar graphs	a. Activities to explore and explain ways of calculation b. Activities to compare sizes of decimal numbers and fractions c. Activities to investigate the relationships among the units d. Activities to construct geometrical figures such as equilateral triangles e. Activities to organize data and to represent the data in tables

	<b>A. Numbers and Calculations</b>	<b>B. Quantities and Measurements</b>	<b>C. Geometrical figures</b>	<b>D. Mathematical Relations</b>	<b>Mathematical activities</b>
Grade 4	<p><b>Representation of integers</b> The units such as hundred million (oku in Japanese) and trillion (cho in Japanese), Round numbers · round numbers, rounding, estimate the results of four basic calculation (moved from grades 5 and 6)</p> <p><b>Division of integers</b> · division in the cases where the divisor is a 2-digit number. Acquisition and utilization of the four basic operations with integers</p> <p><b>Calculation of decimal numbers</b> · addition and subtraction of decimal numbers (the tenths and hundredths places, etc.) · multiplication and division of decimal numbers (decimal number <math>\times</math> integer, decimal number <math>\div</math> integer) (moved from grade 5) Calculation of fractions · addition and subtraction of fractions with like denominators (proper fraction, [improper fraction]), etc. (moved from grade 5) Japanese Abacus · addition and subtraction</p>	<p><b>Area</b> · Units of area (<math>\text{cm}^2</math>, <math>\text{m}^2</math>, <math>\text{km}^2</math>, [a, ha] and measurements · finding areas of squares and rectangles Units of angles (degree (<math>^\circ</math>))</p>	<p><b>Geometrical figures</b> · relationships of parallelism and perpendicularity of lines (moved from grade 5) · parallelogram, rhombus, trapezoid (moved from grade 5) · cube, rectangular parallelepiped (moved from grade 6) · [representing the location of an object]</p>	<p><b>relationships between two numbers/quantities as they vary simultaneously</b> · represent how the numbers/quantities vary on a broken-line graph and to interpret the features of their variation Representation in algebraic expressions · algebraic expressions that contain some of the four basic operations and, expressions with (<math>()</math>), formulas · [expressions with <math>\square</math>, <math>\triangle</math>] Properties of the four basic operations (moved from grade 5) Gathering and organizing data · table with two viewpoints, broken-line graphs</p>	<p>a. Activities to estimate the result of calculations and to make proper decisions b. Activities to explore and explain ways to determine the area of geometrical figures c. Activities to actually measure the area d. Activities to investigate features of geometric figures, such as parallelograms, by tessellation. e. Activities to investigate how quantities in everyday life relate to each other</p>
Grade 5	<p><b>Properties of integers</b> · even and odd numbers, divisors and multiples (moved from the six grade), [prime numbers] Numeration system for integers and decimal numbers Calculation of decimal numbers · multiplication and division of decimal numbers (the tenths and [hundredths] places, etc.)</p> <p><b>Calculation of fractions</b> · addition and subtraction of fractions with different denominators (proper fraction, [improper fraction]), (moved from grade 6) · multiplication and division of fractions (fraction <math>\times</math> integer, fraction <math>\div</math> integer)</p>	<p><b>Area</b> · finding areas of triangles and parallelograms · [finding areas of rhombi and trapezoids]</p> <p><b>Volume</b> (moved from grade 6) · units of volume (<math>\text{cm}^3</math>, <math>\text{m}^3</math>) and measurement · finding volumes of cubes and rectangular parallelepipeds</p> <p><b>Mean of measurements</b> <b>Per-unit-quantity</b> (example: population density) (moved from grade 6)</p>	<p><b>Geometrical figures</b> · [polygons and regular polygons] · [congruence of figures (partially moved from lower secondary schools)] · properties of geometrical figures · ratio of circumference of a circle to its diameter (<math>\pi</math>) · prisms, cylinders (moved from grade 6)</p>	<p>Simple proportional relations Observation and examination of Mathematical Relations · studying a relation of two quantities that are represented by a simple algebraic expressions Percentage <b>Pie charts, Bar graphs</b></p>	<p>a. thinking about ways of calculating and explaining them b. thinking about ways to calculate areas and explaining them c. drawing and constructing congruent figures d. explaining properties of geometrical figures inductively and deductively e. selecting a graph or a table depending on each objective and applying them</p>
Grade 6	<p><b>Calculation of fractions</b> · multiplication and division of fractions ([calculations that involve both fractions and decimal numbers], etc.) Consolidation and utilization of the four basic operations with decimal numbers and fractions</p>	<p><b>Approximate shape</b> · approximate areas, etc.</p> <p><b>Area</b> (moved from grade 5) · finding area of a circle</p> <p><b>Volume</b> · [finding volume of prisms and cylinders (moved from lower secondary school)]</p> <p><b>Speed</b> · meaning and representation, finding speed [Metric system, how it works]</p>	<p><b>Geometrical figures</b> · [reduced figures and enlarged figures (moved from lower secondary schools)] · [symmetric figures (moved from lower secondary schools)]</p>	<p><b>Ratio</b> proportional relationships and [inversely proportional relationships (partially moved from lower secondary schools)] [algebraic expressions by using letters such as a, x (partially moved from lower secondary schools)] <b>Ways to examine data</b> · the average of data · [“frequency distribution”] [Possible outcomes (moved from lower secondary schools)]</p>	<p>a. Activities to explore and explain the meaning of calculations b. Activities to find how the units are related c. Activities to find reduced figures, enlarged figures and symmetric figures d. Activities to solve problems by using the proportional relationships.</p>

# Chapter 2. Objectives and Content of Mathematics

## Section 1: Objectives

### 1. Objectives of Mathematics

#### (1) About establishing objectives

The objective of lower secondary schools is to provide education as a part of the compulsory education based on the foundation established by elementary school education. Therefore, the objective of lower secondary school mathematics is to further develop students based on the foundation established by elementary school mathematics.

In this revision, added emphasis is placed on considering lower secondary schools as the final stage of the compulsory education process.

It is expected that lower secondary school mathematics will help students master fundamental and basic knowledge and skills, and foster their ability to think mathematically, while knowing the values of mathematics, deepening their understanding of relationships between mathematics, science, and technology, and nurturing their ability and inclination to examine phenomena mathematically.

Moreover, mathematical activities play important roles in helping students master fundamental and basic knowledge and skills, improve their ability to think mathematically, and experience the purpose and joy of learning mathematics. Therefore, mathematical activities will be further emphasized and specific examples of mathematical activities are discussed in the content of each grade level.

#### (2) About objectives

Considering the background discussed above, the following are the goals of mathematics in the Lower Secondary School Mathematics Course of Study.

Through mathematical activities, to help students deepen their understanding of fundamental concepts, principles and rules regarding numbers, quantities, geometrical figures and so forth, to help students acquire the way of mathematical representation and processing, to develop their ability to think and represent phenomena mathematically, to help students enjoy their mathematical activities and appreciate the value of mathematics, and to foster their attitude toward to making use of the acquired mathematical understanding and ability for their thinking and judging.

In this revision, the phrases “through mathematical activities” and “the ability to represent” are added. This reflects the basic policies for the revision of mathematics courses of study by the Central Education Council, “to further enrich mathematical activities” and “the abilities to think and represent mathematically play important roles in advancing efficient and logical thinking as well as intellectual communication”. Moreover, the phrases “appreciate the values of mathematics” and “to foster their attitude toward to making use of the acquired mathematical understanding and ability for their thinking and judging” respond to the basic policies of revision of mathematics courses of study by the Central Education Council, “it is important that we motivate students to learn mathematics, and students should experience the meaning of learning and utility of what was learned”, and “To help students apply what has been learned to activities in daily life, to the study of other subjects, and to learning more advanced mathematics”.

This objective indicates the knowledge, skills, abilities, and attitudes that must be learned in lower secondary school mathematics, and it is necessary to consider that they must be achieved as a whole composed of several interwoven ideas. In this section, we explain five distinct ideas.

## ① About “through mathematical activities”

Mathematical activities are various activities related to mathematics where students engage willingly and purposefully.

“Students engage willingly and purposefully” means students try to find new properties or new ways of thinking or try to solve a concrete problem.

In order to help students experientially understand quantities and geometrical figures, raise their ability to think, make decisions and express ideas, and let them feel the joy and meaning in learning mathematics through mathematical activities, it is necessary to teach in such a way that students will engage in an activity willingly and purposefully.

Moreover, such teaching through mathematical activities needs to be used in each content domain.

Mathematical activities may also include engaging in trials and errors, collecting and organizing data, observing, manipulating and experimenting; however, simply listening to teachers’ explanations or engaging in simple computational exercises will not be viewed as mathematical activities.

In the lower secondary school mathematics, the following three types of mathematical activities are particularly emphasized: activities to discover and extend properties of numbers and geometrical figures based on mathematics students have learned previously; activities to use mathematics in everyday life and in the society; and activities to explain and communicate logically and with a clear rationale by using mathematical expressions. These three types are discussed explicitly in the content of each grade level and explained in more detail in Sections 2 and 3 of Chapter 2.

In this revision, the phrase “through mathematical activities” is added, and instead of the phrase in the Course of Study 2000, “to know the joy of mathematical activities”, the phrase, “to experience the joy of mathematical activities” is used.

It is necessary that students will not only become able to think and make decisions using mathematics from teaching through mathematical activities, but will also experience the joy of learning mathematics and become further motivated to study mathematics.

## ② As for the objectives, “to help students deepen their understanding of fundamental concepts, principles and rules regarding numbers, quantities, geometrical figures and so forth, to help students acquire the way of mathematical representation and processing”

It is necessary that the lower secondary school mathematics emphasize the mastery of the fundamental and basic content while deepening students’ understanding of underlying principles and rules and developing the knowledge and skills that are based on such understanding.

For example, while learning algebraic expressions with letters and solving equations, it is important that students understand not only that the procedures are based on principles and rules, but also that by using principles and rules, mathematical procedures may be developed.

Furthermore, students should be able to utilize the knowledge and skills built on principles and rules in solving problems in their everyday life and in the society by representing and then processing phenomena mathematically.

Moreover, it is important to help students become able to use mathematical concepts, principles and rules as well as mathematical representations and procedures while solving problems. Considerations must be given to teach through mathematical activities so that students can, through experiences, deepen their understanding and master procedures.

### ③ About “to develop their ability to think and represent phenomena mathematically”

In mathematics education, improving students’ ability to think and represent phenomena mathematically is emphasized throughout Elementary, Lower Secondary, and Upper Secondary Schools. In Elementary School mathematics, ideas about numbers, quantities, and geometrical figures are studied in relation to phenomena in students’ everyday life. In lower secondary schools, teaching will aim at furthering students’ ability to think about and represent mathematically various phenomena, not limited to those that are found in daily life.

#### **To think about phenomena mathematically**

We think about phenomena mathematically primarily in two situations. The first situation is where we model phenomena from daily life or society mathematically, process them by using mathematical procedures, then interpret the results in the actual contexts. The other situation is where phenomena in the mathematical world are processed efficiently by selecting representations that are concise and easy to process, and expand on the results.

An example of thinking about phenomena from daily life or in the society is when we make predictions based on data obtained from experiments and actual measurements. For example, if you want to know how long it will take to heat water to a specific temperature, you can graph the relationship between the time and temperature. By noticing that the data points form a roughly linear pattern, you can use a linear function to predict the amount of time.

As an example of situations involving the mathematical world, we can consider the following, which involves properties of numbers. Take any 2-digit natural number. Students may discover that if they add the number to the 2-digit number obtained by reversing the numerals, their sum is always a multiple of 11. Then, they can investigate what happens to the difference of those two numbers. They will conjecture that the difference is always a multiple of 9, and as they try to explain why, they may discover new properties.

It is necessary to help students improve their ability to think about phenomena mathematically by grasping the characteristics of each situation. In the process of thinking about phenomena mathematically, representations become necessary for the following purposes: to concisely express properties of numbers and geometrical figures that are conjectured or discovered; to explain the validity of the conjectures logically with clear rationales; and to orderly and effectively explain the process of using previously learned mathematics.

By representing, it often becomes possible to more effectively and logically advance an argument, to develop more advanced representations that are more concise, and to notice new aspects.

Moreover, representations make it easier to reflect and check the results of one’s thinking processes and decisions. Through these types of experiences, it is important to help students understand the roles of representation.

Furthermore, through representations, it becomes possible for ideas and thoughts to be communicated, shared, and even further refined qualitatively. It is necessary to keep in mind that representations support communication, and intellectual communication will lead to an increased quality of representations and enriched opportunities to learn collaboratively.

#### ④ About “to help students enjoy their mathematical activities and appreciate the value of mathematics”

Both the joy of mathematics and the value of mathematics are to be experienced. That is for the purpose of emphasizing the process of learning mathematics by further emphasizing the affective side of learning and increasing students’ motivation to study mathematics while making it possible for students to actively engage in mathematical activities, That is, students should not just learn completed mathematics. Rather, an added emphasis is placed on the importance of learning mathematics by discovering properties by observing phenomena and through activities involving identifying and extending properties of numbers and geometrical figures by thinking inductively on data obtained from actual manipulation and experiments. Moreover, it is important that opportunities be provided so that students can experience mathematics learning through activities and taste the joy of thinking and learning through surprises and excitement in the process. It is hoped that the process will include mathematical knowledge and skills and mathematical ways of observing and thinking so that their quality will also be raised.

#### The joy of mathematical activities

As mathematical activities, three types of activities are indicated in the content of each grade. These activities are generally set up as problem solving activities. In the process, appropriate mathematical activities are conducted to engage in trials and errors, to collect and organize data, to manipulate, to experiment, and to observe.

Thinking as we move objects or confirming our ideas through experiments leads to an increased intellectual satisfaction. Therefore, it is important that activities involving manipulation of concrete objects and activities focused on thinking and explaining should be connected so that they can reinforce each other.

Moreover, since lower secondary school students are developmentally ready to start thinking more logically and abstractly, not only activities involving manipulation of concrete objects but also activities focused on thinking and explaining should be incorporated so that students can engage in such activities actively and purposefully.

For these reasons, as for the joy of mathematical activities, it is necessary to pay attention to the qualitative side of mathematical activities, which deals with what intellectual growth may result from those activities, not just to enjoying the activities.

#### Value of mathematics

The real purpose of experiencing the “value of mathematics” is to help students study mathematics with motivation. The “value of mathematics” here includes the value of mathematical representations and procedures — for example, “if you represent the quantitative relationship in equations, we can find the answer by processing it procedurally” — the value of fundamental concepts, principles and rules about numbers and geometrical figures, and the value of mathematical ways of observing and thinking.

The value of mathematics also includes understanding that mathematics has practical use and supports and develops science and technology.

In order to experience the value of mathematics, it is important that students recognize as they reflect on the process of studying mathematics that mathematical knowledge and skills, as well as mathematical ways of observing and thinking, will make it possible to process efficiently, represent concisely and clearly, and grasp phenomena appropriately.



## ⑤ About “to foster their attitude toward to making use of the acquired mathematical understanding and ability for their thinking and judging”

Here, “acquired mathematical understanding” means fundamental concepts, principles, and rules about numbers, quantities, and geometrical figures; mathematical representations and procedures; and the ability to observe and represent phenomena mathematically. All of these as a whole are considered “mathematics”.

In order to use mathematics appropriately, it is necessary that students master mathematical processes such as how to write equations, how to explain and prove, and so forth.

It is also necessary to help students understand the need and usefulness of mathematics as well as the reason for using mathematics.

An understanding of the need and usefulness of mathematics relates deeply to the attitude to think and make decisions by using mathematics.

In order for students to feel the desire to use mathematics they learned, it is important that students understand experientially the need and usefulness of mathematics.

Therefore, it is important to make it possible for students to study mathematics actively through experiences, and consideration must be given to helping students become motivated to think and make decisions using mathematics.

## 2 Objectives for each grade

### (1) Ideas behind setting the grade level objectives

The Objectives of Lower Secondary School Mathematics discuss the goals of mathematics instruction in lower secondary school as a whole; therefore, they are general and inclusive. In order to achieve those objectives in actual instruction, more specific objectives will be necessary. Grade level objectives are developed by considering both the coherence of mathematics as a discipline and the developmental level of students. These grade level objectives address the main contents of each grade level as well as the main points of emphasis in instruction.

Therefore, it can be said that the grade level objectives are the instantiation of the objectives of mathematics, and the contents are there to achieve the grade level objectives.

Although grade level objectives are often considered only in the specific grades, it is necessary to pay attention to the coherence of the objectives as something that will be gradually developed across three years of lower secondary school. Considerations must be given to the relationship to “Approaches to Content Organization” discussed later.

Although they are not explicitly stated in the grade level objectives, it is necessary to treat the following ideas in the “Objectives of mathematics” carefully in each grade level: to experience “the joy of mathematical activities” and “the value of mathematics” and “to foster their attitude toward to making use of the acquired mathematical understanding and ability for their thinking and judging”.

The content to be taught in each grade level is organized in the following domains: A. Numbers and Algebraic Expressions, B. Geometrical Figures, C. Functions, D. Making Use of Data, and “Mathematical Activities”.

The aims of instruction in these domains are closely interrelated. For example, the study of positive and negative numbers in the domain “A. Numbers and Algebraic Expressions” aims to enrich students’ concept of numbers and to increase the effectiveness of algebraic expressions by developing the ability to summarize subtraction using addition in algebraic expressions. However, at the same time, this study is useful in the domain “C. Functions”.

Also, it is necessary to develop the ability to think logically not only in the domain “B. Geometrical Figures” but also in the domain “A. Numbers and Algebraic Expressions”. Therefore, considering these ideas, it is necessary to always think about the relationships between the “Objectives of mathematics” and the grade level objectives, and also the relationships among the content domains.

The grade level objectives are stated as (1), (2), (3) and (4) corresponding to the four content domains, A. Numbers and Algebraic Expressions, B. Geometrical Figures, C. Functions, and D. Making Use of Data, respectively.

Although there is no grade level objective for “mathematical activities”, providing opportunities for students to engage in activities itself is deeply connected to reaching the goals of each domain, A. Numbers and Algebraic Expressions, B. Geometrical Figures, C. Functions, and D. Making Use of Data. It is important to achieve the objectives for each content domain in each grade level by carefully considering the three specific types of mathematical activities discussed in Section 2 of this chapter.

## (2) Objectives of Each Grade

[Grade 1 of lower secondary school]

- (1) To expand the range of numbers to positive and negative numbers, and to deepen students’ understanding of the concept of numbers. To understand the necessity and meaning of the use of letters and of equations, to cultivate students’ ability to represent and process the relationships and rules of numbers and quantities in a general and simple manner, and to make use of linear equations with one unknown.

In Elementary School mathematics, the meaning of the four arithmetic operations with whole numbers, decimal numbers, and fractions is taught, and computational fluency is developed.

Moreover, understanding of whole numbers, decimal numbers and fractions as numbers is deepened.

In lower secondary school mathematics, teach students to understand the thinking involved in expanding the range of numbers to include positive and negative numbers and further deepen their understanding of number concepts. Moreover, help students realize that both the set of numbers and the possibilities of arithmetic operations are expanded. Help students become able to consider situations more broadly by uniformly representing numbers and quantities using positive and negative numbers.

Furthermore, in Elementary School Mathematics, students’ ability to represent and interpret relationships and rules among numbers and quantities generally and simply — by using algebraic expressions using words, symbols such as  $\square$  and  $\triangle$ , and letters such as  $a$  and  $x$  — is gradually developed.

Building upon and further extending the study of Elementary School mathematics, in Grade 1 of lower secondary school help students to become able to represent relationships and rules of numbers and quantities generally and simply by using letters in algebraic expressions and to interpret such algebraic expressions. Although students have learned algebraic expressions with letters in elementary school, in teaching this idea in lower secondary school pay careful attention to students’ level of mastery of elementary school mathematics, and help them develop the procedural fluency to transform algebraic expressions purposefully.

Also, teach students to represent relationships among numbers and quantities using the equal sign and inequality symbols. Help them understand that of those relationships of numbers and quantities that can be represented using the equal sign, those that can be satisfied under certain conditions are equations. Students are to understand the meaning of variables and solutions in equations. Students are also to understand that equations are useful for solving problems efficiently because equations can be solved procedurally by transforming them.

- (2) Through activities like observation, manipulation and experimentation with plane and space figures, to deepen students' intuitive ways of viewing and thinking about geometrical figures and to cultivate their ability to think and represent logically.

In Elementary School mathematics, through activities such as observing and constructing various shapes, students' sense of geometrical figures is enriched and their understanding of basic plane and space figures is developed.

Continuing on their study in elementary schools, in Grade 1 of lower secondary school students' understanding of basic properties and compositions of plane and space figures is deepened through empirical activities like observation, manipulation, and experimentation with geometrical figures.

Through this study, increase students' interest and motivation toward what they will study in Grade 2 of lower secondary school and beyond, such as ways to develop and express logical arguments. That is, nurture students' ability to identify the essence behind the basic properties of geometrical figures through activities like basic construction and transformation of plane figures and manipulations of space figures such as representing them with nets. Also, cultivate the ability to observe and express those properties logically.

The phrase in the previous revision, "through observation, manipulation, and experimentation", has been changed to "through activities like observation, manipulation, and experimentation". This is done to more clearly indicate that observing, manipulating, and experimenting are examples of the activities being emphasized. The intention remains the same.

- (3) Through examining concrete phenomena, to deepen students' understanding of direct proportion and inverse proportion, and to cultivate their ability to find out, represent and think functional relationships.

In the Elementary School Mathematics, students are taught to examine two quantities that vary together and to identify their characteristics and patterns. Students use tables, algebraic expressions, and graphs to examine proportional relationships and apply them in problem solving.

In Grade 1 of lower secondary school mathematics, students will deepen the ways developed in elementary school to observe and think about two co-varying quantities and their understanding of direct and indirect proportion. That is, cultivate their ability to identify functional relationships between two quantities in a situation and to examine their correspondences and variations. Students will be taught to view direct and indirect proportion as functions.

It is important that students can actively engage in problem solving and recognize direct and indirect proportional relationships from tables, algebraic expressions and graphs.

The ability to identify and represent functional relationships isn't limited to this domain. Considerations must be given so that students' abilities to use functions will be nurtured so that they can use such ways of observing and thinking even in situations involving geometric figures.

- (4) To cultivate students' ability to collect and organize data according to their purpose, then read trends in the data.

In Elementary School mathematics, the basic ability to examine and represent statistically is cultivated through collecting and organizing data purposefully, using various tables and graphs, and examining averages and distributions.

In Grade 1 of lower secondary school mathematics, help students to understand the meaning and necessity of representative values and histograms and to be able to interpret the trend in data by identifying and explaining it by using those ideas.

[Grade 2 of lower secondary school]

- (1) To foster students' ability to calculate and transform algebraic expressions using letters according to their purpose, and to cultivate their ability to understand and use simultaneous linear equations with two unknowns

In Grade 1 of lower secondary school, the range of numbers has been expanded to include positive and negative numbers, and understanding of properties of calculation is developed with respect to the expansion of the meaning of four arithmetic operations.

In addition, students are taught to represent relationships and rules among quantities using algebraic expressions and to interpret algebraic expressions.

In Grade 2 of lower secondary school, teach students to understand that the four arithmetic operations may be used to calculate with algebraic expressions just as with numbers, and to master the basic manipulation of algebraic expressions.

Moreover, teach students to use algebraic expressions purposefully and appropriately and to examine and process quantitative relationships generally and efficiently by using algebraic expressions. In addition, help students understand simultaneous linear equations with two variables and cultivate their ability use simultaneous linear equations in concrete situations.

- (2) Through activities like observation, manipulation and experimentation, to deepen students' understanding of the properties of basic plane figures, and to understand the necessity, meaning and methods of mathematical reasoning in considerations of the properties of geometrical figures, and to foster their ability to think and represent logically.

In Grade 1 of lower secondary school mathematics, students' understanding of geometrical figures and space is deepened through empirical activities and manipulation. Their interest in and motivation to examine and reason about geometrical shapes and to represent their thinking are increased.

Based on these experiences, in Grade 2 of lower secondary school, teach students to identify properties of triangles and quadrilaterals through observation, manipulation and experimentation, then to verify those properties logically.

In particular, teach the necessity of proof in order to verify that properties observed in several examples may be true in general.

Moreover, teach students to make correct arguments logically and coherently by clarifying the relationship between the hypothesis and the conclusion, the definition of terms used, and by providing an explicit rationale as they try to establish the validity and generality of properties of and relationships among geometrical figures.

- (3) Through exploring concrete phenomena, to understand linear functions, and to foster students' ability to find out, represent and think about functional relationships.

The ways to observe and think with functions fostered in the study of direct and indirect proportions in Grade 1 of lower secondary school will be further deepened. That is, in Grade 1, students' abilities to identify, examine and represent functional relationships among two covarying quantities is cultivated by investigating ways quantities change and correspond in direct and indirect proportional situations using tables, algebraic expressions and graphs.

Based on these experiences, in Grade 2 of lower secondary school, teach students to understand linear functions as they examine various concrete phenomena. In addition, help students to be able to consider linear equations in two variables as functional relationships between the two variables, and to examine various phenomena whose relationships are represented by equations.

(4) To cultivate students' ability to understand and use probability through exploring uncertain phenomena.

In Elementary School mathematics, students learned to examine possible outcomes in concrete phenomena by organizing them systematically.

Based on those experiences, teach students to understand the importance of the idea of probability to consider uncertain phenomena through repeated observations and repeated experiments. Help students to be able to consider and explain uncertain phenomena using the idea of probability.

**[Grade 3 of lower secondary school]**

(1) To understand the square roots of positive numbers and deepen students' understanding of the concept of numbers. To develop students' ability to calculate and transform algebraic expressions according to the purpose, and to cultivate students' ability to understand and use quadratic equations.

Through Grade 2 of lower secondary school, the range of numbers is rational numbers, and phenomena are examined and processed within that range. In Grade 3 of lower secondary school, help students to understand the new types of numbers, irrational numbers.

Deepen students' understanding of ways to expand the range of numbers by understanding the necessity of a new type of numbers to express the length of the diagonal in a square and by noticing the existence of square roots as they examine the inverse operation of squaring.

Teach students to understand that, by using irrational numbers, those quantities that could not have been expressed exactly may be expressed exactly. Expand the range of phenomena that can be examined using numbers.

Through Grade 2 of lower secondary school, students have learned basic calculation and transformations of algebraic expressions. In Grade 3 of lower secondary school, help students understand more advanced ways of thinking and treating algebraic expressions.

That is, help students to deal with algebraic expressions efficiently by understanding how formulas may be used to factor and expand expressions. Moreover, help students to understand the meaning of letters and solutions of quadratic equations so that they can deepen their ways of viewing equations.

In addition, help students to understand and use ways to solve quadratic equations by devising their own methods.

(2) Through activities like observation, manipulation and experimentation, to understand the similarity of geometrical figures, the relationships between inscribed angles and its central angle in a circle, and the Pythagorean Theorem, and to develop students' ability to use them in thinking about the properties of geometrical figures and measuring geometrical figures, as well as their ability to logically think about and represent geometrical figures with a prospect in mind.

Teach properties and measurements of similar figures.

As for the relationships between an inscribed angle and its central angle and the Pythagorean Theorem, help students to identify and understand those relationships through observation, manipulation and experimentation. At the same time, deepen students' understanding of circles and right triangles.

In both cases, deepen students' understanding of the necessity, meaning, and methods of reasoning fostered in Grade 2 of lower secondary school. Extend not only students' intuition and ability to observe geometrical figures but also their ability to examine them logically and to represent their thinking.

It is important to emphasize deep examinations of properties and measurements of geometrical figures and the use of properties and theorems in concrete situations.

(3) Through exploring concrete phenomena, to understand the function, and to develop students' ability to find out, represent and think about functional relationships.

Through Grade 2 of lower secondary school, students' ways to observe and think about functions have been deepened. They also learned about direct and indirect proportions and linear functions and their properties.

Just as in Grades 1 and 2 of lower secondary school, in Grade 3, help students understand the property of functions,  $y = ax^2$ , through the examinations of concrete phenomena. Help students identify the commonalities and differences with linear functions they studied in Grade 2.

Furthermore, help them understand that there are different functional relationships from those that have been studied so far in various phenomena. Extend students' ability to identify, represent, and examine functional relationships.

(4) To cultivate students' ability to read trends in a population by selecting samples out of the population and exploring its trends.

In Grade 1 of lower secondary school, students are taught to identify trends in data by using representative values and histograms.

In Grade 2 of lower secondary school, students are taught to consider and explain uncertain phenomena using the idea of probability.

In Grade 3 of lower secondary school, teach students the necessity of sampling. Help students understand the methods of sampling from the population and the idea that trends in a population may be inferred by examining trends in a sample, by conducting simple sampling.

## Section 2: Content

### 1. Approaches to Content Organization

#### (1) About the Content of Lower Secondary School Mathematics

A detailed explanation of the content of each grade level will be done in Section 3. Here, in this section, we will re-affirm the goals of studying mathematics in relationship to mathematical activities. Then, from the perspective discussed so far, we will explain the general framework of the content of lower secondary school mathematics.

Mathematical activities and the purpose of studying mathematics

In studying mathematics, it is important for learners to deeply experience various phenomena through activities such as observing, manipulating and experimenting. It is also important for students to refine their understanding through activities such as reflecting, representing or expressing themselves in mathematical language, and through repeated close examination. The study of mathematics can be considered as a process of coming to know mathematics and mathematical structures by doing these activities.

This process is nothing more than reconstructing one's own knowledge by gradually raising one's own mathematical knowledge while reflecting on activities such as observing, manipulating and experimenting.

Although there is much value in the knowledge gained through these experiences, the methods of obtaining this new knowledge and the perspectives that constitute the creation of new knowledge are also important. They may become effective tools for problem-solving; they may lead to the discovery of new problems and they may also become the source of furthering new knowledge. Since the developing of new knowledge and the deepening of mathematical understanding are made possible through one's own mathematical experiences, it is important to help students engage actively in problem-solving through enriching mathematical activities.

Framework of Lower Secondary School Mathematics

The basic policy of revising the lower secondary school mathematics curriculum is to structure its content in such a way that students can deepen their knowledge as they progress through grades as discussed above. The following is the summary of the framework:

- ① Concept of numbers and expanding the range of numbers
- ② Euclidean Space
- ③ Functions
- ④ Uncertain phenomena
- ⑤ Algebraic expressions with letters
- ⑥ Mathematical reasoning
- ⑦ Explaining and communicating

Of these items, ① through ③ are about ideas in the mathematical realm in which definitive phenomena are mathematically considered, while ④ discusses the ideas in the real world in which uncertain phenomena are considered. Items ⑤ through ⑦ touch on ideas that support the learning of items discussed in ① through ④. They are explained below.

### ① Concept of numbers and expanding the range of numbers

When dealing with phenomena in the real world, numbers learned in elementary school may not be sufficient in some situations.

In order to deal with real world situations, lower secondary school mathematics will expand the range of numbers used in the classroom to include positive and negative numbers, and eventually to include irrational numbers.

The expansion of the numerical range in the lower secondary school is carried out through observation, manipulation, and experimentation, not through an axiomatic treatment, and the process will continue in upper secondary school.

In lower secondary school mathematics, numbers that were learned in elementary school are re-examined from a more mathematical perspective. The following terminology will be learned: *natural numbers*, *integers*, *rational numbers*, and *irrational numbers*.

In elementary schools, the term, “*integers*” has been used to describe the set of natural numbers and 0. In Lower Secondary School, the same term is used to label the mathematical concept that includes positive and negative numbers.

Moreover, fractions are re-examined as rational numbers, and they will be considered as fraction representations of rational numbers.

The current revision emphasizes a smooth articulation across elementary and lower secondary schools. Therefore, it is necessary to give full consideration to instruction that re-conceptualizes previously learned ideas through reflection and reexamination from a new perspective.

### ② Euclidean Space

Geometry in Euclidean Space is a model in the mathematical world that logically examines the relationships among shapes observed in the real world by grasping them with the lines and planes that compose them.

It starts with defining the terms that are necessary to describe shapes and spatial relationships observed in various relationships in the real world. They are then organized to form geometric structures.

It is necessary to position shapes as objects of mathematical thought by focusing on properties and describing them with formal terms, and also to see geometrical manipulations such as superimposing them as mathematical objects.

By examining geometrical figures and manipulations that have been conceptualized as mathematical objects, students can be guided in identifying and describing new relationships among geometrical figures and to explore the properties and relationships among them.



### ③ Functions

In the world of mathematics, we examine not only static objects such as geometrical figures but also dynamic objects such as the way quantities correspond and vary.

Function is an abstract concept used to examine dynamic objects.

Functions are effective not only in the world of mathematics but also in the real world as we try to grasp the relationships between two quantities in phenomena that are changing simultaneously.

In the real world, if we can identify the relationship between two quantities, we can then understand the correspondences and changes of those quantities within the range where the relationship holds true and then may be able to predict what will happen in the future.

However, in general, functional relationships cannot be observed visually.

Therefore, in order to grasp functional relationships, we use tables, algebraic expressions and graphs. By processing the phenomena using these mathematical representations and examining the relationships among them, we can examine the real world relationships among quantities in the world of mathematics.

These experiences are deeply related to the activities of using mathematics.

### ④ Uncertain phenomena

The objects of mathematics vary significantly, and we examine not only deterministic phenomena but also uncertain phenomena. For example, in order to express the likelihood of uncertain events such as the number rolled on a die, the concept of probability was developed and we examine those phenomena mathematically by assigning the value between 0 and 1 inclusively to each phenomenon. In addition, methods used to grasp the characteristics of population and trends in data were also developed.

For example, by clarifying the characteristics of data by using representative values such as the mean, median and mode, we made it possible to examine the data mathematically. Furthermore, for those cases where examining the entire population is impossible, sampling methods based on random selection was developed in order to grasp the characteristics of the population.

In our everyday life and in our society, we often encounter information about uncertain phenomena.

In those situations, it is necessary to respond appropriately by understanding the characteristics of the information about the uncertain phenomena.

### ⑤ Algebraic expressions with letters

As a method of describing real world phenomena in the mathematical world, there are algebraic expressions with letters.

There are two aspects in the ways algebraic expressions with letters are used:

- a. They represent real world phenomena and relationships using letters and symbols.
- b. They may be modified into a form that are more easily interpreted.

The role of aspect (a) is to make it possible to examine real world phenomena and relationships in the world of mathematics. The main contents include polynomials, equations and functions.

Here, it is necessary that students master the methods used to express real world phenomena and relationships in the forms appropriate for the world of mathematics such as numbers and algebraic expressions.

Once the real world phenomena and relationships are represented in the world of mathematics, we can generate algorithms to process those representations as a part of a symbol manipulation system.

The second aspect points out that these algebraic expressions with letters can further extend our thinking and foster creativity.

By utilizing algebraic expressions with letters, we can capture the true relationships among quantities more clearly and concisely. By modifying them, we may discover a new relationship or gain an insight into a new way to solve a problem.

It is necessary that students understand these roles of algebraic expressions with letters and utilize them.

## ⑥ Mathematical reasoning

The primary forms of mathematical reasoning are induction, analogy, and deduction. They play an important role as we identify properties of numbers and geometrical figures, use mathematics, and explain and communicate our ideas mathematically.

Induction is the form of reasoning that derives a more general conclusion based on observation, manipulation, and experimentation on specific cases.

Analogy is the form of reasoning that predicts a new proposition based on the idea that similar situations will produce similar results.

Although both induction and analogy are important forms of reasoning in identifying properties of numbers and geometrical figures, the resulting propositions may not be valid. It is deduction that can determine whether or not the conclusions resulting from induction or analogy are generally valid.

Deduction is the form of reasoning that draws out necessary conclusions from previously established propositions following the rule of logic.

For example, if you inductively conclude that “the sum of all interior angles of a triangle is  $180^\circ$ ” based on actual measurements, you do not know whether or not “the sum of all interior angles in every triangle is  $180^\circ$ ”.

In order to validate the latter statement, we must deduce the conclusion based on ideas such as properties of parallel lines.

Validating the conjectures derived from induction or analogy using deductive reasoning deepens our understanding of the content. It is also useful in relating and organizing our knowledge.

Each different type of reasoning should be used appropriately and purposefully, and it is not appropriate to make light of induction and analogy just because deduction has been learned.

## ⑦ Explaining and communicating

Activities for explaining and communicating play important roles in the process of grasping deterministic phenomena such as the concept of and the expanding range of numbers, Euclidean Space, and functions, as well as grasping uncertain phenomena.

Moreover, using algebraic expressions with letters and understanding mathematical reasoning are necessary to explain and communicate more effectively.

Furthermore, activities used for explaining and communicating support the activities used to discover the properties of numbers and geometrical figures, as well as support the activities in using mathematics.

During the process of mathematical activities, one must face himself by examining his own thinking and feelings.

By expressing one's own ideas and thoughts in his own words, he can reaffirm his own thought processes. The ideas expressed in one's own words can generate reflective thinking and will become more refined as a result.

This process of internal dialogue can be further facilitated by the communication with others and increases the possibility of raising the quality of thinking. Explaining and communicating with others may generate new perspectives that had gone unnoticed during the internal conversation, and by being asked to explain the reasoning behind one's own thoughts, and the need to explain logically develops by making the rationale supporting one's thought explicit.

This creates the opportunity to experience the values of mathematical knowledge, skills and representations.

Finally, we want to discuss the relationships among contents. The concept of numbers and the expansion of the range of numbers are deeply connected to the use of algebraic expressions with letters. Moreover, the understanding of Euclidean Space may be supported by the concept of numbers, and, in turn, the concept of numbers may be deepened through the understanding of geometrical figures.

Mathematical reasoning plays an important role in the process of discovering properties of numbers, geometrical figures and in clarifying how to verify and explain the properties clearly and appropriately.

Explaining and communicating support each of the items ① through ⑥ discussed above and contribute to qualitative improvement.

## (2) Organization of Content Domains

### About the 4 Content Domains

The contents of lower secondary school mathematics have been organized from two perspectives:

- a) Contents are necessary for living autonomously in society and it is important for anyone to be able to use them.
- b) Contents are desirable as the common foundation for more specialized education beyond compulsory education in the future.

At the same time, considerations were given so that lower secondary school mathematics is positioned as an extension and broadening of the elementary school mathematics which summarizes properties and laws from everyday experiences and is a preparatory step toward upper secondary school mathematics.

When we focus on the structural similarities of the main contents of lower secondary school mathematics, much of it takes place in the world of linear relationships which can be solved by using the four arithmetic operations. However, in order to know the special characteristics of the world of linear relationships, one must also discuss the world of quadratic relationships such as quadratic equations, the Pythagorean theorem, and function  $y = ax^2$ .

Experiencing the necessity of mathematics and understanding things through mathematical investigation means to broaden the viewpoints in the study of mathematics, such as going from linear equations to quadratic equations.

Behind the viewpoint of grasping the contents of lower secondary school mathematics as discussed above is the aim to help students experience the ideas in the world of quadratic relationships. In addition, we want students to experience the following two ideas:

First, mathematics is not a closed or static discipline, rather, it is creative and ever-growing.

Second, is the necessity and usefulness of learning the contents in a way which is centered around the world of quadratic relationships.

As students try to grasp various phenomena encountered in their daily lives and in society mathematically, they can experience the necessity and usefulness by realizing that there are worlds of quadratic and cubic relationships.

As the goals of lower secondary school mathematics indicate, numbers, quantities, and geometrical figures are clearly identified as objects of study in mathematics. Therefore, it is obvious that Domains “A. Numbers and Algebraic Expressions” and “B. Geometrical Figures” are mentioned as the universal and fundamental domains. In the current COS, many hours are dedicated to the instruction of these two domains.

Previously, the domain, “C. Mathematical Relations”, was also established as a domain to extend students’ ability to use their ways of viewing and thinking about phenomena mathematically. Therefore, it has been argued that teaching needs to provide opportunities for students to fully utilize their ways of viewing and thinking mathematically and to use many different relationships as objects of mathematical examination.

In the current revision, it has been decided to establish a new domain, “D. Making Use of Data”, in order to discuss uncertain phenomena. As a result, the ideas related to the functions that have been discussed in “C. Mathematical Relations” were included in a new domain, “C. Functions”.

The purposes of the old domain, “C. Mathematical Relations”, discussed above have been passed on to the new domain.

For domain “D. Making Use of Data”, in addition to the ideas related to probability that have been discussed in “C. Mathematical Relations” of the previous COS, the domain is organized to include ways of examining data mathematically to identify properties and patterns; and to make inferences about the population through sampling. These ideas should be taught by doing activities that will incorporate them by examining and making judgments about phenomena from our everyday life and in society.

The relationships among the content domains, “A. Numbers and Algebraic Expressions”, “B. Geometrical Figures”, “C. Functions”, and “D. Making Use of Data”, may be explained as follows: If we focus on the nature of the object of study as being a definitive phenomena or an uncertain phenomena, we can divide these four domains into two groups, “A. Numbers and Algebraic Expressions”, “B. Geometrical Figures”, and “C. Functions”, in one group and “D., Making Use of Data”, in the other. Furthermore, by focusing on the nature of the methods of examination as being static or dynamic, we can put “A. Numbers and Algebraic Expressions”, and “B. Geometrical Figures” in one group and “C. Functions”, in the other.

Ways of viewing and thinking with functions developed in “C. Functions” may deepen the understanding of contents in “A. Numbers and Algebraic Expressions” and “B. Geometrical Figures”. On the other hand, understanding of the content in “A. Numbers and Algebraic Expressions” and “B. Geometrical Figures” are crucial for understanding the content of “C. Functions”.

As it was stated in 2-(1) of Section 1 of this chapter, the four content domains are closely related and instruction must consider that fact.

In the current revision, an added emphasis is placed on the cohesiveness of compulsory education; therefore, further collaboration and articulation across elementary and lower secondary schools are demanded.

Therefore, it is necessary that the relationships across elementary and lower secondary school mathematics are well understood.

The content domains of elementary school mathematics and the content domains of lower secondary school mathematics are related as shown in the table below.

Content Domains and Main Contents in Elementary School Mathematics		Content Domains of Lower Secondary School Mathematics
Numbers and Calculations	<ul style="list-style-type: none"> <li>· Concept of numbers</li> <li>· Calculation with whole numbers, decimal numbers and fractions</li> </ul>	A. Numbers and Algebraic Expressions
Quantities and Measurements	<ul style="list-style-type: none"> <li>· Quantities and measurements necessary in daily life</li> <li>· Calculating the measurements of geometrical figures such as length, area, and volume</li> </ul>	
Geometrical Figures	<ul style="list-style-type: none"> <li>· Properties of geometrical figures</li> </ul>	
Mathematical Relations	<ul style="list-style-type: none"> <li>· Algebraic expressions using symbols such as <math>\square</math>, <math>\triangle</math>, <math>a</math> and <math>x</math></li> </ul>	A. Numbers and Algebraic Expressions
	<ul style="list-style-type: none"> <li>· Relationships of quantities that change simultaneously</li> <li>· Direct and indirect proportional relationships</li> </ul>	C. Functions
	<ul style="list-style-type: none"> <li>· Possible cases</li> <li>· Organizing data</li> </ul>	D. Making Use of Data

There are four content domains in elementary school mathematics: “A. Numbers and Calculations”, “B. Quantities and Measurements”, “C. Geometrical Figures” and “D. Mathematical Relations”. Domain “A. Numbers and Algebraic Expressions” of lower secondary school corresponds to “A. Numbers and Calculations”, and a part of “D. Mathematical Relations” of elementary school mathematics.

Domain “B. Geometrical Figures” of lower secondary school mathematics corresponds to a part of “B. Quantities and Measurements” and “C. Geometrical Figures”, of elementary school mathematics.

In elementary school mathematics, the measuring of geometrical figures and the investigating of the properties of geometrical figures were placed into two separate domains. However, in lower secondary schools, those two ideas are positioned as the two primary viewpoints in examining geometrical figures in “B. Geometrical Figures”.

Domains “C. Functions”, and “D. Making Use of Data”, corresponds to a part of “D. Mathematical Relations” of elementary school mathematics.

### About Mathematical Activities

Mathematical activities are the various activities related to mathematics where students engage willingly and purposefully.

The “various activities related to mathematics” that are emphasized in lower secondary school mathematics are activities used to discover properties of numbers and geometrical figures; activities designated to use mathematics and activities used to explain and communicate ideas using mathematical representations.

Of course, as a principle, these mathematical activities are carried out as problem solving, and throughout the process students may engage in trial and error, collecting and organizing data, experimenting, observing and other mathematical processes. However, in the “Mathematical Activities” section, the three that are mentioned above are discussed.

Opportunities for students to engage in mathematical activities autonomously and with initiative should be established intentionally and strategically.

To be responsive to students’ developmental levels and also to the nature of mathematical contents, mathematical activities for grade 1 and for grades 2 and 3 are shown separately as can be seen in the table below.

	Grade 1	Grades 2 & 3
a. Activities to discover properties of numbers and geometrical figures	Activities to discover properties of numbers and geometrical figures based on previously learned mathematics	Activities to discover and then extend properties of numbers and geometrical figures based on previously learned mathematics
b. Activities to use mathematics	Activities to use mathematics in situations from everyday life	Activities to use mathematics in situations from everyday life and from society
c. Activities to explain and communicate ideas mathematically	Activities to explain and communicate ideas using mathematical representations in one’s own way	Activities to explain and communicate ideas logically using mathematical representations while making the rationale clear

In (a), the emphasis in Grade 1 is “to discover properties of numbers and geometrical figures”, while in Grades 2 and 3, the emphasis is not only “to discover” but also “to extend” what was discovered, expecting qualitative improvements.

In (b), the scope of activities is limited to “daily life” but in grades 2 and 3, it is expanded to include “society”.

As for (c), the activities used to explain and communicate, in Grade 1 the emphasis is “in one’s own way” while in Grades 2 and 3, an increased quality in terms of “logically” and “making the rationale clear” is expected.

Also, it is necessary to attend to the fact that, in many cases, activities in (a) and (b) are carried out in conjunction with activities in (c).

When establishing opportunities to have students engage in mathematical activities, it is important to pay attention to the flow of a series of activities. At the same time, it is necessary for teachers to be clear about what the focus is.

In elementary school mathematics, the “Mathematical Activities” section is included in the content of each grade level, and more concrete examples are discussed in each of the domains, “A. Numbers and Calculations”, “B. Quantities and Measurements”, “C. Geometrical Figures”, and “D. Mathematical Relations”.

## **2. Overview of the Content in Each Domain**

### **A. Numbers and Algebraic Expressions**

#### **(1) Purposes of teaching “Numbers and Algebraic Expressions”**

The contents of “Numbers and Algebraic Expressions” are being used in many situations in daily life and in society.

Moreover, they occupy an important position as the foundation for many contents in all other domains in lower secondary school mathematics.

Since numbers and algebraic expressions are so closely related to each other, they have been placed into a single content domain. However, here, in order to consider their structures, we will discuss them separately.

#### **① About numbers**

In elementary school mathematics, starting with counting objects in their surroundings, students learn the concepts of non-negative integers, decimal numbers and fractions as well as the meaning of the four arithmetic operations.

Moreover, students are expected to master basic calculations.

As the range of numbers is expanded, students are to deepen their understanding of calculations and learn to apply calculations appropriately to solve problems found in their surroundings.

In lower secondary school mathematics, the following two goals are kept in mind as the instruction on numbers take place.

#### **a. Expanding the range of numbers and understanding the concept of numbers**

The range of numbers is expanded by introducing negative and irrational numbers. Students are to understand not only the thinking involved as the range of numbers is expanded but also the sets of numbers and the possibility of arithmetic operations with them.

The aim is to help students deepen their understanding of the concept of numbers and also to represent and process a wide variety of phenomena generally and clearly using these numbers.

#### **b. Understanding the meaning and ways to calculate with the newly introduced numbers and mastering calculations**

Students are to understand the meaning and ways to calculate with numbers, including negative and irrational numbers. Teachers will help students to become fluent in calculation with those numbers.

## ② About algebraic expressions

With respect to algebraic expressions, students in elementary schools learn, for example, to use expressions with  $\square$  and  $\triangle$ , such as  $5 + \square = 8$  and  $3 \times \triangle = 24$ ; they learn to grasp the relationship between addition and subtraction or multiplication and division. They also learn to represent quantitative relationships using expressions with words, such as “speed  $\times$  time = distance”, or to interpret such expressions.

Moreover, in order to prepare for the study of algebraic expressions with letters in lower secondary schools, students learn to use letters such as  $a$  and  $x$  in place of  $\square$  and  $\triangle$ .

By using letters and algebraic expressions with letters, we can represent quantities and their relationships concisely, clearly and generally.

Once relationships are represented in algebraic expressions with letters, they can be formally processed to suit different purposes. In this way, developing the skills to represent quantities and their relationships in phenomena using algebraic expressions with letters, ways to observe and think more generally are nurtured, and the disposition to discover new relationships by processing formally are fostered. These are all significant purposes for learning.

The content related to algebraic expressions, therefore, is just as important as numbers because they are fundamental knowledge and are skills that relate to the study of mathematics as a whole.

In lower secondary school mathematics, the following are the three goals of teaching of algebraic expressions:

### a. Understanding the meaning of letters, in particular as variables

The aim here is to help students understand that the letters are not just symbols but they can represent different values and also, it is to deepen students’ understanding of variables by helping them understand the set of values a letter may assume.

The understanding of variables may be deepened by the following: by helping students become conscious of the fact that a letter may stand for different numbers by substituting different numbers for the letter to determine the values of algebraic expressions, or, as the example on the right illustrates, by expressing each odd number as shown, confirms that odd numbers can be represented generally as an algebraic expression,  $2n + 1$ .

$1$	$=$	$2 \times 0 + 1$
$3$	$=$	$2 \times 1 + 1$
$5$	$=$	$2 \times 2 + 1$
$\dots$		
(odd number)	$=$	$2 \times n + 1$

### b. Nurturing the ability to represent and interpret algebraic expressions with letters

The aim is to enable students to represent quantities and their relationships in algebraic expressions with letters and to interpret those expressions.

To reach this aim, it is necessary to help students see the connections to their daily life and to society as they represent and interpret algebraic expressions with letters.

A typical example is to solve a real-world problem by capturing quantitative relationships in the situation as algebraic equations, and by interpreting the solutions of the equations in context to find the solution of the problem.



### c. Developing the ability to calculate and process algebraic expressions with letters

The aim is to enable students to calculate with algebraic expressions with letters such as performing arithmetic operations with them or factoring them.

This means to transform a given algebraic expression into a form that may be easier to interpret. By doing this, students may understand that once quantitative relations are expressed in algebraic expressions with letters, results can easily be obtained by formally processing them.

It is through these studies that help students understand that letters stand for numbers and algebraic expressions with letters can be used in calculations by applying the same rules of computations with numbers.

As discussed above, calculating and transforming algebraic expressions with letters are important basic skills in the study of mathematics. However, considerations should be given so that the study does not become meaningless computation practice with unnecessarily complicated expressions.

As a goal of grade 2 of lower secondary school mathematics states, “To foster students’ ability to calculate and transform algebraic expressions using letters according to their purpose...” manipulation of algebraic expressions must be done for purposes, not just to calculate.

Instruction should proceed by keeping in mind that students will become aware of the fact that algebraic expressions with letters are used as tools, for example, in explaining properties about numbers or geometrical figures.

## (2) Overview of Content

### ① About Numbers

#### Treatment in Elementary School Mathematics

In elementary school mathematics, the primary focus is to incorporate manipulative and experiential activities that are connected to phenomena from the surroundings into the study of numbers and calculations.

The main topics in elementary school mathematics are as follows.

- a. By the end of grade 4, the meaning and properties of the four arithmetic operations have been studied, and computational fluency and the ability to use calculations have been fostered.

Moreover, students are deepening their understandings of the commutative law, associative law, and the distributive property. In addition, students are expected to understand and be able to use the meanings and representations of decimal numbers and fractions, addition and subtraction of decimal numbers and fractions with like denominators, and multiplication and division of decimal numbers by whole numbers.

- b. In grade 5, by examining the numeration system, students are to deepen their understanding of whole numbers and decimal numbers. They study about even and odd numbers, factors, the greatest common factor, multiples, the least common multiple, and prime numbers.

Furthermore, students are to deepen and utilize their understanding of multiplication and division of decimal numbers and addition and subtraction of fractions with unlike denominators.

- c. In Grade 6, students are to deepen and use their understanding of multiplication and division of fractions. They are also expected to solidify their understanding of and the ability to calculate with decimal numbers and fractions.

### Treatment in Lower Secondary School Mathematics

Based on the treatment of numbers in elementary school mathematics, the following content will be discussed in lower secondary school mathematics.

a. Grade 1 of lower secondary

To help students understand positive and negative numbers through the examination of concrete situations, and enable them to carry out the four arithmetic operations with integers and to represent the examined situations by using positive and negative numbers.

In grade 1 of lower secondary school, the range of numbers is expanded to include positive and negative numbers, and students are helped in order to develop a unified perspective of numbers and computational fluency.

Moreover, students are expected to be able to use positive and negative numbers to deal with concrete situations.

b. Grade 2 of Lower Secondary School

There is no new content on numbers in grade 2.

The focus is on deepening the understanding of positive and negative numbers as students apply what they have learned in concrete situations or through the study of equations, and functions. Another focus is the mastery of calculations.

c. Grade 3 of Lower Secondary School

Square roots of positive numbers will be taught. Enable students to use square roots to represent and examine phenomena.

Square roots are critical in the study of quadratic equations and the Pythagorean Theorem. Therefore, considerations should be given to the use of square roots in those situations.

Also, in conjunction with the discussion of factoring of polynomials, prime factorization of natural numbers will be discussed.

## ② About Algebraic Expressions

### Treatment in Elementary School Mathematics

The study of algebraic expressions in elementary schools focuses on representing and interpreting expressions in the forms of numerical expressions, expressions with words, and formulas. The main contents are as follows:

a. By the end of grade 4, students have learned about concisely representing quantitative relationships and properties of numbers using numerical expressions or algebraic expressions with words or with symbols like  $\square$ ,  $\triangle$ . They also have learned to interpret expressions.

In addition, the ideas related to formulas and the use of formulas is also being studied.

b. In grade 5, students are expected to deepen their understanding of representing quantitative relationships using symbols such as  $\square$ ,  $\triangle$ . In addition, they learn to examine the relationships of quantities that are expressed in simple algebraic expressions by focusing on how the quantities correspond or change.

c. In grade 6, students learn to use letters such as,  $a$  and  $x$  in place of words or symbols such as  $\square$ ,  $\triangle$ , which stand for numbers in algebraic expressions. They also substitute different numbers for letters to investigate the results.

## Treatment in Lower Secondary School Mathematics

The central focus of instruction on algebraic expressions in lower secondary school mathematics is expressions with letters. The contents are as follows.

### a. Grade 1 of lower secondary school

- To cultivate the ability to represent quantitative relationships and properties using algebraic expressions with letters, to interpret such expressions and, to enable students to carry out calculations involving algebraic expressions.
- To help students understand equations and examine linear equations with one variable. In teaching algebraic expressions with letters, as well as equations in grade 1 of lower secondary school, it is necessary that instruction considers carefully the connections to elementary school mathematics.

### b. Grade 2 of Lower Secondary School

- To foster the ability to identify quantitative relations in concrete phenomena, to represent those relationships using algebraic expressions, to interpret algebraic expressions in concrete situations and to enable students to calculate with algebraic expressions with letters.
- To help students understand simultaneous linear equations with 2 variables and use them in investigations.

### c. Grade 3 of Lower Secondary School

- To enable students to expand or factor simple polynomials with letters and extend their ability of transforming and interpreting algebraic expressions to serve different purposes.
- To help students understand quadratic equations and to be able to use them in investigations.

As shown above, instruction on algebraic expressions in lower secondary school mathematics begins with an introductory treatment of literal expressions and deals with contents up to quadratic equations. Instruction should help students represent quantitative relationships and properties using algebraic expressions with letters or equations so that they can solve problems efficiently.

Through such a study, students' algebraic processing ability will be heightened, which will be utilized in the study of other domains.

The current revision shows that students' abilities of representing quantitative relationships and properties in algebraic expressions with letters and of interpreting such expressions are to be cultivated in grade 1, while in grades 2 and 3, their abilities of using algebraic expressions with letters in explaining quantitative relationships, of transforming algebraic expressions purposefully, and of interpreting algebraic expressions are to be extended.

These ideas have been in place in previous revisions, however, in the current revision, enrichment of linguistic activities across subject matters is emphasized. Therefore, the importance of representing and interpreting, as well as explaining and communicating, is re-affirmed.

The phrase, "to transform algebraic expressions purposefully and to interpret algebraic expressions," means that students can interpret quantities and quantitative relationships represented by algebraic expressions as they try to explain the validity of relationships among quantities or geometrical figures by modifying algebraic expressions with letters and also while creating equivalent expressions as they solve equations.

The differences between domain "A. Numbers and Algebraic Expressions" of the current and the previous revisions are summarized here:

In grade 1 of lower secondary school, it was decided to discuss the following: sets of numbers and the possibility of understanding arithmetic operations; representing size relationships using inequalities, and solving simple proportions in relationship to the study of linear equations with one variable.

In grade 3 of lower secondary school, it was decided to deal with rational and irrational numbers and also to solve quadratic equations using the quadratic formula.

## B. Geometrical Figures

### (1) Purposes of teaching “Geometrical Figures”

Often times, we examine objects in our surroundings by focusing on their “shape”, “size”, and “positional relationship” while ignoring such characteristics as composition, weight, and color.

When objects are observed from such a perspective, we are dealing with geometrical figures. One of the important goals of lower secondary school mathematics is to enable students to examine geometrical figures.

The purposes of teaching geometrical figures in lower secondary school mathematics possess the following two aspects:

- To foster the desire and attitude in students of deepening their understanding of the basic concepts and properties of plane and solid figures and to use them in reasoning and making judgments because it is common to examine phenomenon in our surroundings from the perspective of “shape,” “size,” and “positional relationship.”
- To further extend students’ abilities to reason and express logically because both intuitive and logical ways of observing and reasoning about geometrical figures, are developed through the study of properties of geometrical figures mathematically, and play important roles not only in lower secondary school mathematics but also in many other fields of study.

### ① About the concept formation of geometrical figures and the understanding of their properties

The following are three goals of instruction in order to foster the understanding of concepts and properties of geometrical figures and the disposition of using those ideas in reasoning and making decisions.

#### a. Understanding the concepts and properties of basic geometrical figures

The properties of plane figures such as triangles, quadrilaterals, and circles, the transformations of geometrical figures, the concepts of congruence and similarity, positional relationships of lines and planes in space, the concepts and properties of solid figures such as prisms, pyramids, and spheres are studied.

Some of these contents are dealt with in elementary schools however, in lower secondary schools, these topics will be treated cohesively and systematically.

#### b. Mastering the ability to draw and construct geometrical figures accurately

In the study of geometrical figures, it goes without saying that the skill to draw and construct geometrical figures is expected. In general, construction refers to basic geometric construction with a compass and a straight edge. However, in this case, other drawings which can be used are also considered, such as those that may be used as we examine transformations of geometrical figures or structures of solid figures.

Therefore, it is important to consider not only the ability of drawing figures accurately but also the ability and disposition of reflecting on the appropriateness of the completed drawings.

These abilities depend on the understanding of concepts and properties of geometrical figures discussed in (a). However, it is also the case that the concept of geometrical figures will be more firmly established through the study of drawing and construction discussed here.

Therefore, the ideas discussed in (a) and (b) are not independent of each other and it is necessary that they are studied in connection with each other for more effective learning.

This goal also includes the development of the ability of representing solid figures. Since we cannot represent solid figures on a plane as they are, we must use drawings such as sketches, nets, and projections that are appropriate for the purposes.

#### c. To extend the ability of using knowledge and skills related to geometrical figures

The knowledge about the concepts and properties of basic geometrical figures learned through (a), along with the ability of representing them in drawings, will become the foundation for not only the later study of geometrical figures but also for the study of lower secondary school mathematics in general. The goal here is to develop the ability to use knowledge and skills about geometrical figures.

## ② About the development of logical reasoning

The following are the goals of instruction with respect to the development of logical reasoning.

- a. To extend the ability of gaining insights about geometrical figures

In elementary school mathematics, geometrical figures were handled empirically, and, through the study of geometrical figures, students' intuitive ways of observing and thinking about geometrical figures developed to a certain level. The lower secondary school mathematics aims to broaden students' abilities and to also deepen the contents.

Intuition and perception about geometrical figures allow us to detect the essence behind the properties of geometrical figures. They should be backed up by logical reasoning, but they can also guide logical reasoning. The aim of the study of geometrical figures in lower secondary school mathematics is to improve these abilities.

- b. To help students understand mathematical reasoning and extend the ability to express logically

Forms of reasoning include inductive reasoning, analogical reasoning, and deductive reasoning. Naturally, these are taught starting in elementary school.

In lower secondary school mathematics, the major goal is for students to understand the differences among these forms of reasoning and to be able to use them appropriately. In general, inductive and analogical reasoning are important methods used to discover general properties from concrete experiences.

However, students are to understand that conclusions obtained through inductive or analogical reasoning are not always valid and deductive reasoning is necessary to validate them.

For example, we may inductively conclude that “the sum of the three interior angles of a triangle is  $180^\circ$ ” by examining several triangles. However, it is impossible to show that this is true with all triangles inductively, and we must use deductive reasoning. As in this example, students are to understand that inductive or analogical reasoning is important to discover new ideas while deductive reasoning is important to explain the validity of the discovery. It is necessary that students can use appropriate reasoning.

Students learn these three forms of reasoning in elementary schools, too.

For example, students discover inductively that “the sum of three interior angles of a triangle is  $180^\circ$ ”. Then, based on this idea, students learn to explain the sum of angles in a quadrilateral using deductive reasoning, making the rationale explicit.

A major unique feature of lower secondary school mathematics is the use of deductive reasoning in the content domain, “B. Geometrical Figures”.

This is because of the following reasons: the contents about geometrical figures are suitable for the study of deductive reasoning, many problems may be examined, and the process of reasoning can be illustrated visually in geometrical figures.

Another benefit of deductive reasoning is that concepts and properties about geometrical figures may be organized systematically instead of just as a collection of unrelated ideas. Moreover, students in lower secondary schools are in the developmental stage where they are interested in deductive reasoning and their ability to carry out such reasoning is also increasing.

Therefore, an important objective of instruction of geometrical figures is the understanding of the meaning and the need for mathematical reasoning and their methods.

In order to achieve this objective, students must first understand the meaning of definitions, which is the foundation of reasoning, and also the methods of reasoning. It is necessary to teach them about what ideas can be used as rationale so that students may understand the meaning and methods of mathematical reasoning.

Reasoning is a process of validating a certain idea first to oneself and then to convince others. Therefore, through the instruction of reasoning about geometrical figures, it is important to develop students' ability not only to convince oneself but to convince others by making a logical argument.

Expressing one's own thinking process logically contributes to the development of the ability to reason.

In order to develop such an ability, it is important for students to be able to experience the value of expressing their ideas mathematically through activities in which they must explain and communicate their thinking processes and ideas by using mathematical representations. Instruction needs to proceed gradually, first listening to students' own ways of explanation, then by emphasizing how to make a logical argument, and finally by enabling them to explain their argument to others coherently. With respect to this instruction process, teaching of geometrical figures in grade 1 of lower secondary school requires special consideration.

The teaching of geometric construction and solid figures at this level is a step in cultivating the ability to logically examine situations. Therefore, it is necessary that teaching will not just focus on manipulation and procedures.

## (2) Overview of Content

### Treatment in Elementary School Mathematics

In elementary schools, the study of geometrical figures focuses on manipulation and intuitive treatments. Through activities such as observation and construction, ideas such as constituent parts of geometrical figures and positional relationships are discussed. The main content in domain, "C. Geometrical Figures", are as follows.

- a. Geometrical figures that have been studied by the end of grade 4 are triangles, quadrilaterals, squares, rectangles, right triangles, isosceles triangles, equilateral triangles, circles, spheres, parallelograms, trapezoids, cubes, and rectangular prisms (cuboids).

In addition, lines, right angles, sides, vertices, faces, angles, centers, radii, diameters, diagonals, and planes are discussed as constituent parts of geometrical figures.

Moreover, as a way to view and examine geometrical figures, activities used to observe and construct figures, compare the lengths of sides, focus on the shapes of angles and relationships of lines such as perpendicular and parallel, drawing sketches and nets, and representing the positions of objects are studied.

- b. In grade 5, polygons, regular polygons, and the congruence of geometrical figures are discussed.

Furthermore, various properties of geometrical figures and the construction of geometrical figures using those properties are learned. The meaning of the ratio of circumference ( $\pi$ ), prisms, and cylinders are also discussed.

The geometrical figures and ideas discussed include bases, lateral faces, congruence, the ratio of circumference, polygons, regular polygons, prisms, and cylinders.

- c. In grade 6, reduced and enlarged drawings and symmetrical shapes are discussed. We also discuss geometrical figures in domain "B. Quantities and Measurements." The primary contents are as follows:

- d. By the end of grade 4, the following topics related to the comparison and measurement of quantities are discussed: length, area, direct comparison and measurement of volume, how to determine the area of rectangles and squares, and angle measurements.

- e. In grade 5, ways to calculate the area of triangles, parallelograms and trapezoids are discussed. Units of volume ( $cm^3$ ,  $m^3$ ) and how to determine the volume of cubes and rectangular prisms are also discussed.

- f. In grade 6, estimating the area of shapes from our surroundings by approximating them, and calculating the area of circles, using 3.14 as an approximation for  $\pi$ , are discussed.

How to determine the volume of prisms and cylinders as well as the structure of the Metric System are also discussed.

### As for the treatment in lower secondary school mathematics in the current revision

It should be noted that the phrase, “through activities of observing, manipulating and experimenting”, is inserted in every grade level.

The purpose of this phrase is to proceed with the study of geometrical figures through activities used to observe, manipulate, and experiment in order to resolve the wondering, questioning, or immediate problems, to predict outcomes with good foresight, to devise a plan to solve the problem at hand, and to confirm the anticipated results.

In elementary school mathematics, the study of geometrical figures and their measurements are studied in two separate domains, “B. Quantities and Measurements” and “C. Geometrical Figures.”

However, in lower secondary school mathematics, because it is important to examine both properties of geometrical figures and their measurements in relationship to each other, both properties of geometrical figures and their measurements are studied in domain, “B. Geometrical Figures.”

The main contents in each grade are as follows:

#### a. Grade 1 of lower secondary school

- By using activities of observation, manipulation and experimentation, students’ understanding of plane figures are deepened by constructing geometrical figures with foresight and investigating relationships of geometrical figures, and by cultivating the ability to logically examine and express their own ideas.
- Activities of observation, manipulation and experimentation, examine and deepen students’ understanding of solid figures and extend students’ abilities to measure geometrical figures.

In grade 1 of lower secondary school, the focus will be to deepen students’ abilities to empirically view and think about geometrical figures through activities of observation, manipulation and experimentation. Students’ abilities of examining and expressing their own ideas logically should be cultivated.

First, by focusing on the similarities of geometrical figures, discuss the basic geometric construction such as bisecting an angle, drawing the perpendicular bisector of a segment, and drawing a perpendicular line. Students are to think about the steps of construction on their own by thinking about symmetry of geometrical figures and their defining characteristics, and they should be able to explain the steps coherently.

Also, enrich students’ ways of viewing geometrical figures by understanding the transformations of geometrical figures (translation, reflection, and rotation) and by examining the relationships of two figures from the perspective of geometric transformation.

Furthermore, students are to know positional relationships of lines and planes in space. In addition, forming solid figures by moving lines or plane figures, representing solid figures on a plane, and interpreting properties of solid figures from their representations on a plane are also discussed. As for measurement of geometrical figures, arc length and area of sectors, the volume and surface area of basic prisms and pyramids as well as spheres are discussed.

b. Grade 2 of lower secondary school

- Through activities of observation, manipulation and experimentation, students are to discover properties of basic plane figures and to verify them using properties of parallel lines.
- Help students understand congruence and deepen their ways of perceiving geometrical figures. Foster their ability to examine and express their own ideas logically by verifying properties of geometrical figures based on the congruence conditions of triangles.

Starting in this grade level, properties of geometrical figures will be examined through reasoning. In this grade, basic plane figures are the primary objective.

Through activities of observation, manipulation and experimentation, students are to discover the properties about angles in triangles and other polygons, and verify those properties using the properties of parallel lines.

Students are to understand the meaning of congruence of plane figures, and also to verify properties of triangles and parallelograms by using ideas such as the congruence conditions of triangles.

They will also study about discovering new properties by reading proofs of properties of geometrical figures.

c. Grade 3 of lower secondary school

- Students are to verify properties of geometrical figures based on such ideas as the similarity conditions of triangles. Extend their ability to examine and express their own ideas logically and enable them to examine by using properties of similar figures.
- Through activities of observation, manipulation and experimentation, students discover the relationship between inscribed angles and the central angles. They are to become able to use the relationships in their future examination of geometrical figures.
- Discover the Pythagorean Theorem through activities of observation, manipulation and experimentation, and enable students to use it in their future examination of geometrical figures.

In this grade, the students' abilities of reasoning mathematically that was cultivated in grade 2 of lower secondary school will be extended. The aim is to enable students to logically examine geometrical figures. Topics such as proving properties of geometrical figures using the similarity conditions of triangles, comparing lengths of segments in relationship to parallel lines, the relationship of inscribed angles and central angles, and the Pythagorean Theorem, are particularly appropriate in extending this ability.

We will summarize the difference from the prior COS in the domain, "B. Geometrical Figures." It was decided that line and point symmetry and volume of prisms and cylinders that were discussed in grade 1 of lower secondary schools has been shifted down to elementary schools. Volume and surface area of spheres have been shifted down to grade 1 of lower secondary school from upper secondary school.

Geometric transformation (translation, reflection, and rotation) and projection of geometrical figures are discussed in this grade.

The relationship between inscribed angles and central angles has been shifted from grade 2 to grade 3 of lower secondary school.

In grade 3 of lower secondary school, the relationship between inscribed angles and central angles has been shifted to grade 3 from grade 2. The converse of the Central Angle Theorem is also discussed.

Moreover, the ratios of area and volume of similar figures has been shifted from upper secondary school to this grade level.



## C. Functions

### (1) Purposes of teaching “Functions”

In examining natural and social phenomena, it is effective to concisely grasp and represent various relationships such as correspondence relationships, dependence, and causal relationships in those phenomena. In lower secondary school mathematics, an aim is to enable students to capture relationships and rules in various phenomena mathematically so that they can be mathematically examined and processed.

Therefore, in lower secondary school mathematics, it is important to gradually increase students’ abilities to identify two quantities in the phenomena and to represent and examine their functional relationships during the three years of lower secondary school.

The following two aspects may be considered with respect to the purposes of teaching functions in lower secondary school mathematics:

- Functional ways of viewing and thinking are often necessary to examine and understand phenomena in our surroundings.
- Functional ways of viewing and thinking, which have developed through the study of various functions, play an important role in reflecting on what has been learned as well as future learning in various fields of study.

### ① Functions, and their tables, algebraic expressions, and graphs

Based on elementary school mathematics, lower secondary school mathematics aims to help students understand the concept of functions in relationship to the expansion of the range of numbers and algebraic expressions with letters. At the same time, it aims to develop students’ abilities to identify, represent, and examine functional relationships as they try to grasp and explain concrete phenomena, as well as extend the ability to view and think using functions. The following are two goals of teaching functions in lower secondary school mathematics:

- a. To help students understand the basic concepts and properties of functions.

Through the examination of variation and correspondence of two quantities, help students understand the unique features of graphs, rates of change, and other properties of functions by representing the relationships, such as direct and indirect proportions, linear functions, and functions in the form of  $y = ax^2$ , in the form of algebraic expressions with letters. At the same time, also help students to understand the concepts of coordinates, variables, and domains: the fundamental concepts related to functions.

- b. To extend students’ abilities to investigate functions by connecting tables, algebraic expressions, and graphs.

Students are expected to understand that tables, algebraic expressions, and graphs are useful in identifying unique features of functions. Enable students to investigate features of functions efficiently by representing variations and correspondences of two quantities that are changing simultaneously by using tables, algebraic expressions, and graphs. In lower secondary school mathematics, examination of functions through the focusing on forms of algebraic expressions is emphasized.

Students are to represent the relationships between two quantities in tables and then investigate the way the quantities vary using a table. They may then discover a pattern of correspondence and be able to represent it in an algebraic expression. Moreover, students are to investigate the way of variation by creating a table from an algebraic expression, and also to determine the rate of change from an algebraic expression. They are to also investigate the way quantities vary by drawing graphs from an algebraic expression or a table.

As shown here, the emphasis is to extend students’ abilities in using tables, algebraic expressions, and graphs by making connections among them instead of treating them individually.

## ② Grasping and explaining phenomena using functions

For this idea, the lower secondary school mathematics will aim at the following:

- a. To extend students' abilities in explaining the usage of functions

Enable students to use functions in explaining daily and mathematical situations.

Functions were developed to efficiently record and examine natural and social phenomena. The understanding of functions can be deepened through explaining to others the things that became clear by representing them in tables, algebraic expressions, and graphs.

With respect to the use of functions in our daily life, it is important to capture relationships in phenomena by idealizing or simplifying them, that is, to assume quantitative relations in phenomena to be known functions, so that problems may be processed and the obtained results are interpreted by examining them in context.

It is important to note that the results obtained may be applicable within the range in which our assumptions are valid. Moreover, the understanding of previously learned contents may be deepened by re-examining them more dynamically from the viewpoint of functions, by identifying what stays constant among the things that are changing, or by explaining those ideas to others.

- b. Nurture the disposition of using ways to observe and think with functions to grasp phenomena

Nurture the ways of observing and thinking with functions by identifying, representing, and examining functional relationships.

For example, nurture the attitude to solve various problems by making predictions about unknown situations using known situations or by transforming the situations to make them easier to think about.

Help students understand the merits in observing and thinking with functions by using them.

## (2) Overview of Contents

### Treatment in the Elementary School Mathematics

In the elementary school mathematics, students studied two co-varying quantities based on their experiences. The main contents are as follows:

- a. By the end of grade 4, students have learned to investigate two quantities that vary simultaneously, to represent and interpret the way quantities change from broken line graphs, to identify co-varying quantities from their surroundings, and how to investigate quantitative relationships by representing them in tables or graphs.

They have also learned how to represent positions of objects.

- b. In grade 5, students are to examine relationships between two quantities by using tables, and they also study simple cases of direct proportional relationships. In addition, they are to deepen their understanding of algebraic expressions that represent quantitative relationships. They learn to focus on the way quantities correspond and change in order to examine quantitative relationships that can be represented in simple algebraic expressions.

- c. In grade 6, students are to understand direct proportional relationships and use them to solve problems. Also, in order to deepen their understanding of direct proportional relationships, they study inverse proportional relationships.

In the current revision, the domain, "Quantitative Relations," is established starting from grade 1 instead of grade 3, as was the case in previous revisions, so that it can be studied in all 6 years of elementary school. Moreover, as the example of the treatment of direct proportional relationships in grades 5 and 6 shows, a curriculum based on the spiral approach is being emphasized.

It was decided that inverse proportional relationships would be taught in grade 6.

### Treatment in Lower Secondary Schools

In the lower secondary school mathematics, students are taught to identify, represent, and examine functional relationships through concrete phenomena. The differences from the elementary school mathematics are the inclusion of negative numbers in the domains of functions, the use of the coordinate plane for graphing, and the use of letters in algebraic expressions.

In this revision, the meaning of functional relationships is taught in grade 1 of lower secondary school while various phenomena and functions are discussed in grade 3.

In addition, in every grade level, the emphasis is placed on grasping and explaining concrete phenomena using functions while making connections among tables, algebraic expressions, and graphs.

a. Grade 1 of Lower Secondary School

- Deepen students' understanding of direct and inverse proportional relationships through the examination of how 2 quantities identified in concrete phenomena change and correspond. Cultivate the ability to represent and examine functional relationships.

In grade 1 of lower secondary school, students are expected to understand what it means to say, "... and ... are in a functional relationship," or "... is a function of ...". As a basic model of quantitative relationships, direct and inverse proportional relationships studied in elementary schools will be re-conceptualized as functions. For that reason, ideas such as, when one value is determined, the other value is also determined, variables and the range of values they can assume, and coordinate plane are studied.

In addition, the domains of direct and indirect proportional relationships studied in the elementary school mathematics were non-negative numbers. However, in the lower secondary school mathematics, the domain will be expanded to rational numbers, including negative numbers.

In elementary school mathematics, features of direct and indirect proportional relationships were examined through tables, algebraic expressions, and graphs. Those features will be re-examined more generally as properties based on algebraic expression in the form of  $y = ax$  and  $y = \frac{a}{x}$ . As this illustrates, in the lower secondary school mathematics, students are expected to understand the usefulness of algebraic expressions with letters in the examination of functional relationships. In the current revision, it was decided that representing direct proportional relationships in algebraic expressions with letters would be studied in elementary school mathematics. To represent and process functional relationships, tables, algebraic expressions, and graphs are used. Students have studied tables, algebraic expressions, and graphs in elementary school mathematics somewhat. However, in lower secondary school mathematics, students are to deepen their understanding of the features of basic functions such as direct and inverse proportional relationships.

As a way to use direct and inverse proportional relationships, students are to grasp and explain concrete phenomena using those relationships. Enable students to decide whether or not concrete phenomena represented in algebraic expressions are direct or inverse proportional relationships, and to solve problems by considering concrete phenomena as direct or inverse proportional relationships. Students should be able to explain the rationale behind their judgments or solutions to others.

b. Grade 2 of Lower Secondary School

- Help students understand linear functions through identifying and examining how two quantities identified in concrete phenomena change and correspond. Nurture the ability to identify, represent, and examine functional relationships.

In grade 2 of lower secondary school, as an extension of the study in grade 1, linear functions will be taught. The aim is to deepen the understanding of the features of graphs and the rates of change by making connections among tables, algebraic expressions, and graphs.

As a way to use linear functions, students are to grasp and explain concrete phenomena using linear functions.

Enable students to decide whether or not concrete phenomena represented in algebraic expressions are linear functions and enable them to make predictions about future situations by considering observations or the results of experiments about concrete phenomena as linear functions. Students should be able to explain the rationale behind their judgments or predictability to others.

In linear equations in the form of  $ax + by + c = 0$ ,  $b \neq 0$ , when the value of  $x$  is determined, the value of  $y$  is also determined. Therefore, they can be considered as an algebraic expressions that describe functional relationships between variables  $x$  and  $y$ .

Through this type of perspective, help students develop a unified understanding of functions and equations.

c. Grade 3 of Lower Secondary School

- Help students understand functions of the form,  $y = ax^2$  through the examination of how two quantities identified in concrete phenomena change and correspond. Further extend the ability to identify, represent, and examine functional relationships.

In grade 3 of lower secondary school, the study of linear functions is further extended. As a typical example of functions beyond direct and indirect proportional relationships and linear functions that may be found in concrete phenomena encountered by students in everyday situations, functions of the form  $y = ax^2$  will be taught.

Deepen the understanding of features of functions such as rates of change and characteristics of graphs by making connections among tables, algebraic expressions, and graphs.

As a way to use functions of the form  $y = ax^2$ , students are to grasp and explain concrete phenomena using linear functions.

It is important for students to decide whether or not concrete phenomena represented in algebraic expressions are functions in the form,  $y = ax^2$ , and to make predictions about future situations by considering observations or the results of experiments about concrete phenomena as functions in the form,  $y = ax^2$ .

Students should be able to explain the rationale behind their judgments or predictability to others.

Also, the fact that there are phenomena that cannot be grasped by the previously learned functions will be discussed.

Through these studies, the meaning of function as the correspondence of one unique value to each value in the domain will be clarified so that it can become the foundation for future study. The differences from the prior revision in domain "C. Functions" are summarized here.

In grade 1 of lower secondary school, terms related to functions are introduced instead of waiting until grade 2, as was the case previously.

In grade 3 of lower secondary school, various phenomena and functions that were discussed in Mathematics I of upper secondary school are now included.

## D Making Use of Data

### (1) Purposes of teaching, "Making Use of Data"

In our rapidly developing information society, we are often demanded to make judgments by collecting data purposefully and interpreting trends in the data even about situations where definitive answers are difficult to obtain.

The purpose of this domain is to cultivate statistical and probabilistic perspectives and reasoning through understanding and using the basic methods of data analysis to identify and explain trends in data.

Data here refer to the statistical and probabilistic data obtained from various phenomena. In our daily life, we often have to make judgments about uncertain phenomena.

In those situations, it is necessary to make appropriate judgments by using data. In the past revisions, the study of probability and statistics in lower secondary schools tended to emphasize "organizing" data. In this revision, this domain is named, "Making Use of Data," in order to make it clear that the emphasis is on thinking and making judgments using organized data.

#### ① About the treatment of uncertain phenomena

The objects of mathematics are not just those ideas in which definitive answers and results are possible, such as those found when solving equations and proving properties of geometrical figures. This domain aims at enabling students to understand that situations such as features of an entire population or events that are influenced by chance are also objects of mathematical examination.

Also, it should be kept in mind that one of the features of this domain is that the results of this examination are not always one correct conclusion.

When grasping an entire population is difficult, we can still examine it by using histograms, representative values, or sampling techniques. We can use probability to examine the events that are influenced by chance.

As is the case in the other domains, numbers, quantities, and geometrical figures are objects of study, but the results obtained are trends about phenomena.

#### ② About engaging in problem solving

Although it is important for students to be able to actually make histograms or determine representative values or probabilities, it is just as important to make sure that students can use those ideas to predict or make judgments about concrete phenomena.

For example, enable students to not only be able to determine that "the probability is  $\frac{1}{3}$ " but also understand what it means that "the probability is  $\frac{1}{3}$ " and use it to make judgments or explain things.

Therefore, in order to teach these topics for students to become aware of these ideas, it is important that they engage in purposeful activities such as taking advantage of problems found in everyday life or society, constructing problems, exploring solution methods, and seeking the answers.

It is necessary to teach students to value not only "the probability is  $\frac{1}{3}$ " but also "because probability is  $\frac{1}{3}$ ..." In addition, it is important to help students reflect on the predicted results or decisions in light of the concrete phenomena so they can evaluate and improve them.

#### ③ About grasping the objects and explaining

As stated in ① and ②, in this domain, it is important to enable students not only to make histograms or determine probability, but also to use them to examine phenomena or to identify trends.

In doing so, it is necessary that students experience a series of activities: identifying problems from their everyday life or society, collecting data necessary to solve those problems, processing the collected data using tools such as computers, identifying trends, and then explaining their ideas.

As we teach this domain, considerations must be given to the unique feature of this domain, that is, uncertain phenomena are being discussed, so students can not only find the correct answers but also can explain their predictions or decisions as they make the rationale clear.

Moreover, help them improve the quality of their explanations through activities of explaining and communicating their own ideas.

## (2) Overview of Contents

### Treatment in Elementary School

As contents related to "Making Use of Data", in elementary school mathematics, the sorting and organizing of data, representing data in graphs and tables, and relative quantities, which are the basis of frequency distribution, are taught. The main contents are as follows:

- a. By the end of grade 4, students have learned the following topics: collecting, sorting, and organizing data purposefully, using tables and graphs to represent data for easier understanding, and making and reading bar graphs and broken line graphs.
- b. In grade 5, students are expected to understand averages of measurements and percentages. In addition, they study the collection, classification and organization of data purposefully, the representation of data in circle graphs or percentage bar graphs, and the exploration of patterns in data. Furthermore, they learn to use tables and graphs by selecting appropriate format purposefully.
- c. In grade 6, students learn about averages, and tables and graphs that represent frequency distribution. They also examine possible outcomes for concrete situations by organizing them systematically.

### Treatment in Lower Secondary School

In lower secondary school mathematics, based on the treatment in elementary schools, the following contents on probability and statistics are taught:

- a. Grade 1 of Lower Secondary School
  - To enable students to identify trends in data collected purposefully by organizing them in tables and graphs, possibly with the assistance of computer, and also by focusing on representative values and the distributions of data.

In grade 1, based on the study of statistical analysis and representations through the examination of averages and distribution of data in elementary school mathematics, students are helped to understand the need for and the meaning of histograms and representative values, and to use those ideas to identify and explain trends in the data.

On the surface, it might appear as if the same contents in elementary school mathematics are discussed. However, in lower secondary school mathematics, data is obtained from a wider range of situations including society in general and the amount of data will be much larger.

Students are expected to understand statistical methods for organizing and processing such data like histograms and representative values. Appropriate use of those methods and common errors are also studied.

An emphasis is on identifying and explaining trends in data using statistical methods. Considerations should be given so that instruction will not focus solely on making histograms or determining representative values.

Although making histograms or determining representative values by hand may be useful to understand why they are needed or what they mean; however, in instruction focusing on the efficient processing of data and identifying trends from the results, tools such as computers should be used actively.

In addition, as students study these contents, they are to learn about the meaning of errors and approximations as well as representing numbers in the form of  $a \times 10^n$ .

#### b. Grade 2 of Lower Secondary School

- Through activities of observation and experimentation with uncertain phenomena, students are helped to understand probability and use it to examine and represent the phenomena.

Based on the study of possible outcomes in elementary school mathematics, through activities of observation and experimentation with uncertain phenomena, students are helped to understand the meaning of empirical probability and theoretical probability and their relationship.

As is the case in grade 1 of lower secondary school, the emphasis is on grasping and explaining uncertain phenomena, and to avoid making the goal of learning solely on determining probability.

In the current revision of the COS, examination of possible outcomes by organizing them systematically is a topic to be studied in grade 6 of elementary schools; however, determining probability based on the analysis of possible outcomes is taught for the first time in this grade.

#### c. Grade 3 of Lower Secondary School

- Help students understand that trends in a population may be identified by collecting data from sample groups, perhaps by using computers, and by examining trends in the samples.

In grade 3, students are to understand the need for and the meaning of sampling, and in simple cases, will conduct sampling to investigate trends in a population.

It is not necessary to discuss errors associated with sample statistics. Rather, it is important that students can experientially understand that trends of a population may be estimated by collecting sample data and processing this data set.

Also, in relation to the idea of random sampling, it is possible to revisit the need for and the meaning of probability, which is in the content of grade 2.

Computers and other tools should be used to generate a large number of random numbers necessary in the sampling process.

It is also possible to use various information and communication networks, such as the Internet, to gather data or to investigate sampling processes.



## [Mathematical Activities]

### (1) Purposes of teaching "mathematical activities"

Mathematical activities are the various activities related to mathematics where students engage willingly and purposefully. As discussed in the explanation of "through mathematical activities" in the goal of mathematics, "various activities related to mathematics" include a wide range of activities. It is necessary that teaching through mathematical activities should take place in every content domain.

In this revision, the following three types of mathematical activities are particularly emphasized: activities for discovering and extending properties of numbers and geometrical figures based on previously learned mathematics; activities using mathematics in everyday life and in society; and activities explaining and communicating logically and with clear rationale by using mathematical expressions.

The purposes of further emphasizing these mathematical activities, particularly in light of the characteristics of mathematical activities and the basic policy for the revision of COS, are as follows:

#### ① Characteristics of mathematical activities

In principle, mathematical activities are carried out as problem solving. That is, they are a sequence starting with generating wonder and questions, formulating problems by formalizing them, understanding the problems, planning, implementing and reflecting on solution processes, generating new wonder and questions, generating conjectures, and formalizing problems.

Experiencing these series of activities provides opportunities for students to feel the joy of thinking and learning mathematics as well as the necessity for and the usefulness of mathematics. In addition, because these activities require students to persist, they provide opportunities for students to heighten their own self-esteem.

Furthermore, by listening to and adopting different ideas from others, these activities can promote students to understand each other better.

Therefore, mathematical activities are not processes of learning only mathematics but also particular content in the sense of learning to engage in mathematical activities.

Moreover, mathematical activities are a purpose of learning mathematics because we aim for students to use them in their future learning and in everyday life. By balancing these characteristics, it is desired that we nurture students' abilities to think mathematically while experiencing the joy of mathematical activities as they learn the contents of each domain and make connections among them.

## ② Mathematical activities and the basic policies of the revision of the Course of Study

Considering education and schooling are intentional and planned activities used to achieve wishes and purposes, the involvement of teachers is necessary, and it is an invitation for the self-reliance of students.

Therefore, teachers' involvement is sometimes direct but gradually becomes, as appropriate, more individualized and indirect. Eventually it is necessary that opportunities are given so that students can act independently and autonomously.

In the report by the Central Education Council, with respect to curriculum and instruction, "mastery," "use," and "inquiry" are all emphasized. This is because the Council was concerned that "mastery education" and "inquiry education" have been considered as opposing perspectives. Thus, it aims to correct such a perspective.

"Mastery" means that important ideas are to be taught well.

"Inquiry" means freely solving problems using what has been mastered. Moreover, in that process, opportunities will arise for students to discover properties of numbers and geometrical figures or to realize the need for using mathematics.

As those discoveries and realizations are explained and communicated with each other, the motivation for mathematical activities will be strengthened.

In the process of "inquiry," it is necessary to "use" what has been "mastered."

It should be kept in mind that "mastery," "use" and "inquiry" do not always take place in this order; rather, they develop mutually and influence each other.

By "using" knowledge and skills, "mastery" can be furthered. In the process of "inquiry," we reflect on what it means to "master" or "use." We must keep this in mind.

From this background, it is clear that "mastery," "use," and "inquiry" should be pursued in a lesson through mathematical activities or within context. Of course, opportunities to practice should be provided so that students can solidify their mastery of knowledge and skills.

## (2) Overview of Content

In elementary school mathematics, teaching through mathematical activities is emphasized. In lower secondary school mathematics, based on those experiences, an increased emphasis is placed on engaging students in mathematical activities more autonomously.

Mathematical activities include activities used to discover the properties of numbers and geometrical figures based on what has been learned in the world of mathematics as well as activities used to examine and process concrete phenomena from outside of the world of mathematics such as those found in daily life and in society by connecting them to the ideas in the world of mathematics.

Moreover, activities for explaining and communicating using mathematical representations are critical in order to further refine those mathematical activities and to share problems and the results of the examinations. Therefore, it is necessary that activities used to explain and communicate with mathematical representations should be done in conjunction with the two types of activities discussed above.

Based on these ideas, the following three types of mathematical activities have been positioned as the prototypical mathematical activities in lower secondary school mathematics, and opportunities to engage those activities are to be provided in each grade.

Therefore, it is necessary that none of the three types of activities is omitted in the teaching of contents in each grade. Moreover, as opportunities to engage in mathematical activities are set up, it is important to clarify which type of activities are to be emphasized in teaching.

The reason the contents of mathematical activities in grade 1 of lower secondary school is different from those of grades 2 and 3 is because of students' developmental and academic levels. Also, those differences reflect the broadening and rising complexity of the contents in the four domains.

### **① Activities to discover and extend properties of numbers and geometrical figures based on mathematics students have learned previously**

Activities used to discover and extend properties of numbers and geometrical figures based on mathematics learned previously are extensive and creative activities. Mathematical ways of perceiving and thinking play important roles.

Mathematics generated through those activities range from mathematical facts such as concepts, properties and theorems to procedures such as algorithms. Mathematical reasoning such as induction, analogy, and deduction, is also refined.

In grade 1 of lower secondary school, instruction should emphasize activities used to discover properties of numbers and geometrical figures, while in grades 2 and 3, also set up opportunities to engage in activities to extend those properties.

### **② Activities to use mathematics in everyday life and in society**

These are the activities in which problems from daily life and society are solved by representing them formally by idealizing or simplifying them, processing those representations in the world of mathematics, and then interpreting the results obtained in the context of daily life and in society.

Help students experience the purpose for using mathematics by engaging them in activities of examining and processing situations from their daily lives or from society. Students can also experience the necessity and the role that knowledge and previously studied skills studied play, as well as mathematical ways of perceiving and thinking.

In grade 1 of lower secondary school, instruction should emphasize the activities which use mathematics in daily life, while in grades 2 and 3, the scope is broadened to set up activities for the use of mathematics in society as well.

### **③ Activities to explain and communicate logically, with clear rationale by using mathematical expressions**

These are activities in which the facts and procedures about numbers and geometrical figures as well as processes of thinking and rationale for judgments are appropriately represented using words, numbers, algebraic expressions, diagrams, tables, and graphs. They also include activities used to communicate and share ideas using mathematical representations as well as activities used to explain mathematically what was discovered, the process of discovery and the rationale for judgment.

In grade 1 of lower secondary school, instruction should emphasize activities used to explain and communicate using one's own words, while in grades 2 and 3, activities should be set up to give students opportunities to explain and communicate logically and clearly by making the rationale explicit.

It should be noted that the third type of activity should be carried out in conjunction with the first two types of activities. Therefore, it is necessary to clarify which type of activity will be the focus as opportunities to engage in mathematical activities are set up.

## Section 3: Contents of Each Grade

### Grade 1 of Lower Secondary School

#### A. Numbers and Algebraic Expressions

- (1) To be able to understand positive and negative numbers in concrete situations, and to perform the four fundamental operations with these numbers, and to be able to use, represent and consider by using positive and negative numbers.
  - (a) To understand the necessity and meaning of positive and negative numbers.
  - (b) To understand the meaning of the four fundamental operations with positive and negative numbers through relating the meaning to the four fundamental operations with numbers learned in elementary school.
  - (c) To perform the four fundamental operations with positive and negative numbers.
  - (d) To represent and process by using positive and negative numbers in concrete situations.

[Terms and Symbols]

natural number, sign, absolute value

[Handling the Content]

- (1) In connection with (1) under “A. Numbers and Algebraic Expressions” in “Content,” the concept of number sets and the possibility of the four fundamental operations should be dealt with.

In elementary school mathematics, by the end of grade 4, the meaning and properties of the four arithmetic operations have been taught and students have been expected to master those ideas. In grades 5 and 6, the students’ understanding is deepened through investigating that the commutative law, associative law and distributive law hold true with decimal numbers and fractions.

The meaning of and calculation with decimal numbers are studied in grade 5 while the meaning of and calculations with fractions are studied in grade 6. Thus, the students’ number sense and perceptions are broadened. In grade 6, the focus is on mastery and applications of what have been learned.

In lower secondary school mathematics, based on the study in elementary school, the range of numbers is expanded to include positive and negative numbers in grade 1. Students are expected to understand the need for and the meaning of positive and negative numbers. In addition, students are to understand the meaning of the four arithmetic operations and to develop computational fluency with positive and negative numbers. Another aim is to enable students to use positive and negative numbers to represent and process concrete situations.

#### Need for and meaning of positive and negative numbers

As for the need for positive and negative numbers, help students understand them by making connections to prior experiences or everyday situations where positive and negative numbers are used, such as showing the difference in the high temperature from yesterday.

As students develop the understanding of positive and negative numbers, they should also use positive numbers and negative numbers so that they may know their merits such as the following:

- Opposite directions or characteristics may be represented by numbers
- Size may be compared
- Number lines may be used as a representation
- Subtraction is always possible
- Addition and subtraction may be unified

The range of numbers has been expanded gradually since grade 1 of elementary school.

In grade 1 of lower secondary school, by expanding the range of numbers to include positive and negative numbers, it is necessary to reconsider the set of numbers. For example, in elementary school mathematics, integers are referred to as 0 and positive integers. However, in lower secondary school mathematics, negative integers will be included in this set of numbers, and integers as a mathematical concept is defined. Then, we aim to deepen students' understanding of the concept of numbers by discussing the possibility of arithmetic operations with numbers in this set.

#### **Four arithmetic operations with positive and negative numbers and their meaning**

The four arithmetic operations with positive and negative numbers become possible as their meaning is expanded from those of elementary school mathematics. For addition and multiplication, commutative and associative properties still hold true, as does the distributive law.

Here, the emphasis is to help students to understand the meaning of the four arithmetic operations and to develop the computational fluency with positive and negative numbers. To expand the range of numbers and think about the meaning of the four arithmetic operations will be important in grade 3 of lower secondary school when square roots are taught.

In elementary school mathematics, by thinking about multiplication and division of fractions, it was concluded that division could be thought of as multiplication by using reciprocals. Similarly, in lower secondary school, by considering addition and subtraction of positive and negative numbers, we can see subtraction as addition. Suppose we are given the calculation,  $3 - 2$ . If we consider the symbol “ $-$ ” as an operation symbol, it is subtraction. However, if we represent the expression as  $(+3) + (-2)$ , we can see it as addition. By seeing addition and subtraction from a unified perspective, we can consider an algebraic expression with both addition and subtraction as the sum of positive and negative terms. We can then calculate it more efficiently. In addition, if  $a > b$ , then the distance between the two points representing  $a$  and  $b$  on a number line can be represented as  $a - b$ .

Mastery of this way of seeing and calculating algebraic expressions is important as it also plays an important role in combining algebraic expressions or solving equations, which are to be studied later. The mastery should be developed not only in the study of positive and negative numbers but also in the study of algebraic expressions with letters and equations, which are to follow.

## Representing and processing with positive and negative numbers

Positive and negative numbers are not only the objects of calculation but are also helpful in representing various phenomena and changes easily and also make efficient processing possible. For example, we can represent how well we are achieving the previously set goal by representing the difference between the target and actual values using positive and negative numbers. We can also calculate the average (arithmetic mean) efficiently by using an estimated average. Help students to deepen their examination of phenomena by using positive and negative numbers in concrete situations, and enable them to understand the need for positive and negative numbers.

- (2) To cultivate the ability to represent the relationships and rules of numbers and quantities in algebraic expressions using letters, to read the meaning of these expressions, and to be able to calculate algebraic expressions using letters.
  - (a) To understand the necessity and meaning of using letters.
  - (b) To know how to express multiplication and division in algebraic expressions using letters.
  - (c) To calculate addition and subtraction with simple linear expressions.
  - (d) To understand how to represent the relationships and rules of numbers and quantities in algebra.

[Terms and Symbols]

term, coefficient,  $\leq$ ,  $\geq$

[Handling the Content]

- (2) In connection with (2)-(d) under “A. Numbers and Algebraic Expressions” in “Content,” representing magnitude relationships by using inequalities should be dealt with.

In elementary school mathematics, by the end of grade 4, students have learned about representing quantitative relationships and rules concisely by using algebraic expressions with numbers, words, and symbols such as  $\square$ ,  $\triangle$ , interpreting algebraic expressions, and using formulas. In grade 5, students learn ways to perceive and investigate relationships expressed in simple algebraic expressions. In grade 6, the use of letters such as  $a$  and  $x$  in place of words or symbols representing quantities in algebraic expressions are learned. They also learn to evaluate algebraic expressions by substituting numbers in those letters.

In lower secondary school mathematics, by studying the following ideas about algebraic expressions help students understand the benefits of using letters: representing quantitative relationships and rules using algebraic expressions with letters; interpreting the meaning of algebraic expressions; and, calculating with algebraic expressions with letters. In teaching these ideas, give full consideration to the levels of understanding of the materials in elementary school mathematics and carefully discuss the generality afforded by letters so that students can gradually develop an understanding of the use of letters without too much confusion. For example, incorporate activities like representing quantitative relationships and rules using algebraic expressions with numbers, words, and symbols such as  $\square$ ,  $\triangle$ , interpreting algebraic expressions, and evaluating algebraic expressions by substituting numbers in words and symbols.

## Need for and meaning of the use of letters

Algebraic expressions with letters are needed to represent quantitative relationships and rules concisely, clearly, and generally. For example, if we are to state the commutative law of addition in words, we will say, "the sum of two numbers will remain the same when the addends are reversed." If we use specific numbers, we can say it concisely, for example, " $2 + 3 = 3 + 2$ ," but we cannot express that the commutative law holds true in general. However, by using letters  $a$  and  $b$ , we can express the commutative law concisely, clearly, and generally, by saying " $a + b = b + a$ ."

Furthermore, by using algebraic expressions with letters, we can examine quantitative relationships abstractly, by considering them as relationships of numbers, without being bound by the specific contexts. For example,  $s = ab$  may be representing the relationships such as "area of a rectangle = length  $\times$  width", "cost = unit price  $\times$  number of units," or "distance = speed  $\times$  time", and in every situation, we can transform the equation into  $a = \frac{s}{b}$  to examine the quantitative relationships.

Another benefit of algebraic expressions with letters is that we can communicate our thought processes to others accurately using algebraic expressions with letters. For example, if we arrange match sticks as shown in Figure 1, the number of match sticks you need to make  $n$  squares may be represented as  $4n - (n - 1)$  or  $2n + (n + 1)$  as shown in Figures 2 and 3, respectively.

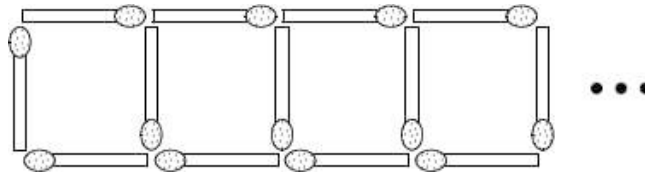


Figure 1

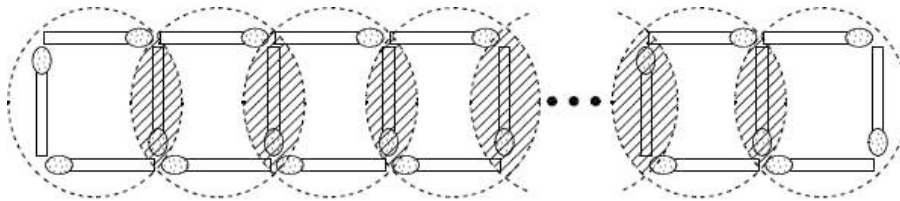


Figure 2

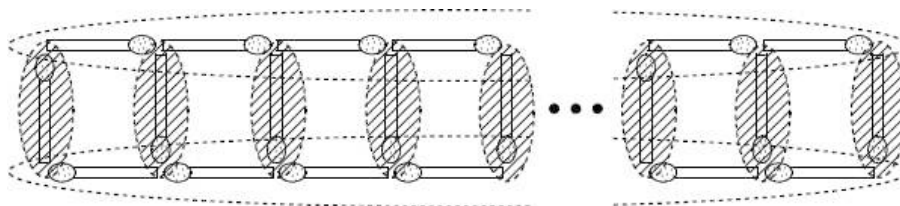


Figure 3

## To know about how to express multiplication and division of algebraic expressions with letters

Students are to learn that, when expressing quantitative relationships and rules using algebraic expressions with letters, the multiplication symbol, "×", is usually omitted between two letters or between a letter and a number, and division is represented by the fraction form instead of using the division symbol, "÷", unless it is specifically needed. For example, the following are typical notations:

$$a \times b = ab, \quad a \div b = \frac{a}{b}, \quad a \times a = a^2$$

Through these conventions, algebraic expressions can be written more concisely and their manipulation becomes more efficient. It is important to note that the expressions such as  $ab$ ,  $\frac{a}{b}$ ,  $a + b$ , and  $a - b$ , represent not only the operations to be performed but also the results of the operations. It is necessary that teachers pay close attention, especially at the beginning of the study of algebraic expressions with letters, because students often feel strange leaving operation symbols in expressions, for example,  $3a + 2$  or  $5x - 5$ .

## Addition and subtraction of linear expressions

As for calculations with algebraic expressions with letters, addition and subtraction of linear expressions will be taught. The goal is to master simple calculations necessary in solving linear equations with one variable. Therefore, the focus is on combining like terms involving linear expressions with one letter such as  $(2x-3)+(x+1)$  or  $2(3x+4) - 3(x-5)$ .

To develop an understanding of the ways to calculate with algebraic expressions with letters, it is important to make connections to the world of numbers, for example, calculations with letters also involve terms that calculations with numbers do, and properties of operations remain valid.

Moreover, enable students to use calculations with specific numbers or daily situations to understand calculations of algebraic expressions. For example,  $a - (b + c) = a - b - c$  may be related to  $5 - (3 + 2) = 5 - 3 - 2$ , or to the calculation of the amount of change with  $a$ -yen when a  $b$ -yen item and a  $c$ -yen item are purchased.

## Writing and interpreting algebraic expressions with letters

Algebraic expressions with letters are a powerful form of representation. It is important for students to experience their benefits by writing algebraic expressions with letters to represent quantitative relationships and rules and by interpreting those expressions. It is also important that students develop the disposition to utilize algebraic expressions actively.

In order to write and interpret algebraic expressions, quantities represented by letters and their relationships must be understood. For example, if the admission fee for an adult at an art museum is  $a$ -yen and the admission for a child is  $b$ -yen, the total cost for an adult and two children can be represented as  $a+2b$ . In the same situation, the expression,  $a - b$ , can be interpreted as the difference of the admission fees for an adult and a child.

To represent with algebraic expressions with letters, we must decide what operations to perform with quantities that are represented by letters. Grasping the relationships among quantities will be easier if we consider specific numbers instead of just thinking with letters.



As for algebraic expressions that represent quantitative relationships, the size relationships of quantities will be represented in equations or inequalities. For example, in the previous art museum example, the relationship, "the total cost of admission for one adult and two children were 1000-yen," can be represented by,  $a + 2b = 1000$ . Alternately, we can express the relationship in,  $2b = 1000 - a$ , or,  $a = 1000 - 2b$ . The equal sign here is used to represent the equality of quantities, not as a symbol to indicate an operation and its result. That is, the equation,  $a + 2b = 1000$ , means that " $a + 2b$  and 1000 are equal (because they are both representing the total cost of admissions, therefore, they are balanced)," not "when you perform the calculation  $a + 2b$ , the answer is 1000". Being able to interpret algebraic expressions this way is very closely related to the study of linear equation. In addition, if we say, "we received change of 1000-yen when we paid the admission for one adult and two children," it can be interpreted that "the total amount we paid was less than 1000-yen". Therefore, we can represent the relationship as  $a + 2b < 1000$ . Here the aim is to help students write and interpret quantitative relationships represented in algebraic expressions with inequality symbols and to deepen their understanding of algebraic expressions with letters.

A letter may stand for different values, and in order to help students develop that understanding, it is helpful to have them evaluate the algebraic expressions with letters by substituting different numbers in the expressions. This is also important to understand the meaning of the solutions of equations. When evaluating algebraic expressions, make sure that students can correctly substitute negative values as well. In addition, avoid making the evaluation of algebraic expressions just computation practice by connecting the expressions to concrete situations.

- (3) To understand equations, and to be able to look at them by using linear equations with one unknown.
- (a) To understand the necessity and meaning of equations, as well as the meanings of letters within equations and their solutions.
  - (b) To know how to solve equations based on the properties of equalities.
  - (c) To solve simple linear equations with one unknown and make use of the linear equations in concrete situations.

[Terms and Symbols]

transposing terms

[Handling the Content]

- (3) In connection with (3)-(c) under "A. Numbers and Algebraic Expressions" in "Content", solving simple proportional equations should be dealt with.

In lower secondary school mathematics, based on the study of algebraic expressions with letters in grade 1, the necessity and the meaning of equations and the meaning of their solutions and methods to solve linear equations with one variable based on the equality relationships will be taught. Students are also expected to understand the benefits of algebraic manipulations through the study of these ideas.

## The necessity and the meaning of equations and the meaning of their solutions

Equations are algebraic expressions with variables (unknowns) that represent the conditions for the equality relationships, and they are needed to determine the value that will satisfy the conditions accurately. The solutions of equations are those values that satisfy the conditions. For example, the equation,  $x + 3 = 5$ , is a mathematical representation of, "the sum of  $x$  and 3 equals 5," but it can also be thought of as the condition for the value of  $x$  that will satisfy the equality relationship. Whether or not the condition is satisfied depends on the value the variable  $x$  in the equation will take. If we consider the domain of  $x$  to be the set of integers, we can substitute  $\dots, -3, -2, -1, 0, 1, 2, 3, \dots$  into  $x$ . We will then find out that when, and only when, the value of  $x$  is 2, the equality relationship is satisfied. Therefore, 2 is the solution of this equation. This way of determining the solution may be important in understanding the meaning of the solution of an equation, but it is not an efficient procedure. Equations can be solved by procedural manipulation based on the properties of equality relationships; therefore, they are useful in obtaining solutions in concrete situations efficiently.

### Properties of equality relationships

To solve linear equations with one variable, we transform the given equations using the properties of equality relationships into the form,  $x = \alpha$ . The four properties of equality that are used in the process are as follows:

① If  $a = b$ , then  $a + c = b + c$

② If  $a = b$ , then  $a - c = b - c$

③ If  $a = b$ , then  $ac = bc$

④ If  $a = b$  and  $c \neq 0$ , then  $\frac{a}{c} = \frac{b}{c}$

Properties ① and ② may be considered the same from a unified perspective of addition and subtraction using positive and negative numbers. Similarly, ③ and ④ may be considered the same, as well.

It is important that students understand the properties of equality relationships with a concrete image, perhaps developed through activities that use a pan balance, and to be able to use them in solving equations.

In studying how to solve equations using the properties of equality relationships, students should understand not only the merit of being able to solve equations through manipulation but also the fact that the properties of equality relationships justify the transformation of equations. It is particularly important that teachers help students to understand and to be able to explain the transposition of terms, which is a useful technique in solving equations, based on properties ① and ② above. It is also possible to extend students' abilities to perceive and think mathematically based on what have been validated previously through this type of instruction.

## Solving linear equations with one variable

We teach how to solve linear equations with one variable,  $ax + b = cx + d$ , by transforming it into the form,  $x = \alpha$ , by using the properties of equality relationships.

That is, by using properties ① and ②, we generate the idea of transposing terms, and by transposing terms, we transform the given equations into the form  $Ax = B$  ( $A \neq 0$ ). Then, by using properties ③ and ④, we change the coefficient of  $x$  to 1 to find the solution. Students are expected to understand that equations can be solved by transforming the given equation step by step through a series of equivalent equations into the form,  $x = \alpha$ . By examining the process, help students summarize the process to solve equations generally and become proficient at solving equations. It is necessary to note that transforming equations in the solution process is different from the transformation of algebraic expressions during the calculation, such as addition and subtraction of numbers or linear expressions, in the following way: Calculation of numbers and algebraic expressions means that the given expression is being transformed into a more simplified expression. Transformations of equations during the solution process not only change the given equation into more simplified equations but also equations with the same solutions. Help students understand that transforming the given equation into a series of equivalent equations is different from the transformation of algebraic expressions studied before this point.

As for solving linear equations with one variable, enable students to solve equations necessary to solve concrete problems so they can actually use what they learn.

$$\begin{array}{rcl} & 5x + 3 - (2x - 6) & 5x + 3 = 2x - 6 \\ = & 5x + 3 - 2x + 6 & 5x - 2x = -6 - 3 \\ = & 5x - 2x + 3 + 6 & 3x = -9 \\ = & 3x + 9 & x = -3 \end{array}$$

## Use of linear equations with one variable

To solve problems by using equations, the following series of activities are performed:

- ① Identify the quantity to be determined and represent it with a letter
- ② Identify the quantity that can be represented in two different ways based on the quantitative relationships in the problem situation, and represent the relationship in algebraic expressions with letters and numbers
- ③ Create an equation by connecting the two expressions with the equal sign, and solve the equation
- ④ Examine the solution in the context of the original problem to determine the answer

Step ② closely relates to the study of writing and interpreting quantitative relationships using algebraic expressions with letters. In step ④, the solution of the equation is examined in the context of the original problem to determine if it is appropriate as the answer to the problem. This means to re-examine the conditions that could not be incorporated into the equation. It is important to develop the disposition to reflect on the results obtained through mathematical manipulation in terms of their appropriateness through a study like this one.

In our daily life, we often encounter problems involving ratios. As possible types of situations to use linear equations, we can think of those situations in which simple proportions must be solved. For example, consider this problem: two types of solutions will be mixed in the ratio of 3 : 5 in terms of their weights. If we use 150 g of solution B, how many grams of solution A are needed? If we consider that  $x$  g of solution A is used, we can establish a proportion,  $\frac{3}{5} = \frac{x}{150}$ . This can be considered as a linear equation with one variable, and the solution of the equation is  $x = 90$ .

In other words, we need to mix 90 g of solution A. In this manner, in those situations where ratio is used to determine quantities, we can solve the problem by solving the equations obtained as proportions.

## B. Geometrical Figures

1. (1) Through activities like observation, manipulation and experimentation, to deepen the understanding of plane figures by constructing them with a prospect in mind and exploring the relationship between geometrical figures, and to cultivate the ability to think and represent logically.
  - (a) To understand the fundamental methods for constructing figures like the bisector of an angle, the perpendicular bisector of a line segment and perpendicular lines, and to make use of them in concrete situations.
  - (b) To understand parallel translation, symmetric transformation and rotational transformation, and to explore the relationship between two geometrical figures.

[Terms and Symbols]

arc chord //  $\perp$   $\angle$   $\triangle$

[Handling the Content]

- (4) In connection with (1)-(a) under “ B. Geometrical Figures ” in “ Content, ” the fact that any tangent of a circle is perpendicular to the radius passing through its point of tangency should be dealt with.

In elementary school mathematics, students’ ability to focus on the constituent parts of geometrical figures is gradually developed through activities to observe and to construct shapes of objects. By the end of grade 4, students are expected to understand triangles, quadrilaterals, isosceles and equilateral triangles, parallelograms, trapezoids, and rhombuses. In grade 5, students learn congruent figures, and in grade 6, they are expected to understand enlarged and reduced drawings, as well as the symmetry of geometrical figures. In this way, students’ ways to view geometrical figures are enriched by examining constituent parts of geometrical figures and in their measurements and positional relationships.

In lower secondary school mathematics, students will learn to construct geometrical figures with a prospect in mind by focusing on the symmetry of geometrical figures. They are also expected to use methods of construction in concrete situations. Through this study, deepen students’ understanding of geometrical figures, and nurture their ability to perceive and think about geometrical figures empirically. At the same time, cultivate their ability to examine and represent logically. Students are also expected to understand about transformations of geometrical figures, and by examining relationships between two geometrical figures, they are expected to enrich their ways of viewing geometrical figures.

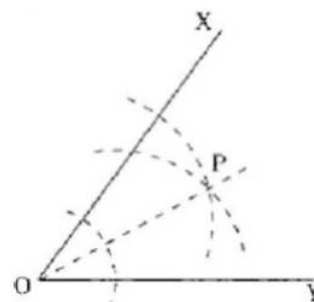
## Fundamental geometrical constructions and their use

The act of drawing figures is an important fundamental skill in the study of geometrical figures. At the same time, it can increase students' interest in and curiosity about geometrical figures. Because it can also deepen their ways of viewing and thinking about geometrical figures intuitively, it also serves an important purpose for facilitating logical examination of geometrical figures.

The following basic geometric construction will be discussed based on the symmetry of plane figures studied in elementary school mathematics: angle bisectors, perpendicular bisector of segments, perpendicular lines to given lines. In geometric construction, rulers are used as a tool to draw a line that goes through two points and compasses are used to draw circles and to copy lengths.

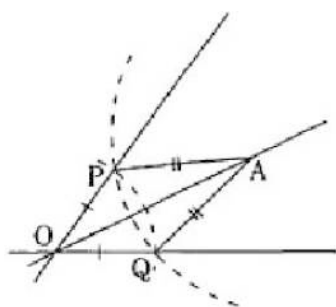
Instead of simply giving directions to complete particular constructions, students should focus on the symmetry or constituent parts of the given figures and think about the steps of construction on their own. It is also important that they can explain the steps of construction in an orderly fashion.

For example, let us consider the construction of the angle bisector of  $\angle XOY$ . This construction requires us to determine the two points the angle bisector will pass through, and then draw a line (a ray) through those two points. Help students to recognize that the angle bisector is the line of symmetry for the given angle, perhaps by having students fold the paper so that the two sides of the angle,  $OX$  and  $OY$ , will coincide. Since the bisector will go through the vertex of  $\angle XOY$ , what we really need is another point,  $P$ , which will be on the angle bisector. If the construction of the angle bisector is considered from this perspective, students can have the prospect that what they need to do is to find a point,  $P$ , which is on the angle bisector of  $\angle XOY$ . In teaching this construction, cultivate students' abilities to examine and express their ideas logically by having them explain the prospect they developed or the steps of construction using their own words.

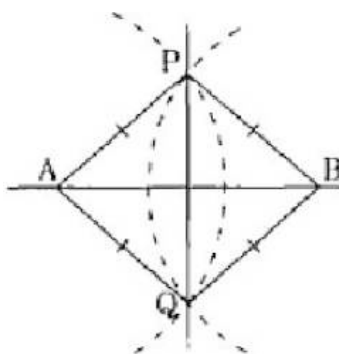


If we focus on the symmetry of figures, the construction of the angle bisector, the perpendicular bisector of a given segment, and the perpendicular line to a given line can be considered as the same process.

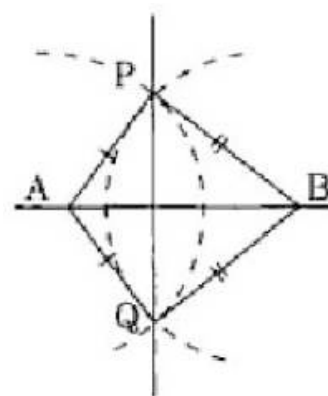
In the construction of the angle bisector, since  $OP=OQ$  and  $AP=AQ$ , the points of intersection of circles centered at  $O$  and  $A$  are points  $P$  and  $Q$ . In the construction of the perpendicular bisector, the points of intersection of circles with the same radius centered at  $A$  and  $B$  are points  $P$  and  $Q$ . In the construction of a perpendicular line to the given line, the points of intersection of circles centered at  $A$  and  $B$  are points  $P$  and  $Q$ .



< the way of viewing of the angle bisector >



< the way of viewing of the perpendicular bisector of a line segment >



< the way of viewing of the perpendicular line >

]

In all cases, the fact that the two circles are symmetric about the line connecting the two centers. Help students to see that symmetry of geometrical figures plays an important role in considering geometric construction from a unified perspective by reflecting on the methods of construction.

The method of constructing a tangent line to a circle can also be considered by focusing on the symmetry of a circle. By translating the line perpendicular to a line of symmetry, one can draw a tangent line. Using this idea, it is possible to understand the way to construct the tangent line through a point on a circle. From this, one can verify that a tangent line is perpendicular to the radius of the circle.

As for the use of geometric construction, discuss the fact that, unlike drawing lines and angles using a ruler (marked straight edge) or a protractor, it is possible to draw geometrical figures accurately without relying on their measurements.

For example, have students construct angles measuring  $30^\circ$  or  $45^\circ$  using only a straight edge and compass, without using rulers or protractors. Or, have them copy sides and angles of a triangle to draw a congruent triangle.

The idea behind drawing congruent triangles by copying the constituent parts is that it is possible to do so by copying three sides, by copying two sides and the angle in between them, or by copying a side and two angles on its end points. Understanding these methods can become the foundation for logical examination and similar activities are also included in elementary school mathematics to lay the groundwork. It is possible to summarize these ideas as the congruence conditions for triangles, but it is also possible to study them in grade 2 of lower secondary school, as specified in the COS.

### **Translations, reflections, and rotations**

Although symmetry of geometrical figures as a relationship between two figures is discussed in grade 6 of elementary school mathematics, the examination of symmetry from the perspective of transformations of geometrical figures is discussed for the first time in lower secondary school mathematics. In grade 1 of lower secondary school, by overlapping two figures by transforming one of them or comparing the original figure and the transformed one, students are expected to identify properties of geometrical figures.

With respect to transformation of geometrical figures, students have examined their properties by "sliding," "turning," and "flipping," and naturally have grasped that the shape and the size of a geometrical figure do not change through those motions. In grade 1 of lower secondary school, three rigid motions, translation, reflection, and rotation, are discussed.

Under a transformation of a geometrical figure, it is moved to a different position by moving all points on the figure following a specific rule.

Under a translation, a figure is moved in a particular direction for a particular distance. Thus, translation is defined by that direction and distance. A reflection will move a geometrical figure around a line as the axis of reflection. Therefore, it is defined by the position of the axis of reflection. Under a rotation, a figure is moved around a given point for a specified angle, and it is defined by the position of the point, the center of rotation, and the measure of the angle. When the angle of rotation is  $180^\circ$ , it is the point transformation <sup>1</sup>.

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<sup>1</sup>Under a point transformation, each point on a plane is moved in the direction of the center of point transformation the distance equal to the distance between the point and the center of transformation.

As these ideas are being taught, it is important to help students identify properties of geometrical figures or enrich their ways of viewing geometrical figures by having them compare geometrical figures before and after a transformation, for example, comparing the positions of line segments, measures of corresponding angles, and congruence of the given figure and its image. In addition, as a way to observe tessellations, it is possible to have students investigate what motion will overlap two geometrical figures in a tessellation or creating a tessellation by transforming a given geometrical figure as the starting point.

As transformations of geometrical figures are discussed, students will be drawing the images of given figures under transformations. Through such activities, help students become used to the use of the straight edge and compass.

Also, in order to help students understand the meaning of geometric constructions, it is important to verify the accuracy of the methods and the results of the basic construction from the perspective of geometric transformations.

By connecting the study of transformations with the study of the content of geometrical constructions, students' understanding of plane figures may be deepened and the important connection to the study of congruence of geometrical figures in grade 2 of lower secondary school.

- (2) Through activities like observation, manipulation and experimentation, deepen the understanding of space figures, and develop the ability to measure geometrical figures.
- (a) To know the positional relationship between straight lines and planes in space.
  - (b) To grasp space figures as objects constructed by the movement of straight lines and plane figures, and to represent space figures on a plane and read their properties in the representation.
  - (c) To find the length and area of a sector, as well as the surface area and volume of basic cylinder, pyramid and sphere.

[Terms and Symbols]

solid of revolution, skewed position,  $\pi$

[Handling the Content]

- (5) In connection with (2)-(b) under "B. Geometrical Figures" in "Content," sketches, developed drawing and projection drawing should be dealt with.

In elementary school mathematics, starting with grade 1, students develop their ability to grasp geometrical figures as an abstract object by observing and sorting solid figures from their surroundings. Starting in grade 2, solid figures are discussed by focusing on their constituent parts. In grade 3, spheres are introduced rectangular prisms, pyramids, cylinders and cones are discussed by the end of grade 5. Their understanding of solid figures is deepened by drawing their sketches and developed drawings<sup>2</sup>.

In lower secondary mathematics, based on what the study was in elementary school, help students deepen their understanding of space figures. In lower elementary school mathematics, the objects that were treated as solid figures in elementary school mathematics must be recognized as space figures, or, figures made up of lines and faces in space. To help students' empirical understanding of space figures, and also to cultivate their ability to examine and represent space figures, instruction should be built on the foundation of activities to observe, manipulate and experiment such as the following: to think about solid figures by building their models, to think about ways to represent the solid figures on a plane, or identify properties of the solid figures by examining their representations on a plane. The study of measurement of geometrical figures is positioned as a way to understand geometrical figures, not just to develop ways to calculate them. It should be noted that pyramids are discussed for the first time in lower secondary school.

<sup>2</sup>"Developed drawings" are also known, perhaps more commonly, as "nets" or "patterns."

## Position relationships of lines and planes in space

In elementary school mathematics, students have learned about parallel and perpendicular relationships of lines and planes in conjunction with the study of solid figures such as squares. However, their examination was limited to the relationships of constituent parts of specific solid figures. In lower secondary school mathematics, students are to examine space relationships of lines and planes as abstract objects while reflecting on their study of solid figures in elementary school mathematics and also while examining concrete space figures.

A line in space is considered to be extending in both directions forever just as is the case in a plane. Similarly, students are to understand that a plane in space extends forever in every direction. Students are to understand that 2 points in space will define a line, and 3 non-collinear points, a line and a point not on the line, and two intersecting lines all define a plane. Moreover, it is important that students can enrich their spatial sense by extending ideas from a plane to space by analogy. For example, just as a plane can be split into two regions by a line, space may be split into two parts by a plane.

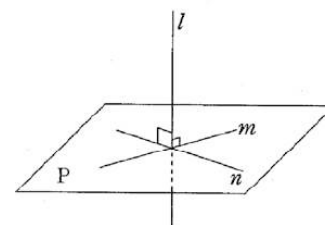
As for the positional relationships of lines and planes in space, students are to examine the positions of lines and planes and how they intersect each other.

There are two possible cases for the positional relationships of two lines in space: when the two lines intersect, and when the two lines do not intersect. When two lines do not intersect with each other, the lines may be parallel, or may not be parallel. When the lines are not parallel (yet do not intersect each other), we say that they are in a skewed position. When two lines intersect with each other, or when they are parallel to each other, the two lines will define a plane. In other words, those two lines will be on the same plane.

As for the relationship of a line and a plane in space, there are three possibilities: the line is included in a plane, the line and the plane will intersect, or the line and the plane are parallel to each other. When the line and the plane intersect, it is necessary to specify that the line is perpendicular to every line that passes through the point of intersection between the line and the plane in order for them to be considered perpendicular to each other, or the line is not slanted in any direction with respect to the plane.

However, if you remember that, "a plane is defined by two intersecting lines," if a line is perpendicular to 2 other perpendicular lines at their point of intersection, then the first line is perpendicular to the plane defined by the 2 perpendicular lines. In other words, to check whether or not a line is perpendicular to a plane, one needs to check whether or not the line is perpendicular to two intersecting lines.

When two planes in space are considered, they may either intersect with each other or they may not. A special case of intersecting planes is the perpendicularity, and when the two planes do not intersect, they are parallel to each other. When two planes intersect with each other, they intersect in a line.



The positional relationships of lines and planes discussed above are the foundation for examining space figures, and we will not be able to analyze space figures without them. When these ideas are taught, as the COS specifies that, "through activities of observation, manipulation and experimentation," incorporate situations to examine concrete space figures where positional relationships of lines and planes are essential and avoid simply giving those relationships formally. For example, it is necessary to help students understand the positional relationships of lines and planes by actually building solids and observing them, or through activities of explaining their ideas using the solids. Through such investigations, help students recognize that two lines that are parallel to a third line are parallel to each other themselves or two lines that are perpendicular to the same third line may not necessarily be parallel to each other.



## Composing space figures by moving plane figures

When examining space figures, if you focus on their constituent parts, they may be considered as a result of moving plane figures such as a line segment, polygon or circle. This perspective is important in developing creativity and intuition about space.

The idea that a plane is composed as a result of moving a line segment will be taught. For example, the lateral surfaces of a prism may be formed as a result of translating a line segment in space. Or, the lateral surfaces of right cylinders or right cones may be formed by rotating segments around a fixed line (axis of rotation).

We will also teach the idea that solid figures are formed as a result of movements of plane figures. For example, prisms and cylinders can be thought of as a result of translating their bases in space. Right prisms can be considered as the result of rotating a rectangle using one of the sides as the axis of rotation, and a right cone may be formed by the rotating of a right triangle around one of its legs as the axis of rotation. Spheres can be viewed as the result of rotating a semi-circle around its diameter. To help students understand these ideas, instruction should make connections to everyday situations such as a potter's wheel or the structure obtained by stacking cards or blocks. It is important to deepen students' understanding of space figures through activities of observation, manipulation and experimentation with space figures, such as actually rotating a right triangle around one of its legs or sorting solids that can be obtained by moving line segments or plane figures.

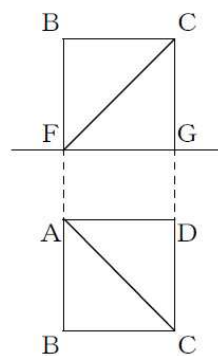
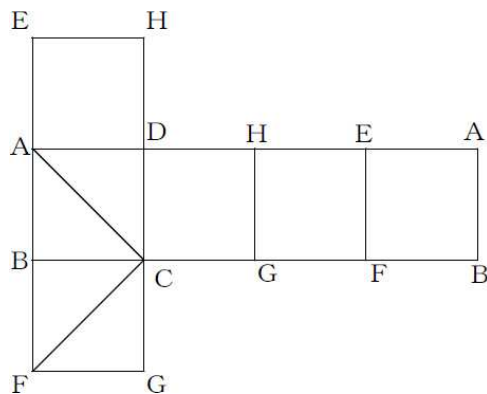
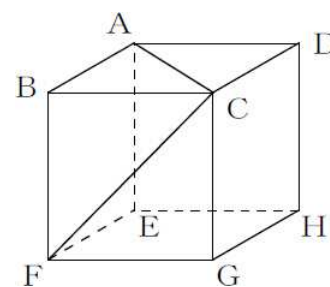
## Representing and interpreting plane representations of space figures

While examining space figures, considering their components as plane figures or relating them to plane figures are important facets of understanding space figures along with considering space figures as the result of the motions of plane figures. Students will be taught about representing space figures on a plane in order to understand their characteristics. They will also learn to identify characteristics of space figures from their representations on a plane.

By drawing sketches of space figures or identifying their characteristics from sketches, students can examine space figures without having actual shapes on hand. With sketches, they can also examine the space figures considering their interior, too.

Projections of space figures are studied in lower secondary school mathematics for the first time. By representing a space figure by how the figure will look when seen from directly above and seen from directly in front, students will examine the characteristics of the space figure.

In this way, students will learn to develop the ways to analyze objects not from a single viewpoint but from multiple viewpoints. In some cases, when a space figure is represented as a sketch or a projection drawing, characteristics of the original space figure may not be preserved. In the sketch of a cube on the right, students might mistakenly think that the diagonals (AC and CF) are not equal by inspection. However, by examining a net or a projection drawing of the same cube, students can reason mathematically, "since the two segments are the diagonals of congruent squares, their lengths must be equal."



While learning to interpret plane representations of space figures, it is important that sketches, nets, and projection drawings are connected to each other so that students can understand space figures more realistically.

Thinking about nets in order to build models of cubes or tetrahedron can lead students to examine those figures more analytically and think about the relationships between faces or edges as well as their positional relationships. In this way, students' understanding of solid figures may be deepened. Or, when calculating the surface area of cylinders or cones, how to determine the areas of curved lateral faces can be a problem. However, if they are represented as nets, those lateral faces are rectangles and sectors, respectively, and their areas can be easily calculated. In this way, thinking about nets will lead to the deepening of understanding about space figures.

As discussed above, by investigating components of concrete space figures by using sketches, nets, and projection drawings, students' ability to think and represent logically is to be cultivated.

### **Length of an arc and area of a sector**

In elementary school mathematics, circumference and area of circles are taught. In lower secondary school mathematics, have students reflect on what they studied in elementary school and express area and circumference using  $\pi$ . In addition, enable students to calculate the length of an arc and the area of the sector by understanding the proportional relationship between the length of an arc and the length of the corresponding arc.

### **Surface area and the volume of prisms, pyramids, and spheres**

In elementary school mathematics, the volume of cubes, cuboids, prisms and cylinders are dealt with. The formula to calculate the volume of prisms, area of base  $\times$  height, has been studied in elementary school. In lower secondary school mathematics, students can deepen their understanding of how to calculate the volume of prisms and cylinders by connecting the view of prisms and cylinders as solids obtained by translating their bases.

The volume of pyramids, which is discussed for the first time in lower secondary mathematics, is  $\frac{1}{3}$  of the volume of a prism with the congruent base and height. The volume of a sphere is  $\frac{2}{3}$  of the volume of a cylinder in which the sphere can fit snugly. To help students develop a concrete understanding of volume of pyramids and spheres, have them estimate the relationship between the volume of the pyramid or the sphere and the volume of a prism or a cylinder. Then, through demonstration or experimenting with models, verify the estimation.

The surface area of prisms and pyramids should be taught through activities such as drawing nets of the given solid figures. It is also important that students understand the values of nets. As for the surface area of spheres, help students develop a concrete understanding by using models or by measuring the results of experiments. As the measurement of solid figures is taught, have students think about measurements of what parts of the solids are needed or what types of drawings may be helpful. By treating them more holistically, we can help students further deepen their understanding of space figures.

It should be noted that we only discuss those space figures with bases whose area formulas have already been learned, such as triangles and circles.

## C. Functions

- (1) Through finding out two numbers/quantities in concrete phenomena and exploring their changes and correspondence, deepen the understanding of relationships of direct proportion and inverse proportion, and cultivate the ability to find out, represent and think of functional relationships.
  - (a) To understand the meaning of functional relationships.
  - (b) To understand the meaning of direct proportion and inverse proportion.
  - (c) To understand the meaning of coordinates.
  - (d) To represent direct proportion and inverse proportion into tables, algebraic expressions, graphs and so on, and to understand their characteristics.
  - (e) To grasp and explain concrete phenomena by using direct proportion and inverse proportion.

[Terms and Symbols]

function, variable, domain

In elementary school mathematics, the following topics are taught between grades 4 and 6: representing quantitative relationships using algebraic expressions using symbols such as  $\square$ ,  $\triangle$ ,  $a$ ,  $x$ , substituting various numbers in symbols to evaluate algebraic expressions, representing changes in quantities using broken line graphs and identifying characteristics from such graphs, proportional relationships and their applications, and inverse proportion relationships.

In grade 1 of lower secondary school mathematics, based on the study in elementary school, the contents related to functional relationships are further enriched to enable students to identify two quantities in concrete situations that vary simultaneously and understand the meaning of their functional relationships by focusing on their changes and correspondences.

The study of direct and inverse proportional relationships is the foundation of exploration of quantitative relationships in our daily life. In this study, cultivate students' abilities to identify, represent, and examine functional relationships through observation of concrete phenomena, and avoid approaching these topics just formally and generally. It is also important to re-conceptualize direct and inverse proportional relationships, which students studied in elementary school, as functions based on the expansion of the range of numbers.

### Meaning of functional relationships

A functional relationship exists between two quantities when the value of one quantity is determined, the value of the other quantity will also be fixed to one and only one value. The aim here is to help students deepen their understanding of functional relationships by focusing on the way quantities change and correspond, and capture the relationships using expressions, "... and ... are in a functional relationship," or, "... is a function of ..."

The instruction in grade 1 of lower secondary school will center around the study of direct and inverse proportional relationships, as they are examples of functions. In the early stage of the study of functions, some students might mistakenly think that only direct and inverse proportional relationships are functions. Therefore, care should be taken so that students can experience the expansion of the concept of functions and cultivate their abilities to identify, represent and examine functional relationships.

Even in elementary school mathematics, students learn to examine quantitative relationships by representing them in tables, algebraic expressions, and graphs. However, the nature of algebraic expressions or graphs representing quantitative relationships differ in lower secondary school mathematics when compared to elementary school mathematics. As for algebraic expressions, in elementary school mathematics, quantitative relationships are represented in algebraic expressions with numbers, words, symbols such as  $\square$  and  $\triangle$ , and letters. In lower elementary school mathematics, algebraic expressions are generalized in those using letters, which are used for variables and constants. In lower secondary school mathematics, the coordinate system becomes the foundation of graphing, which is not the case in elementary school mathematics. Moreover, in lower secondary mathematics, the ranges of values that variables can take will be expanded to positive, negative numbers and 0.

In lower secondary mathematics, the purpose of representing quantitative relationships in tables, algebraic expressions and graphs is to examine functional relationships by grasping the characteristics of changes and correspondences from those representations.

When representing quantitative relationships in tables, it is important to capture the way two quantities correspond to each other. Therefore, arranging the values for one of the variables (independent variables) in a purposeful order in a table is important.

Representing quantitative relationships in algebraic expressions, variables and constants must be distinguished. It is necessary to clearly define which quantity will be considered as  $x$  and which will be  $y$ . When quantitative relationships are represented in algebraic expressions, it can be seen that when the value of one variable is fixed, the value of the other variable is also fixed. Moreover, tables and graphs can be easily constructed based on algebraic expressions. Care should be taken to help students realize that there are functional relationships that cannot be represented in an algebraic expression.

To graph quantitative relationships in graphs, points whose coordinates represent the values of corresponding quantities must be plotted. Help students understand that they can determine from graphs that when the value of variable  $x$  is fixed, the value of variable  $y$  is also fixed.

When tables, algebraic expressions, and graphs are used, it is important that connections are made among them, instead of treating them in parallel or independently, so that students can understand them more holistically. For example, while examining a quantitative relationship represented in a table, by representing the same relationship as an algebraic expression or a graph, students can deepen their understanding of the quantitative relationship by seeing that other correspondences not shown on the table can be gathered from the alternative representations. Also, while examining a quantitative relationship represented in an algebraic expression, if one represents the relationship in a table or graph, the way quantities change and correspond can be more concretely captured. In this way, those representations might make it easier to understand characteristics of the quantitative relationship.

In the previous COS, functional relationships and functions were introduced in grade 2 of lower secondary school. The reason these ideas are introduced in grade 1 in the current COS is to cultivate students' abilities to identify, represent, and examine functional relationships by re-conceptualizing direct and inverse proportional relationships, which were studied in elementary school mathematics, as functional relationships.

## Meaning of direct and inverse proportional relationships

In elementary school mathematics, simple cases of direct proportional relationships are discussed in grade 5, and, in grade 6, students are expected to understand direct proportional relationships based on the study in grade 5. They also learn to investigate characteristics of direct proportional relationships by examining their tables and graphs. To help students deepen their understanding of direct proportional relationships, students are expected to know about inverse proportional relationships.

Students have seen the following three meaning of direct proportional relationships in elementary school mathematics:

- When one quantity increases 2, 3, ... times as much, or decreases  $\frac{1}{2}, \frac{1}{3}, \dots$ , the other quantity also increases 2, 3, ... times as much, or decreases  $\frac{1}{2}, \frac{1}{3}, \dots$ , respectively.
- When one quantity becomes  $m$  times as much, the other quantity also becomes  $m$  times as much.
- The ratio of the two quantities (ratio) is constant.

Students have also seen the following three meaning of an inverse proportional relationship:

- Given two quantities, when one quantity increases 2, 3, ... times as much, or decreases  $\frac{1}{2}, \frac{1}{3}, \dots$ , the other quantity becomes  $\frac{1}{2}, \frac{1}{3}, \dots$  times as much or 2, 3, ... times as much, respectively.
- When one quantity become  $m$  times as much, the other quantity becomes  $\frac{1}{m}$  times as much.
- The product of the two quantities remains constant. The range of numbers for the quantities is limited to non-negative numbers.

In lower secondary school mathematics, building upon the study in elementary schools, the range of numbers in direct and inverse proportional relationships including negative numbers and represent them in algebraic expressions with letters. For direct proportional relationships, using  $a$  as the proportion constant, they can be represented generally as  $y = ax$  or  $\frac{x}{y} = a$ . Inverse proportional relationships may be represented generally as  $y = \frac{a}{x}$  or  $xy = a$ , where  $a$  is the proportion constant.

## Meaning of coordinates

In elementary school mathematics, students learn to represent the position of objects in a plane or in space, which leads to the meaning of coordinates. In addition, students learn to represent the changes of quantities using broken line graphs. However, the approach taken there was to connect the top of bars in bar graphs, not based on the idea of coordinates, that is, to represent the position in a plane using a pair of numbers.

In lower secondary school mathematics, based on the study in elementary school, students are expected to understand coordinates and represent quantitative relationships in graphs using the idea of coordinates.

A position on a plane may be represented by a pair of numbers which correspond to two intersecting number lines as axes, generally. This is the concept of coordinates on a plane. In lower secondary school mathematics, students are expected to understand the meaning of coordinates as a way to uniquely represent a position on a plane by using two perpendicular number lines intersecting at the origin,  $O$ . By using coordinates, it is possible to represent graphs as a set of points.

## Tables, algebraic expressions, and graphs of direct and inverse proportional relationships

Students have learned about direct proportional relationships in grades 5 and 6 in elementary school. In lower secondary school mathematics, variables must be clearly defined, and students should be able to identify the relationship between variables  $x$  and  $y$  and represent the relationship as  $y = ax$  or  $\frac{y}{x} = a$ . They are also expected to understand the meaning of  $a$  as the proportion constant.

The range of values the variables can take has been expanded to include negative numbers. Students are to understand that graphs of direct proportional relationships are lines passing through the origin. Students also learn about how graphs will change when the proportion constant,  $a$ , changes. In the current revision of the COS, the aim of teaching inverse proportional relationships in grade 6 of elementary school mathematics is to deepen students' understanding of direct proportional relationships. In lower secondary mathematics, variables must be clearly defined, and students should be able to identify the relationship between variables  $x$  and  $y$  and represent the relationship as  $y = \frac{a}{x}$  or  $xy = a$ . They are also expected to understand the meaning of  $a$  as the proportion constant. Students are expected to understand that graphs of inverse proportional relationships are two curves that do not pass through the origin. They will also learn how graphs change when the values of  $a$  changes. Graphing a curve is a new experience for students; therefore, have them plot as many points as necessary so that students can understand that the graph will be a curve. It is possible to let students use calculators so that the calculations to determine the values of  $x$  or  $y$  become more efficient.

It is important to clarify the difference between tables, algebraic expressions, and graphs studied in elementary school mathematics and those studied in lower secondary school with the expanded range of numbers and the use of coordinates.

### To explain concrete phenomena using the ideas of direct and inverse proportional relationships

It is important that students can explain concrete phenomena using the ideas of direct and inverse proportional relationships.

There are many phenomena related to direct and indirect proportional relationships in our daily life as well as other subject matters, particularly in science. Moreover, it is possible to study direct and inverse proportional relationships using mathematics contents that have already been discussed, such as the length of a side and the area. If we can capture the relationship between two quantities as a direct or inverse proportional relationship, we can capture many characteristics of the way those quantities change and correspond. In addition, it is possible to explain those characteristics more easily by using tables, algebraic expressions and graphs. For example, consider the question, "what would happen to the circumference when the radius becomes 2, 3, ... times as long?", which is about a direct proportional relationship between the radius,  $r$ , and the circumference,  $\ell$ . We can respond to this question more easily if we use the meaning of algebraic expression,  $\ell = 2\pi r$ , without having to calculate with specific numbers. Furthermore, when we consider the question, "if the radius becomes 2, 3, ... times as long, would the area also becomes 2, 3, ... times as much?", we can detect from the algebraic expression  $S = \pi r^2$ , where  $S$  is the area of the circle, the radius and the area are not in a direct proportional relationship. With respect to inverse proportional relationships, some students might mistakenly think that, "since B decreases as A increases, B is inversely proportional to A." It is important that care should be taken so students will not develop this misunderstanding by examining the characteristics of tables, algebraic expressions and graphs of inverse proportional relationships.

Moreover, some daily phenomena may be considered as indirect or inverse proportional relationships even though they might not be, strictly speaking. It is important for students to know that sometimes it is possible to predict changes and correspondences about two quantities by considering them in a direct or inverse proportional relationship, by idealizing or simplifying the situations. It is also important that students understand that, by idealizing or simplifying the situations, we introduce some constraints.

Finally, when dealing with concrete situations, it is important to pay attention the range of values the variables can assume. For example, when considering the relationship between the length and the area using a direct or inverse proportional relationship, negative values will not make sense for either length nor area. Enable students to pay attention to the range of numbers for the variables as they explain concrete phenomena.

## D. Making Use of Data

- (1) To be able to collect data according to a purpose, arrange it into tables and graphs by using a computer and other means, and then read trends in the data focusing on its representative values and its variations.
  - (a) To understand the necessity and meaning of histograms and representative values.
  - (b) To grasp and explain trends in the data by using histograms and representative values.

[Terms and Symbols]

mean, median, mode, relative frequency, range, class

[Handling the Content]

- (6) In connection with (1) under “D. Making Use of Data” in “Content,” calculation errors, approximate values and  $a \times 10^n$  format expression should be dealt with.

In elementary school mathematics, students learn about bar graphs, broken line graphs, circle graphs, and percentage bar graphs. Students also learn to examine and represent data statistically through activities such as representing frequency distribution in tables or graphs and investigating the average and spread of data. In grade 5, the average of measurements, and in grade 6, the idea of examining and representing statistically are taught.

In lower secondary school mathematics, students in grade 1 are expected to understand the importance of efficient and some purposeful ways to collect and organize data as well as some reasonable ways of processing the data. In addition, they are expected to understand ideas such as histograms and representative values for a set of data, and to identify trends in data through using those representations in explaining the data.

### Need for and meaning of histograms

Frequently, we have to make decisions based on data in our daily life. Data collected for different purposes may be qualitative, such as gender distribution in a population survey, or quantitative, such as changes in the temperature at noon in the previous month. In either case, in order to make appropriate decisions, we must process the data purposefully and use the results to interpret the trends in the data.

Frequency distribution tables and histograms are methods for such statistical processing. By using histograms, the spread of data can be captured. By dividing the data into a number of classes and counting the number of data in each class, it is easier to capture such features as the shape of the data, the range of distribution of the data, the location of the peak, and symmetry.

When using histograms to interpret trends in data, it is necessary to pay careful attention to the width of each class of data. For example, Figure 1 shows the records of handball tosses by 100 grade 1 students at a lower secondary school.

Figures 2 and 3 are the histograms for this data set using 3 m and 2 m as the width of classes, respectively.

16,	12,	27,	18,	18,	23,	22,	24,	15,	13
26,	12,	24,	24,	15,	10,	18,	15,	18,	18
18,	18,	15,	16,	21,	11,	12,	20,	26,	27
16,	20,	25,	21,	18,	18,	23,	16,	18,	24
16,	18,	14,	18,	14,	14,	18,	15,	14,	18
23,	23,	23,	14,	14,	21,	21,	27,	25,	23
20,	22,	27,	18,	18,	14,	18,	18,	27,	24
15,	25,	15,	24,	23,	21,	25,	25,	15,	16
24,	11,	25,	23,	13,	13,	20,	15,	20,	26
18,	20,	25,	22,	23,	23,	21,	22,	16,	22

Figure 1

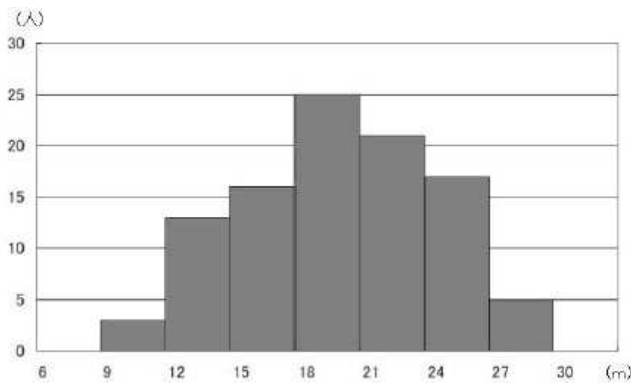


Figure 2

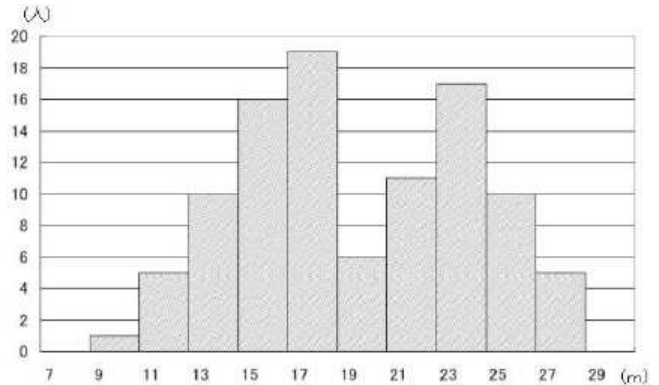


Figure 3

In the histogram in Figure 2, the distribution of the data appears to include one peak, while the histograms in Figure 3 show two peaks. Therefore, the answer to the question, "are there many students who threw the handball between 19 m and 20 m?", may be different depending on which histogram students use.

Thus, depending on the width of classes, histograms may show different trends for the same set of data. Therefore, it is necessary to make several histograms with different widths of classes and compare them when histograms are used to identify trends in data for specific purposes.

Making histograms by hand can deepen students' understanding of the meaning of histograms.

However, for the study such as the one described above, it is important to provide sufficient time for thinking by, perhaps, using computers.

### Need for and meaning of representative values

When identifying trends in data, in addition to frequency tables and histograms, representative values are often used.

A unique feature of representative values is expressing the characteristics of the data distribution in one numerical value from a particular perspective. Mean, median, and mode are most frequently used. By representing a data set in a single value, characteristics of the set can be concisely expressed and the comparison with another set of data becomes easier. On the other hand, it should be kept in mind that some information such as the shape of data distribution may be lost.

Mean can be determined from a data set that has yet to be organized in a frequency distribution table. On the other hand, there are situations where means are not the appropriate representative values.



For example, if the distribution of data is not symmetrical or there are outliers, the mean of the data set is heavily influenced and may not be appropriate as a representative value. In those situations, the median or the mode of the set is used. In other situations, the purpose of the data set suggests that the mean is not the appropriate representative value. For example, if a shoemaker is determining which shoes to produce the most of, they will not simply calculate the mean size of shoes sold this year and produce the shoes of the mean size the most.

Rather, in this situation, they will use the mode, that is the size that sold the most. In this way, when representative values are used, it is important to clarify the nature and the purpose of the data to decide which type of representative value is to be used.

Another method of representing the characteristics of data distribution in a single value is the range of data. The range of a data set is the difference between the largest and smallest data values, and it shows the degree of distribution of the data. Two data sets with an equal mean do not necessarily have the same range. Furthermore, the range can be influenced by even a single outlier. Its use and interpretation requires some care.

### **Need for and meaning of relative frequency**

When comparing two data sets with different number of data, we cannot simply compare the frequency of each class directly. In those situations, we can compare the relative frequency of each class because a relative frequency will show the ratio of the frequency in each class to the whole data set.

A relative frequency is a value showing the ratio of a part (frequency in a class) to the whole (the total number of data), and we can consider it as a frequency of the class. In teaching this idea, it should be kept in mind that this is the foundation for probability discussed in grade 2 of lower secondary school. By using relative frequencies for a single data set, we can more easily determine the ratio of each class as well as the ratio of combinations of classes (above or below a certain class).

### **Grasping and explaining trends in data**

Enable students to grasp and explain trends in data using histograms and representative values.

The purpose of histograms and representative values is not actually to create or calculate them. Rather, when they are used to identify and explain trends in data, they become meaningful. Therefore, when teaching these topics, provide students a series of activities such as the following: identify a problem from everyday situations, gather data necessary to solve the problem, make histograms and calculate representative values – perhaps with the help of computers, identify trends in the data, and explain the solution using the results of the analysis.

For example, let's think about how the number of runners in a relay race between two classes will influence the results. The data from the physical education class may be used, or a new set of data can be gathered for this investigation. By making frequency distribution tables and histograms from the data set, we can predict, "which class will win if we select 10 runners from each class?" or, "what if we increased the number of runners to 20 students from each class?" based on the distribution of the data.

What is important here is not whether or not the prediction turns out to be correct, rather, making clear on what the identified trends and explanations are based. Through activities of explaining and communicating, students can know that even from the same data set several interpretations are possible. By listening to each other's explanations and their bases, students may also deepen their understanding of the ideas being used.

In our daily life, to capture trends of a data set, representative values alone are often used because of their simplicity. However, it is necessary that students can grasp trends of data with a clear understanding that representative values may not capture some information.

## Use of tools such as computers

While dealing with a large number of data, or data that have large numbers or numbers with fractional parts, use tools such as computers to efficiently process the data, and the emphasis should be placed on interpreting the results of statistical processing to identify trends. However, consideration should be given that making histograms or calculating representative values by hand may be important while teaching the need for and the meaning of histograms and representative values.

It is necessary to think about how to use those tools effectively such as having a computer available to each student, and using a computer as a tool to display during the whole class discussion. In addition, when gathering data using various information networks, the data are no longer primary data. Therefore, care should be taken to check for the reliability of the data by, for example, identifying who gathered the data using what technique.

## Errors and approximations

In elementary school mathematics, students learned about approximate numbers and how to use them purposefully. They have also learned about the decimal numeration system. Here, errors, approximation, and the idea of expressing the numbers in the form of  $a \times 10^n$  will be handled.

Every measurement includes an error. For example, suppose you measure a student's height using a tool whose smallest interval is mm. If the student's height measures 157.4 cm, it means his true height is greater than equal to 157.35 cm but less than 157.45 cm.

In other words, if the height of the student is  $x$  cm, then

$$157.35 \leq x < 157.45.$$

Help students understand that measurements always involve errors, perhaps by representing the measurements on a number line, and 157.4 cm is being used as the approximation. Students are to understand experientially the meaning of approximations and errors.

As for the representations of numbers, if a measurement is 2300 m and only the numbers to the tens place are reliable, then the "0" in the ones place is simply serving as a place holder. Then, instead of writing it as 2300, it can be represented as  $2.30 \times 10^3$  m. By doing this, we can clearly indicate the significant digits, and we can estimate the amount of error. The purpose here is to help students know about this way of representing numbers.

## [Mathematical Activities]

- (1) In learning each content of “A. Numbers and Algebraic Expressions”, “B. Geometrical Figures”, “C. Functions” and “D. Making Use of Data”, and in learning the connections of these contents, students should be provided opportunities doing mathematical activities like the following:
  - (a) Activities for finding out the properties of numbers and geometrical figures based on previously learned mathematics
  - (b) Activities for making use of mathematics in daily life
  - (c) Activities for explaining and communicating each other in one’s own way by using mathematical representations

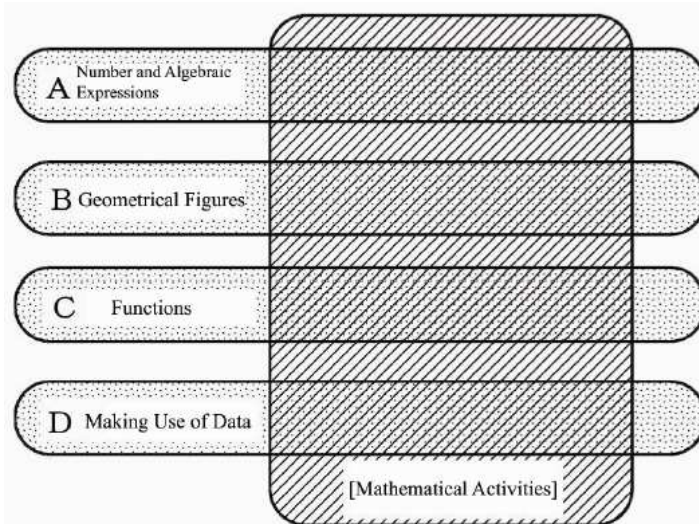
In the Elementary School Mathematics, the purposes of having students engage in mathematical activities are to help them gain the ability and mastery of goal understanding or skills and experience the joy and usefulness of mathematics.

In lower secondary school mathematics, the emphases in grade 1 are, building on those experiences acquired in elementary schools, to have students engage in mathematical activities autonomously, to master basic and foundational knowledge and skills, to raise their ability to think, to make decisions and express ideas, and to let them feel the joy and meaning in learning mathematics.

### **Roles of mathematical activities**

Mathematical activities are listed in all four content domains of the COS, “A. Number and Algebraic Expressions,” “B. Geometrical Figures,” “C. Functions,” and “D. Making Use of Data”; however, their relationships to the content domains are both horizontal and vertical. They are a part of an important structure in lower secondary school mathematics. Mathematical activities have been emphasized in the past COS, and various efforts have been made. However, it appears that the intent of mathematical activities may not be well understood. For example, some associate mathematical activities primarily as activities with concrete materials. Therefore, in order to re-affirm the intent of mathematical activities and to develop a common understanding, mathematical activities are positioned separately from the content of the four content domains in the current revision of the COS. Below, we discuss three types of mathematical activities that cut across all four content domains, from the perspective that mathematical activities are various activities related to mathematics where students engage willingly and purposefully.

These activities are to be implemented in conjunction with the instruction of the contents of the four domains. Mathematical activities are not to be taught independently from the four content domains. In the actual instruction, teachers are required to clarify when each type of mathematical activities may be most effectively used and implement appropriate activities also considering students’ mathematical understanding. It is not intended that all three types of mathematical activities will be implemented in a single lesson. Moreover, “activities for observation, manipulation and experimentation” are not necessarily mathematical activities. It should be remembered that it is necessary that students engage in mathematical activities willingly and purposefully.



### Engaging students in mathematical activities

Another reason that mathematical activities are discussed separately in the "Content" section of the COS is to clarify the purpose of engaging students in mathematical activities. Engaging students in mathematical activities to help them learn basic and foundational knowledge and skills is very important from the perspective of experiential learning. From this perspective, engaging in mathematical activities willingly is a method of learning for students, and it is also a method of teaching for teachers. Furthermore, the willing engagement in mathematical activities is necessary to use knowledge and skills to solve problems and to develop the abilities to think, reason, and make decisions. From this perspective, they are also a content of instruction. Finally, by enabling students to willingly engage in mathematical activities, we are aiming at enabling students to willingly engage in learning activities and think for themselves. Therefore, mathematical activities are a goal of instruction as well.

#### a. Activities used to discover and extend properties of numbers and geometrical figures based on mathematics students have learned previously

Activities used to discover and extend properties of numbers and geometrical figures based on mathematics students have learned previously are activities for thinking extensively and creatively to solve new problems identified by not viewing what has been learned previously too definitively or rigidly. In that process, various mathematical ways of viewing and thinking such as the following play important roles: to engage in trial and error, to change perspectives and think flexibly, to generalize or to consider special cases, to abstract or concretize, to think analytically or to develop a unified perspective. Also in the process, students may not only use mathematical ways of viewing and thinking that they have already learned but also discover and generate new ideas. They may also predict inductively or analogically, and then verify the predictions deductively. By using such mathematical reasoning appropriately, new ideas may be discovered.

What may be discovered in this process may vary from mathematical facts such as concepts, properties, or theorems, to processes and algorithms. Of course, mathematics that has been learned previously plays an important role supporting these discoveries, and students can re-affirm the merits of mathematics.

Here are some examples of, "activities used to discover and extend properties of numbers and geometrical figures based on mathematics students have learned previously" in grade 1 of lower secondary schools. We will also discuss prerequisites necessary for students to willingly and autonomously engage in these activities.

## ○Activities to discover ways to add two numbers with opposite signs

These activities are mathematical activities for grade 1 content, "A. Number and Algebraic Expressions," (1) - b. The aim of the activities are to discover ways to add two numbers with different signs such as  $(+5) + (-2)$  and  $(-4) + (+3)$  based on the study of addition of two numbers with the same signs. Another aim is for students to think about ways to carry out subtraction, multiplication, and division, which will be studied later from a similar perspective.

For this purpose, help students understand the meaning of addition of two numbers with the same sign, perhaps by thinking of addition as the movements along a number line, and carry out the calculations using the meaning. Then, set students up so that students will want to think about how to add two numbers with different signs now that they have learned that they can add two numbers of the same sign.

Set up opportunities for students to engage in activities to think about how to add two numbers with different signs on their own and to summarize the process of calculation using diagrams and words, so that they can understand that two numbers with different signs can be added just as two numbers with the same sign. To help those students who cannot discover ways to carry out the operation, encourage them to reflect on how two numbers with the same sign were added and to think about how that process can be applied in the cases with two numbers with different signs.

## b. Activities to use mathematics in everyday life and in the society

To think and make decisions about events in our daily life by connecting them to mathematics, we must first put those events on the stage of mathematics, that is, to model those events. In that process, we may have to idealize or simplify the situations. For example, we may have to assume that a certain definition can be applied so that we can examine or process the situation mathematically. Then, the problems can be processed in the mathematical world and the results can be obtained. Then, the results must be examined and interpreted in the original context from our daily life to identify the solution to the problem. In this step, it is necessary to help students pay attention that, in the process of idealizing or simplifying, some constraints may have been introduced to the situation.

It is important that students can experience the purpose of using mathematics through activities to examine and process events from our daily life by connecting them to mathematics. In addition, through those activities, students can also experience the benefits of knowledge and skills they have previously learned and the ways to view and think about things mathematically.

Here are some examples of, "activities to use mathematics in everyday life and in society" in grade 1 of lower secondary schools. We will also discuss prerequisites necessary for students to willingly and autonomously engage in these activities.

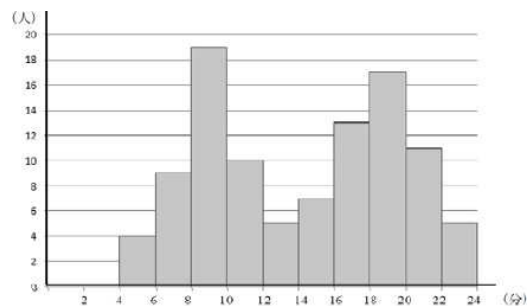
## ○Activities to identify own position in a population by using histograms and representative values

These activities are mathematical activities for grade 1 content, “D. Making Use of Data”, (1) - b. For example, students can answer the question, “can the amount of my commuting time be considered long among other students at my school?” by collecting appropriate data and using tools such as histograms and representative values.

In this process, help students know the benefits of grasping trends in data by using histograms and representative values so that they can use those ideas when they organize data.

Provide opportunities for students to collect data about students’ commuting time at their school, and for them to create histograms and calculate representative values, perhaps using tools such as computers, so that they can base their decisions on the data. Suppose the mean commuting time of all students is 13 minutes and one particular student’s commuting time is also 13 minutes. Students should think about whether or not it is appropriate to conclude that, “since the particular student’s commuting time is close to the mean commuting time, we cannot conclude his/her commuting time is long for the students at the school”. Positions within a population are influenced by the distribution of data, and there are cases we cannot use the mean value to make a judgment.

For example, if the data is distributed as shown in the histogram on the right, the conclusion that “there are many students whose commuting time is similar to that of the particular student’s” is incorrect. If students are making judgment solely based on the mean, encourage them to reflect on the characteristics of mean and to compare with other representative values so that they can also consider the distribution of data as a whole.



Judging whether or not one’s commuting time is long should be based on the median, or perhaps using the relative frequency so that we can say, “since the commuting time for the students is among the top 10%, his/her commuting time can be considered long for the students at the school”.

## c. Activities to explain and communicate logically and with clear rationale by using mathematical expressions

In order to express mathematically, facts and procedures about numbers, quantities, and geometrical figures or own thinking processes and the rationale for their judgment, it is necessary to represent ideas using words, numbers, algebraic expressions, diagrams, tables, and graphs appropriately.

It is also important to help students experience the benefits of expressing mathematically by providing opportunities for students to communicate their thinking and ideas using mathematical representations. In addition, set up opportunities for students to improve their own ideas by communicating with others or to discover something that could not have been discovered by working alone.

It is absolutely necessary in the study of mathematics to communicate what was discovered, to explain the steps of algorithms or solutions of a problem, and to justify their ideas. It is important that students can explain logically as they communicate their ideas.

In grade 1 of lower secondary schools, instead of expecting well-constructed expressions or appropriate interpretations from the beginning, the focus should be on helping students become accustomed to mathematical representations and having students explain and communicate their ideas using their own words. Help them experience the merits of mathematical representations, and gradually refine their expressions.

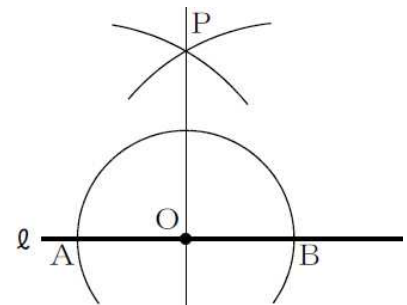
Here are some examples of, “activities used to explain and communicate logically and with clear rationale by using mathematical expressions” in grade 1 of lower secondary schools. We will also discuss prerequisites necessary for students to willingly and autonomously engage in these activities.

○Activities used to explain the process of constructing the perpendicular line to the given line through a given point on the line

These activities are mathematical activities for grade 1 content, "B. Geometrical Figures" (1) a. The purpose is to enable students to justify the steps of constructing the perpendicular line through a given point on the line by referring to ideas such as symmetrical figures and construction of an angle bisector. In this process, cultivate the foundation for explaining ideas by making reasons explicit. Also, through communication activities, help students to recognize different ideas from their own and use them to further improve their own ideas. It is important that students have been taught the processes of constructing angle bisectors and perpendicular bisectors of line segments and the fact those processes can be justified by using symmetries of geometrical figures. Based on those studies, help students understand the process of constructing perpendicular lines through a given point on a line and actually try constructing them.

Have students verify that perpendicular lines can indeed be constructed and provide opportunities for them to explain the steps.

The reasons that perpendicular lines can be constructed include the following: -"Because the process will draw a rhombus or a kite with segment AB as a diagonal and point O as the point of intersection of the diagonals," and "Because we have an isosceles triangle ABP with segment AB as the base and point O is the mid-point of AB." These ideas are using properties of geometrical figures that are symmetrical. It is also possible for students to use the constructions that they have learned previously. For example, "Because we constructed segment AB in such a way that point O is its mid-point, then we constructed the perpendicular bisector of AB," or "Because we are constructing the perpendicular bisector of  $\angle AOB$ , which measures  $180^\circ$ ". For those students who are having difficulty coming up with an explanation, encourage them to reflect on what they have learned previously and facilitate their thinking by considering the similarities of the processes of construction.



The focus here is not to discuss which reason is better, instead it is to check if students' explanations include explicit reasons through activities used to communicate own ideas. Therefore, what is important is if students are using terms such as "line," "segment," and "line symmetry," in their explanations, not the form of their explanation.

## Grade2

### A. Numbers and Algebraic Expressions

(1) To foster the ability to find out relationships of numbers and quantities in concrete phenomena, represent these relationships in algebraic expressions using letters and read the meaning of these expressions, and to be able to calculate the four fundamental operations with expressions using letters.

(a) To calculate addition and subtraction with simple polynomials, as well as multiplication and division with monomials.

(b) To understand how to grasp and explain numbers and quantities and the relationships of numbers and quantities in algebraic expressions using letters.

(c) To transform simple algebraic expressions according to the purpose.

[Terms and Symbols]

similar term

In grade 1 of lower secondary school, the following ideas are taught: using positive and negative numbers to represent quantities and quantitative relationships, using algebraic expressions with letters to represent quantities, quantitative relationships, and rules, interpreting algebraic expressions with letters, and arithmetic manipulation of algebraic expressions with letters. Students also learn to carry out simple calculations such as addition and subtraction of linear expressions so that they can solve linear equations with one variable.

Based on these studies, in grade 2, students are expected to learn to carry out the four arithmetic operations with polynomials with more than one variable. They are also expected to understand that algebraic expressions with letters can be used to capture and explain quantities and quantitative relationships, as well as further develop their ability to write and interpret algebraic expressions with letters. Another goal is to help students experience the merits of using algebraic expressions with letters.

In teaching algebraic expressions with letters in grade 2, consideration should be given to the study of simultaneous linear equations that follows as well as the contents of "B. Geometrical Figures" and "C. Functions."

#### Addition and subtraction of polynomials and multiplication and division of monomials

The goals here are for students to understand the meaning of monomials and polynomials, to carry out simple addition and subtraction of polynomials, such as  $(3x - 2y) - (2x + 5y)$ , to multiply polynomials by numbers, such as  $2(4x - 5y)$ , and to multiply and divide monomials.

Avoid complex practices that may not serve any purpose. Specially for addition and subtraction of polynomials, focus on helping students develop proficiency in carrying out simple calculations necessary to solve simultaneous linear equations, such as  $2(3x - 2y) - 3(2x + 5y)$ .

Also, consider providing opportunities for students to re-learn in connection with grade 1 study of calculations of algebraic expressions with letters. For example, the error below is probably because students did not understand the idea of terms.

$$\begin{aligned} & (4x + 5) - (2x + 3) \quad \dots \quad \textcircled{1} \\ & = 4x - 2x + 5 - 3 \\ & = 2x + 2 \\ & = 4x \end{aligned}$$



This error may be reduced if students deepen their understanding of terms, “terms with  $x$  and  $y$  cannot be combined into one term”, from their grade 2 study of addition and subtraction of algebraic expressions with two letters.

$$\begin{aligned} & (4x + 5y) - (2x + 3y) \quad \dots \quad \textcircled{2} \\ &= 4x - 2x + 5y - 3y \\ &= 2x + 2 \\ &= 3x + 2y \end{aligned}$$

By using this idea, based on the study of calculations such as  $\textcircled{2}$  shown, discuss calculations like  $\textcircled{1}$  above to make it an opportunity for them to recognize and correct the error.

### **Ideas that can be captured and explained by using algebraic expressions with letters**

In the study of mathematics in general, the use of algebraic expressions with letters is very important. In grade 1 of lower secondary schools, students learn to represent quantitative relationships and rules using algebraic expressions with letters. In grade 2, the study will be deepened so that students will understand that algebraic expressions with letters can capture and explain quantities and quantitative relationships and further develop their abilities to write and interpret algebraic expressions.

For example, in explaining, “the sum of two odd numbers is an even number”, the following steps are involved.

- $\textcircled{1}$  Express the two odd numbers as  $2m + 1$  and  $2n + 1$ , where  $m$  and  $n$  are integers.
- $\textcircled{2}$  Calculate their sum,  $(2m + 1) + (2n + 1)$ , and re-write the results,  $2m + 2n + 2$ , in the form,  $2(m + n + 1)$ .
- $\textcircled{3}$  Interpret that the expression obtained in  $\textcircled{2}$  represents  $2 \times (\text{integer})$ .
- $\textcircled{4}$  From  $\textcircled{3}$ , we can conclude that the sum of two odd numbers is an even number.

In order for students to understand that quantities and quantitative relationships can be captured and explained using algebraic expressions with letters, it is important that writing, interpreting, and calculating algebraic expressions with letters are taught in conjunction with each other as students try to justify a conjecture using algebraic expressions with letters.

Through such studies, students will develop their abilities to identify and represent quantitative relationships in phenomena and interpret algebraic expressions in contexts. They will also discover new relationships through induction and analogy and understand the necessity of explaining those discoveries generally using algebraic expressions with letters. They can also develop their ability to use algebraic expressions with letters. These ideas must be learned over time; therefore, instruction should aim at gradual development, keeping in mind the use of algebraic expressions with letters to be studied in grade 3 of lower secondary school.

### **Transforming algebraic expressions purposefully**

Transformations of algebraic expressions may be divided into the following two categories. Transformations in the first category are like those discussed above – transforming the algebraic expressions purposefully to explain properties of numbers and geometrical figures.

The other category involves transformations using the properties of equality relationships to create equivalent expressions. For example, the formula to calculate the area of triangles,  $S = \frac{1}{2}ah$ , can be transformed to a formula to calculate the length of a base by solving it for  $a$ . Transformations of equations are used in many different situations. Therefore, it is important that students can freely transform equations in simple cases such as transforming the formula for the area of triangle to obtain a formula to calculate the length of area. Avoid using unnecessarily complex algebraic expressions whose transformations will not serve any purpose.

In either case, the focus of the study is to help students experience the merits of transforming algebraic expressions purposefully in concrete situations, instead of meaningless manipulation of algebraic expressions.

- (2) To understand simultaneous linear equations with two unknowns, and to be able to consider by using it.
  - (a) To understand the meaning of linear equations with two unknowns and their solutions.
  - (b) To understand the necessity and meaning of simultaneous linear equations with two unknowns and the meaning of their solutions.
  - (c) To solve simple simultaneous linear equations with two unknowns and make use of them in concrete situations.

In grade 1 of lower secondary school, students are expected to understand the meaning of letters and solutions of linear equations with one variable, and to learn to solve those equations.

In grade 2, based on those studies, students are expected to understand linear equations with two variables and the necessity and the meaning of simultaneous linear equations with two variables. They are expected to be able to solve those equations. Moreover, it is aimed that students will develop the ability to use simultaneous linear equations in concrete situations.

### **Meaning of linear equations with two variables and their solutions**

Help students understand that the solution to a linear equation with two variables is a pair of values for the two letters,  $x$  and  $y$ , which will satisfy the given equation. In other words, there is no fundamental difference from the meaning of solutions for linear equations with one variable. The two letters in a linear equation with two variables are both variables, and the pair of values from the range of numbers for each variable that satisfies the equation is its solution.

For example, with the equation,  $2x + y = 7$ , if the range of values for variables  $x$  and  $y$  are natural numbers, then there are a finite number of solutions, namely  $(1, 5)$ ,  $(2, 3)$ , and  $(3, 1)$ . If the range of variables is all integers, then there are an infinite amount of solutions. In this way, solutions to linear equations with two variables differ from solutions to linear equation with one variable because there may be more than one solution with linear equations with two variables.

### **The necessity and the meaning of simultaneous linear equations with two variables and the meaning of their solutions**

A pair of simultaneous linear equations with two variables represents two conditions, each represented by an equation, to be satisfied. To solve a system of simultaneous linear equations with two variables means to find a pair of values that will satisfy both equations. To help students understand the meaning of the solution to a pair of simultaneous linear equations with two variables, it is possible to restrict the range of values for each variable to the set of natural numbers and have students list solutions for each equation so that they can find a common solution. This way of solving simultaneous equations is not efficient in the process of problem solving. However, there are ways to solve simultaneous equations efficiently by transforming the equations as discussed below. Those methods will be useful in the context of concrete problem solving situations.

It is also possible to deepen students' understanding of the meaning of solutions to simultaneous equations by connecting linear functions and graphs of linear equations with two variables.

## **Solving simultaneous linear equations with two variables**

A way to solve simultaneous linear equations with two variables is to eliminate one of the letters so that we can use the methods to solve linear equations with one variable. This way of finding the solution is an example of thinking that will connect new problem solving situations to what has already been learned previously. In addition to enabling students to solve simultaneous linear equations with two variables, help students recognize this way of thinking on their own so that they can understand solution by elimination or by substitution.

Consideration should also be given to provide opportunities to re-learn what it means to, "solve an equation," by making connections to the study of linear equations with one variable from grade 1.

The study of solving simultaneous linear equations should aim at developing sufficient proficiency to solve simultaneous linear equations from concrete problem situations and enabling students to use simultaneous linear equations.

## **Use of simultaneous linear equations with two variables**

While using linear equations with one variable, we can use only one variable to represent quantitative relationships in phenomena. However, in concrete situations, it is often easier to represent relationships using two variables than with only one variable. By using simultaneous linear equations with two variables, the range of problems that can be solved will expand, as well as problem solving becomes easier.

To use simultaneous linear equations, the step of setting up the equations is important. In the process of instruction, having students capture and represent quantitative relationships in algebraic expressions by focusing on a specific quantity such as relationships of length, time or weight. It is also useful to make the relationships more explicit by representing the quantities in a table or a segment diagram.

Moreover, as students attempt to solve concrete problems, help them realize that the number of equations and the number of variables must be the same in order to have a unique solution. Help students use linear equations with one variable and simultaneous linear equations with two variables with a good prospect in mind.

## B. Geometrical Figures

(1) Through activities like observation, manipulation and experimentation, be able to find out the properties of basic plane figures and verify them based on the properties of parallel lines.

(a) To understand the properties of parallel lines and angles, and based on that, verify and explain the properties of geometrical figures.

(b) To know how to find out the properties of angles of polygons based on the properties of parallel lines and angles of triangle.

[Terms and Symbols]

opposite angle<sup>3</sup> , interior angle, exterior angle

In grade 1 of lower secondary schools, construction and transformation of geometrical figures are discussed. The relationships among lines and planes in space are also discussed. Students also learn to think about space figures as the result of moving plane figures, to represent space figures on a plane and to interpret features of space figures represented on a plane. Furthermore, students are expected to be able to calculate the length of an arc, the area of a sector, and surface area and volume of prisms, pyramids and spheres. Through these studies, students' spatial sense is enriched and their knowledge of geometrical figures is deepened. Their ability to examine and represent logically is also cultivated.

In grade 2, enable students to investigate properties of angles in triangles, quadrilaterals, and other polygons using coherent and logical reasoning. The aim is to help students express their reasoning using their own words in a way that is easily understood by others through activities used to observe geometrical figures carefully and to manipulate and experiment such as construction of figures.

### Properties of parallel lines and angles

The study of coherent and logical reasoning starts with the study of properties of vertical angles and properties of parallel lines.

Typically, the following two properties of parallel lines are discussed:

- Corresponding angles formed by a transversal intersecting a pair of parallel lines are congruent.
- If corresponding angles formed by a line intersecting a pair of lines are congruent, then the two lines are parallel.

Students learn about parallel lines in grade 4 of elementary school. For example, 2 lines are defined to be parallel when they are perpendicular to another line. Then, through activities such as drawing parallel lines, the two ideas above are intuitively and empirically recognized. In lower secondary schools, these ideas become the basis of reasoning.

From the proposition, “vertical angles are congruent”, and the two ideas above, the following propositions can be deduced:

- Alternate interior angles formed by a transversal intersecting a pair of parallel lines are congruent.
- When alternate interior angles formed by a line intersecting a pair of lines are congruent, then the pair of lines are parallel.

Then, enable students to deduce that, “the sum of angles in a triangle is  $180^\circ$ ”, or the angles in a parallelogram can be determined when one angle measurement is given.

As for deducing new ideas, considerations should be given to the fact that elementary school mathematics provided students with some foundational experiences. In grade 2 of lower secondary school, it is important to help students become able to explain their ideas in an orderly manner using their own words instead of demanding formal written proofs. For example, while thinking about the congruence of vertical angles, instead of just relying on measurements, it is necessary to incorporate activities to verify this relationship in a coherent manner, making clear what the basis of the argument is.

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<sup>3</sup>Typically called vertical angles

## Properties of angles in polygons

Based on the properties of angles in a triangle, discuss the sum of interior angles and the sum of exterior angles of polygons.

As for the sum of interior angles in polygons, although the conclusion is important, the aim is to help students learn that they can find the sum by dividing polygons into basic figures, namely triangles. This is an example of mathematical thinking, "relating to what was learned previously." For the sum of exterior angles, enable students to be able to find the sum using what they learned about the sum of interior angles. It is also important to reflect on the methods and results of the sum of interior angles in triangles and quadrilaterals, which were discussed in elementary school.

- (2) To understand the congruence of geometrical figures and deepen the way of viewing geometrical figures, to verify the properties of geometrical figures based on the facts like the conditions for congruence of triangles, and to foster the ability to think and represent logically.
  - (a) To understand the meaning of congruence of plane figures and the conditions for congruence of triangles.
  - (b) To understand the necessity, meaning and methods of proof.
  - (c) To logically verify the basic properties of triangles and parallelograms based on facts like the conditions for congruence of triangles, and to find out new properties by reading proofs of the properties of geometrical figures.

[Terms and Symbols]

definition, proof, converse

[Handling the Contents]

- (1) In connection with (2)-(c) under "B. Geometrical Figures" in "Content," the fact that square, rhombus and rectangle are special cases of parallelogram should be dealt with.

In (1), students are to experience deducing properties of angles in polygons using properties of parallel lines and explaining their thinking in a coherent manner while making the reasons explicit. Here, the major goal is to foster students' abilities to verify, examine, and represent properties of geometrical figures deductively, using the congruence conditions of triangles.

Students have examined properties of isosceles triangles in grade 3 of elementary school and parallelograms in grade 4 through doing experiments, measurements, and observations by focusing on angles and sides.

In grade 2 of lower secondary school, enable students to examine properties of geometrical figures logically through coherent reasoning. In addition, an important aim is to enable students to express their ideas appropriately through activities for communicating their thinking processes and results.

The relationship of inscribed angles and central angles has been moved to grade 3 from grade 2 of lower secondary school in this current revision. The reason is to continue the study of properties of geometrical figures in grade 2 and to examine the relationship between inscribed angles and central angles using those properties and through mathematical reasoning. The use of the relationship between inscribed angles and central angles in concrete situations is also emphasized.

## Meaning of congruence and the congruence conditions of triangles

Two geometrical figures are congruent if:

- ① One figure can be moved on top of the other by combination of transformations, or
- ② All corresponding segments and corresponding angles are equal in their measures.

Condition ① defines congruence based on transformations of geometrical figures studied in grade 1 of lower secondary school.

On the other hand, condition ② is a more static definition of congruence of figures that are composed of line segments. The aim in grade 2 is to foster students' ability to verify properties of geometrical figures deductively, based on the congruence conditions of triangles, and to examine and express their ideas logically.

As was the case with the properties of parallel lines, the congruence conditions for triangles are not deduced here. Rather, they are accepted intuitively and empirically. Two triangles are congruent if:

- the length of all three corresponding pairs of sides are equal,
- the length of two corresponding pairs of sides are equal and the measures of angles between the two sides are also equal, or
- the length of a pair of corresponding sides are equal and the measures of corresponding angles on the end points of the side are also equal.

It is important that students understand that of the six constituent parts of a triangle, three sides and three angles, only three are needed to judge for congruence. It is also important that students can use these as the bases of reasoning. The congruence conditions of triangles may be discussed in conjunction with the instruction of geometrical construction in grade 1.

The proposition, "the measures of the base angles of an isosceles angles are equal," and the following congruence conditions for right angles can be deduced from the congruence conditions of triangles discussed above.

Two right triangles are congruent if

- the measures of the hypotenuse and one of the corresponding acute angles are equal, or
- the measures of the hypotenuse and one of the corresponding legs are equal.

The congruence conditions of triangles are used in a wide range of situations, for example, proving the construction for copying a given angle and the construction of the angle bisector. In some cases, only one pair of geometrical figures are congruent while in others, perhaps drawing auxiliary lines, a series of pairs of figures must be shown to be congruent to reach a conclusion. Thus, there is a wide range of complexity in its use, and it is necessary that instruction is adjusted appropriately to the level of students' understanding and development.

## Mathematical reasoning

Instruction on mathematical reasoning to help students understand its necessity and meaning should take place in all content domains necessary, not just in the domain of geometrical figures. However, the domain of geometrical figures is particularly suited for teaching the necessity and the meaning of mathematical reasoning because its process can be visually captured by the use of concrete figures.

There are three types of mathematical reasoning: induction, analogy, and deduction. Induction and analogy are used frequently in elementary school mathematics. These types of reasoning are used when a more general conclusion is derived from several results obtained from observations, manipulations, and experimentations. Deduction is also used in elementary school mathematics. Induction and analogy are particularly important for discoveries. Deduction is important to verify those discoveries. It is necessary that students understand different roles of these types of reasoning and be able to use appropriate reasoning purposefully.

Induction and analogy play an important role in generating hypotheses properties and relationships of geometrical figures based on examinations and processing of specific figures. However, in order to verify the hypotheses generated deductive reasoning is needed. Hypotheses are not always valid. Students should be taught to demonstrate this by using counterexamples. Considerations should be given to discuss how to modify the original hypothesis to derive a valid proposition.

In the process of mathematical reasoning, it is important to make the bases of reasoning clear. In the domain, "B. Geometrical Figures," propositions that are used as the bases of reasoning includes properties of vertical angles, properties of parallel lines, and the congruence conditions of triangles. The study of deductive reasoning, using those propositions as the bases to verify properties of geometrical figures, starts seriously in grade 2 of lower secondary school.

However, reasoning logically and coherently is not a new idea in grade 2. In grade 1, students are expected to explain their ideas using what they have previously learned as the basis in situations such as geometrical construction of plane figures and composition of space figures. Thus, even in grade 1, students partially experienced deductive reasoning.

An important aim in grade 2 is to foster the ability to express the process of reasoning accurately and in a way that is easy to understand. However, this cannot be accomplished all at once. Thus, at first, the emphasis is on expressing the process of reasoning in one's own words so that others can easily understand through activities used to explain and communicate by making the basis of reasoning explicit. Help students become used to terms and symbols such as "because," "alternately," "therefore," "on the other hand," and "as a result." In this way, help students gradually raise their ability to express the process of reasoning accurately and in a way that is easily understood by others.

As for the instruction on writing formal proofs, start by having students plan an overview of a proof for simple conjectures, then write the main points of the proof using their own words, incorporating the terms and symbols discussed above. Then, help them gradually improve their writing. It may also be useful to incorporate activities for evaluating proofs such as comparing two different proofs of the same proposition to compare and contrast them or identify and correct a faulty proof. Teaching of writing proofs will continue on through grade 3 of lower secondary school. Thus, instruction should be done in steps to help students improve their proof writing gradually.

### **The necessity and the meaning of proofs and their methods**

A proposition consists of "hypothesis" and "conclusion." Thus, before proceeding with justification, we must first clearly identify the "hypothesis" and the "conclusion." A proof is a series of statements starting with the "hypothesis" and leading to the "conclusion," supported by the ideas that have already been accepted as true. To disprove a proposition, we only need to find a counterexample. In the instruction of proofs, it is important that students can not only prove but also disprove something. Moreover, if we reverse the "conclusion" and "hypothesis," the new statement is called the "converse" of the original proposition. Even if the original proposition is true, its converse is not necessarily true. Students are expected to understand that.

In order to understand the need for a proof, it is important for students to understand the difference between ideas that are inductively derived from the results of activities of observation, manipulation and experimentation, and those that are derived deductively. It is possible to increase the likelihood of a conjecture derived inductively from the examination of several geometrical figures by testing it against several other geometrical figures that satisfies the conditions. However, we cannot verify whether or not the conjecture is true with all figures that satisfy the condition. Therefore, we need deductive proofs. In this way, students are expected to understand why we need deductive proofs.

The following points should be made clear:

- a) A Proof is a method of showing a proposition is true generally, without exception.
- b) Drawings used in the process of proving are representative of all figures.

Also, in the process of proving, we must make the basis for our reasoning explicit. As discussed above, the following ideas are examples of ideas that can be used as the basis for reasoning: properties of vertical angles, properties and conditions of parallel lines, properties of congruent figures, and the congruence conditions for triangles. In grade 3 of lower secondary school, properties of similar figures and the similarity conditions of triangles are also included.

## Properties of triangles and parallelograms

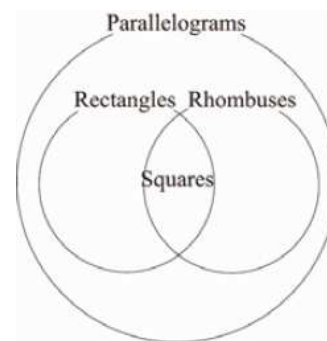
The important goal here is to foster students' abilities to examine and represent their ideas logically, as well as deepen their understanding of geometrical figures by examining properties and conditions of various triangles and parallelograms deductively by using previously learned properties of parallel lines and the congruence conditions of triangles.

The following basic properties and conditions about triangles and parallelograms are discussed:

- The properties of isosceles triangles and congruence conditions of right triangles
- The properties of parallelograms and conditions for parallelograms
- The properties of squares, rectangles, and rhombuses

Since students have seen, "properties of isosceles triangles" and, "properties of parallelograms" in elementary school mathematics, they often wonder, "why do we need to prove something we already know?" and some may lose interest. Therefore, the instruction should emphasize the importance of explaining something that they already understand to others so that they can also understand it. Help them understand the necessity and the meaning of proofs and their methods.

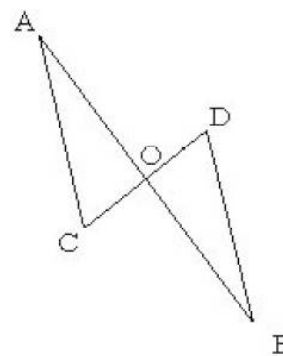
As for, "properties of parallelograms", the "Handling of Content" states that "squares, rhombus and rectangles are special cases of parallelograms that should be dealt with." This statement means that students are to understand the relationships among rectangles, rhombuses, squares, and parallelograms logically by examining the definitions of those figures and, "conditions for parallelograms." For example, drawing a diagram like the one shown on the right may be used. Also, for some of the properties of parallelograms, have students verify that they also are true in squares and rectangles, which are special types of parallelograms.



## Reading proofs and discovering new properties

In the study of proofs of properties of triangles and quadrilaterals, students should not only write proofs but also read them. Reading proofs is necessary when re-examining or evaluating a proof of a property of geometric figures.

For example, consider the proof for the proposition, "given two segments, AB and CD, intersect at point O. If  $AO=BO$  and  $CO=DO$ , then  $\triangle AOC \cong \triangle BOD$ ". After reading the proof, focusing on the segments and angles that were not used in the proof, students may be able to discover new properties,  $AC=BD$ ,  $\angle OCA = \angle ODB$  and  $\angle CAO = \angle DBO$ . Moreover, since  $\angle CAO = \angle DBO$ , students can discover that  $AC \parallel BD$  and quadrilateral ACBD is a parallelogram.



It is important to foster the ability to examine and represent logically by reading proofs. It is also important to emphasize the idea of discovering new properties by reading proofs not only in the domain, "B. Geometrical Figures" but also in "A. Numbers and Algebraic Expressions" as the idea of explaining quantitative relationships by reading algebraic expressions with letters.



## C. Functions

- (1) Through finding out two numbers/quantities in concrete phenomena and exploring their change and correspondence, to understand linear functions, and to foster the ability to find out, represent and think about functional relationships.
  - (a) To know that in concrete phenomena there are some phenomena that can be grasped as linear functions.
  - (b) To understand linear functions by interrelating tables, algebraic expressions and graphs.
  - (c) To recognize linear equations with two unknowns as algebraic expressions representing functions.
  - (d) To grasp and explain concrete phenomena by using linear functions.

[Terms and Symbols]

rate of change, slope

In grade 1 of lower secondary school, students investigated the changes and correspondences of two quantities in concrete phenomena and developed an understanding of functional relationships. They have also re-conceptualized direct and inverse proportional relationships as functional relationships. They were expected to understand variables, ranges of numbers variables can take, and the coordinates. Also, students learned to represent direct and inverse proportional relationships in tables, algebraic expressions, and graphs, and to identify characteristics of the relationships. They also learned to use direct and inverse proportional relationships to grasp and explain concrete phenomena.

In grade 2, like in grade 1, through activities of investigating changes and correspondences of two quantities in concrete phenomena, students will examine linear functions. Through this study, foster students' abilities to identify, represent and examine functional relationships. The study of linear functions is an extension of the study of proportional relationships. At the same time, it is an entrance to a more in-depth study of functions represented in algebraic expressions with letters, such as the focusing on the rate of change.

### Phenomena and linear functions

In grade 2, based on the study of direct and inverse proportional relationships, students are to understand linear functions and deepen their understanding of functional relationships. Students are expected to find the following relationships between two quantities,  $x$  and  $y$ , that are in a functional relationship from concrete phenomena.

- As  $x$  increases by  $k$ ,  $y$  increases by  $ak$ .

Based on studies like this, students are to learn to represent the relationship using algebraic expressions with letters. Students are to understand that, in general, linear functions can be represented as  $y = ax + b$ , where  $a$  and  $b$  are constants. They are also to know that there are phenomena that may be grasped with linear functions.

Building on the study of direct and inverse proportional relationships in grade 1, students identify two quantities that change simultaneously and examine what functional relationships might exist between them and how they may be represented as algebraic expressions and graphs. Direct proportional relationships are special cases of linear functions,  $y = ax + b$ .

## Tables, algebraic expressions, and graphs of linear functions and their relationships

In grade 1, students studied how to use tables, algebraic expressions, and graphs to capture features of changes and correspondences between two quantities in a functional relationships.

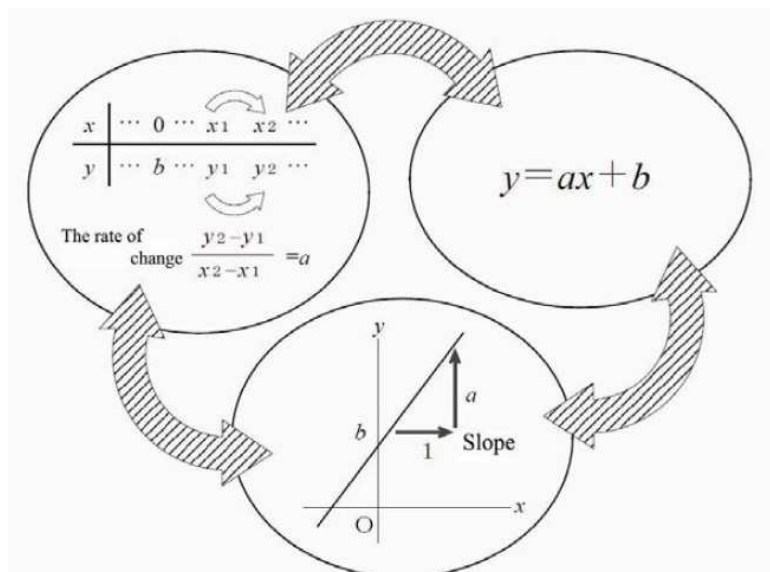
Building upon this, in grade 2, students will capture the characteristics of linear functions by using tables, algebraic expressions, and graphs and by making connections among them so that they can deepen their understanding of linear functions.

In particular, have students think about the way linear functions change. Up to this point, students have often thought about ratios of corresponding values or the increase/decrease of quantities based on tables. Here, in order to capture the way functions change more succinctly, students will learn about the rates of changes of values of corresponding variables.

For linear function,  $y = ax + b$ , when the values of variable  $x$  changes from  $x_1$  to  $x_2$ , that is, changes by  $x_2 - x_1$ , the corresponding values of  $y$  changes from  $y_1$  to  $y_2$ . The rate of change,  $\frac{y_2 - y_1}{x_2 - x_1}$ , is constant and equals  $a$ . This is a unique feature of linear functions and it means that the graph will be a straight line.

By examining rates of changes, help students understand that the coefficient of  $x$ , that is  $a$ , is how much  $y$  will increase when  $x$  increases by 1. Moreover, help students to understand that the amount of increase in  $y$  corresponding to the amount of increase in  $x$  can be determined based on the value of  $a$ .

The graph of linear function,  $y = ax + b$ , is a line and  $a$ , indicates its slope. Therefore, the increase/decrease of values of the functions can be judged by whether,  $a$  is a positive or negative number. Moreover,  $b$  is the value of  $y$  when  $x$  is 0. Therefore, it is the  $y$ -coordinate of the point of intersection between the graph of the function and the  $y$ -axis. Students are expected to understand these ideas as well.



The reason the phrase, "rate of change," is included in "Terms and Symbols" is so that its instruction will not simply focus on calculation of rates of change procedurally. In addition, it is important to enable students to use rates of change in examinations and explanations of phenomena appropriately.

## Linear equations with two variables and linear functions

Given the linear equation with two variables,  $ax + by + c = 0$ , as a representation of the relationship between the two variables  $x$  and  $y$ , then, the combinations of values of  $x$  and  $y$  become the objects of examination.

When determining the pairs of values for  $x$  and  $y$  that will satisfy this equation, if  $b \neq 0$ , then once the value of  $x$  is fixed the corresponding value of  $y$  is also fixed. Therefore, we can consider the algebraic expression  $ax + by + c = 0$  represents a functional relationship between  $x$  and  $y$ .

For example, the linear equations with two variables,  $x - 2y + 6 = 0$ , can be considered as an algebraic expression representing a functional relationships. Moreover, if we transform this expression to  $y = \frac{1}{2}x + 3$ , we can see that  $y$  is a linear function of  $x$ .

Since the graphs of linear equations with two variables are lines, the solution of simultaneous linear equations with two variables can be solved by locating the point of intersection of the two lines in the coordinate plane. Therefore, it is possible to understand visually the meaning of solutions of simultaneous linear equations with two variables.

## Grasping and explaining phenomena using linear functions

As it was taught in grade 1, there are many phenomena that can be grasped as functional relationships. Here, teach students to grasp and explain concrete phenomena using linear functions. When grasping and explaining phenomena, students should be purposeful and have a clear idea of what it is they want to clarify. It is important that students make it clear how phenomena are interpreted and made into objects of mathematics, and explain them using tables, algebraic expressions, and graphs chosen purposefully.

When the relationship between two quantities identified in concrete phenomena appears to be a linear function based on observation and experimentation, how the quantities change and correspond can then be observed and predicted based on the idea of linear functions. For example, students can graph the relationship between time and temperature as water is being heated. By observing how the points are plotted, the relationship between the two quantities can be simplified or idealized. Then, the relationship between these two quantities can be considered as a linear function. By using the algebraic expression for the linear function, we can predict how long it will take to heat the water to a particular temperature, and the reason for the prediction can be explained. Also, by comparing the predictions with the actual results of experiments, through communication activities, students can think about the reasons for the differences and devise a better way to make predictions.

## D. Making Use of Data

- (1) Through activities like the observation of and experimentation on uncertain phenomena, understand probability, and be able to consider and represent by using probability.
  - (a) To understand the necessity and meaning of probability, and find the probability of an uncertain event in simple cases.
  - (b) To grasp and explain uncertain phenomena by using probability.

In elementary school mathematics, how to count the number of all possible events systematically in concrete situations is taught in grade 6.

In grade 1 of lower secondary school, students learn that relative frequencies indicate the ratios of frequency in individual classes to the total number of data, and they can be considered as frequency of each class.

Based on these studies, in lower secondary school mathematics, students are to know that numbers that have been used to describe definitive phenomena may also be used to describe uncertain phenomena such as the tossing of a die in grade 2. Also, enable students to explain uncertain phenomena using the idea of probability.

### The necessity and the meaning of probability

In mathematics lessons, definitive phenomena are typically discussed. However, in reality, uncertain phenomena from our daily life and society are also objects of mathematics, and to describe and understand the likelihood of occurrence, we need the idea of probability.

We cannot predict the result of rolling a die. However, when we roll a die many times and organize the data, the ratio of the number of times each number rolled to the total number of tosses tend to reach a fixed value. Students are to understand that probability is used to describe the likelihood of the occurrence of an event based on, "the law of large numbers."

For example, make the total number of tosses,  $n$ , larger and larger. Then, count the number of times,  $r$ , "1" is rolled and calculate the ratio  $\frac{r}{n}$ . As  $n$  becomes larger,  $r$  also becomes larger. However, the value of  $\frac{r}{n}$  becomes closer to a certain value. This fixed value to which the ratio tends toward is called the probability of rolling "1" when you toss a die.

Also, if we roll a fair die, we should expect the same likelihood of rolling any number. Therefore, for each number, if we roll a die many times, we anticipate the ratio of rolling it will approach  $\frac{1}{6}$ . In reality, the value to which the ratio  $\frac{r}{n}$  discussed above approaches when we toss a die many times is indeed  $\frac{1}{6}$ .

As with this example, when we can expect each event to occur at the same frequency, that is, it is "equally likely," we can determine the probability by counting the number of ways events may occur.

To determine probability, we can save much time and effort by counting the number of outcomes instead of actually conducting experiments many times. However, the true meaning of probability, that is what can be learned about uncertain events, may get lost. For example, if a student thinks, "because the probability of rolling a "1" is  $\frac{1}{6}$ ," "if we roll a die 6 times, we will get a "1" one time," then it indicates that their understanding is incomplete.

In teaching this topic, have students conduct experiments many times so that they can actually experience and understand that the ratio of a particular event occurring will approach a particular value. When probability is determined by counting possible events, it is also important to have students not only check whether or not the answers are correct but also what was understood about the likelihood of the occurrence of events by actually conducting experiments and investigations.

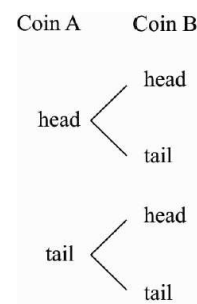
## Determining probability for simple cases

To determine probability based on the counting of all possible outcomes, it is necessary to count accurately the number of equally likely outcomes for the event. Based on the study in grade 6, help students learn to count accurately by organizing outcomes systematically. The events should be simple enough so that students can count outcomes correctly using tools such as tree diagrams and 2-dimensional tables.

An example of a simple case is the tossing of two coins simultaneously and recording the outcomes of each coin. There are four different outcomes possible: (head, head), (head, tail), (tail, head), and (tail, tail), and these outcomes are equally likely. Of these four outcomes, there is only one outcome that will result in both coins landing as heads. Therefore, the probability of getting two heads is  $\frac{1}{4}$ .

As discussed earlier, in this example, it is necessary to note that, “ the probability is  $\frac{1}{4}$  ” does not mean that if we toss two coins simultaneously four times, we will definitely get two heads one time.

Also in the example above, a common error is to think that the outcomes are (head, head), (head, tail), and (tail, tail), and the probability of getting two heads is  $\frac{1}{3}$ . Help students to count all outcomes without any omission or duplication. It is also important that students can understand experientially. Have students actually conduct experiments many times and compare the results obtained with their counts.



## Grasping and explaining uncertain phenomena

We can grasp and explain uncertain phenomena using the idea of probability. Probability is effective as the basis for grasping and explaining uncertain phenomena.

In teaching this topic, the focus should not be solely on determining probability, but an emphasis should be placed on problem solving involving uncertain phenomena. It is important that students can explain phenomena using probability as the basis of their reasoning. It is necessary to incorporate phenomena from our daily life and society so that it becomes clear to students that those phenomena can be explained using probability as the basis of reasoning.

For example, to determine whether or not the order in which you draw will influence the likelihood of winning a lottery, that is, whether or not the lottery is fair, we can explain using the idea of probability. It is also used to instruct that the rules of the lottery have to be clearly stated since the judgment may be influenced by the way the rules are stated.

By grasping and explaining uncertain phenomena using probability, help students understand that numbers can be used to make decisions involving those situations where we cannot say, “this will certainly be \_\_\_.” Help students experience the relationships between mathematics and real-life and in society. It is important to emphasize the meaning of probability when teaching this idea.

## [Mathematical Activities]

- (1) In learning each content of “A. Numbers and Algebraic Expressions”, “B. Geometrical Figures”, “C. Functions”, and “D. Making Use of Data”, and in learning the connection of these contents, students should be provided their opportunities for doing mathematical activities like the following:
- (a) Activities for finding out and developing the properties of numbers and geometrical figures based on previously learned mathematics
  - (b) Activities for making use of mathematics in daily life and society
  - (c) Activities for explaining and communicating each other in an evidenced, coherent and logical manner by using mathematical representations

In grade 1 of lower secondary school, as students study the contents of the four domains and relate them to each other, by providing ”activities for finding out the properties of numbers and geometrical figures based on previously learned mathematics” , ”activities for making use of mathematics in daily life”, and ”activities for explaining and communicating each other in one’s own way by using mathematical representations”, it was intended that students can engage in mathematical activities willingly and autonomously. Other aims help students master basic and fundamental knowledge and skills, develop the ability to think, decide and express, and experience the joy and meaning of studying mathematics.

In grade 2, this fundamental perspective is further emphasized and the quality of mathematical activities is to be raised. This should be done by considering the students’ level of mathematical understanding, their own development as well as the relationships to the grade 2 content in each domain.

By the way, the three types of activities below are identical to those discussed in grade 3. This is because we decided that it is necessary to provide consistent instruction over two years, making appropriate adaptations to the specific content in each grade.

The grade 2 content of, ”Role of mathematical activities” (pp 98) and ” Engaging students in mathematical activities” (pp. 99) does not change from those of grade 1.

### **a. Activities for finding out and developing the properties of numbers and geometrical figures based on previously learned mathematics.**

“Activities for finding out the properties of numbers and geometrical figures based on previously learned mathematics” in Grade 1, emphasize the process of discovering properties based on prior learning. In grade 2, students’ willingness and autonomous engagement with mathematics is further emphasized. Thus, opportunities should be provided for students to further develop the properties they discovered and identify new problems to solve. Therefore, ”activities for finding out and developing the properties of numbers and geometrical figures based on previously learned mathematics” are, as in grade 1, activities used to extend one’s own thinking and to think creatively. Mathematical ways of viewing and thinking play important roles. In the process of a mathematical activity, as the level of the content in each domain increases, the need for verification using deduction and the importance of appropriate use of mathematical reasoning to discover mathematical facts also increases. Thus, the aim is not only to discover mathematical facts or procedures, but also to increase the quality of and to further refine mathematical reasoning such as induction, analogy (to develop predictions and conjectures), and deduction (to verify and justify). For this purpose, it may be necessary to re-examine the discovered properties of numbers and geometrical figures from different perspectives, such as changing the conditions and thinking about the converse, so that students can further extend their discoveries. It is also necessary to consider that mathematical reasoning may also be the starting point for such extension.

Here are some examples of, ”activities for finding out and developing the properties of numbers and geometrical figures based on previously learned mathematics” in grade 2 of lower secondary school. We will also discuss prerequisites necessary for students to willingly and autonomously engage in these activities.

## ○Activities to determine the sums of interior angles and of exterior angles of $n$ -gon

These activities are mathematical activities for grade 2 content, "B. Geometrical Figures," (1)-b. The aim is for students to discover that the sum of interior angles of  $n$ -gon can be represented as  $180^\circ \times (n - 2)$  and the sum of exterior angles is  $360^\circ$ . In the process, students will discover inductively that the sum of interior angles of quadrilaterals and pentagons, and then clarify the reasons based on the fact that the sum of interior angles of a triangle is  $180^\circ$ . Through such experiences, help students to know the usefulness of making connections to previously learned content. Moreover, students may discover that by focusing on exterior angles, instead of interior angles, there will be a new problem to explore. Enable students to use these experiences in their further study of properties of geometrical figures.

For that purpose, through activities, teach students that by dividing polygons into triangles by diagonals drawn from one vertex (Figure 1), the sum of interior angles of  $n$ -gon can be represented as  $180^\circ \times (n - 2)$ . Help students understand that this reasoning is based on the fact that the sum of interior angles of a triangle is  $180^\circ$ . Also, by reflecting on the process of investigation, help students to start thinking about other ways of determining the sum of interior angles focusing on the idea of dividing polygons into triangles.

Based on these studies, students can think about dividing polygons into triangles differently. For example, by drawing segments connecting a point on a side of the polygon to each of its vertices, we can subdivide the polygon into triangles (Figure 2). From this, based on the sum of interior angles of triangles is  $180^\circ$ , set up an opportunity for students to represent the sum of interior angles of the  $n$ -gon. Also, compare the new algebraic expressions for the sum of interior angles with the one obtained by sub-dividing the polygon by diagonals and clarify their relationship. For those students who cannot represent the sum of interior angles using algebraic expressions with letters, facilitate them to think about the relationship between the interior angles of polygons and the interior angles of triangles obtained by subdividing the polygons in quadrilaterals and pentagons inductively.

Then, change the focus of investigation to exterior angles, and think about the sum of exterior angles of  $n$ -gon (Figure 3). Here use previously learned relationships, that is, the sum of an interior angle of a polygon and the related exterior angle is  $180$ , and the sum of interior angles of  $n$ -gon is  $180^\circ \times (n - 2)$ , to derive the algebraic expression to determine the sum of exterior angles. It is also possible to investigate the sum of exterior angles of quadrilaterals and pentagons so that students have generated a conjecture, "the sum of exterior angles is  $360^\circ$ ," first.

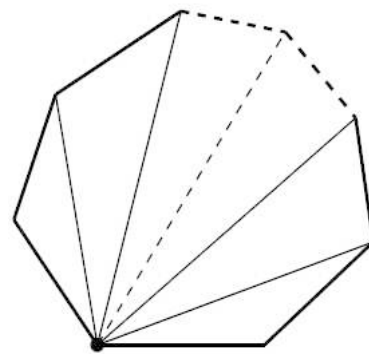


Figure 1

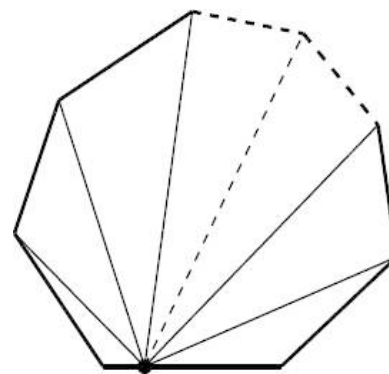


Figure 2

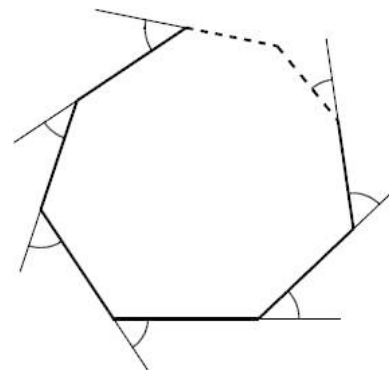


Figure 3

## **b. Activities for making use of mathematics in daily life and society**

”Activities to use mathematics in everyday life” in grade 1 of lower secondary school emphasized on using mathematics in events from students’ surroundings as the objects of examination. In grade 2, the range of situations where mathematics is to be used is broadened to include various phenomena in the society as students engage in this type of activities.

By broadening the range of situations where mathematics may be used, help students to engage in activities involving situations as if they are their own even though first-hand experience might not be possible. Through such experiences, it is important to help students to understand the purpose of using mathematics. Also, by connecting various situations to mathematics, help students understand the roles and merits of previously learned knowledge, skills, and mathematical ways of viewing and thinking. This will increase opportunities for students to experience the merits of mathematics.

The fact that the range of situations to use mathematics is expanded to society in general does not mean to value the use of mathematics in society more than in daily life. It is important to consider carefully the relationship between the content of each domain and situations where mathematics are to be used so that these types of activities are implemented in alignment with their purposes.

Here are some examples of ”activities for making use of mathematics in daily life and society” in grade 2 of lower secondary schools. We will also discuss prerequisites necessary for students to willingly and autonomously engage in these activities.

### **○ Activities to make predictions by considering the relationships between two quantities as a linear function**

These activities are mathematical activities for grade 2 content, ”C. Functions,” (1)-e. The purpose is to predict the amount of time necessary to heat water to a specific temperature based on an experiment in which the relationship between the time and the temperature while water is being heated is examined. In the process, help students to know the merit of considering the relationship in an experiment or an observation as a linear function through idealizing or simplifying so that they can process and make predictions using tables, algebraic expressions, and graphs. Help students to make use of this idea in examination of phenomena.

For this purpose, through activities, teach students to capture unique features of linear functions in tables, algebraic expressions, and graphs, as well as in their relationship with each other. Also, help students understand the meaning of considering a situation as a linear function. For example, they can examine the relationship between the time and the length of an incense stick, and then predict when the stick will be a particular length or at what time the stick will completely burn up.

Based on those studies, provide an opportunity for students to predict when the water will reach a particular temperature. Students can first graph the relationship between the time since the heating of water started and the water temperature. Then, from the fact that the points appear to fall on a straight line, conjecture that the relationship is linear and represent it in an algebraic expression. Then, they can predict at what time the water will reach a particular temperature. For those students who have difficulty representing the relationship between the time and the temperature using an algebraic expression, facilitate them to check the method of determining the equation of a linear function from its graph. Also, discuss the fact that, depending on how you draw a line on the graph, the predicted time may be different, or there are phenomena that cannot be considered as a linear function so that they can make use of this knowledge as they try to identify phenomena from their daily life or the society that can be considered as linear functions.



### **c. Activities for explaining and communicating each other in an evidenced, coherent and logical manner by using mathematical representations**

”Activities to explain and communicate logically and with clear rationale by using mathematical expressions” in grade 1 emphasized students’ explaining and communicating using their own words instead of being too concerned about the forms and simplicity of expressions. In grade 2, set up opportunities for students to engage in activities to clearly and orderly explain and communicate with each other so that they can refine their expressions and make them more substantial.

In order for students to use words, numbers, algebraic expressions, diagrams, tables, and graphs appropriately, to use mathematical representations to describe facts about quantities and geometrical figures or methods of processing them, and to increase the appropriateness of interpretation of mathematical representations, it is important that students use those representations by connecting them to each other. Also, help students to understand the importance of explaining coherently and in an easily understood manner when we communicate using mathematical representations. The goal is to help students experience the merits of mathematical representations.

In the study of mathematics, as the grade levels go up, the need for clearly stating the assumptions and conclusions to communicate what is discovered, showing steps of calculation or solution of equations in an orderly manner, and explaining the validity and reasonableness of one’s idea by making the reason explicit increases. The importance of logical explanation is also raised.

In grade 2, to help students represent mathematically their own thinking processes and the basis of judgment, teach them the need for and the roles of different types of mathematical reasoning, for example, induction, analogy, and deduction so that they can understand each and make use of an appropriate one.

Here are some examples of ”activities for explaining and communicating to each other in an evidenced, coherent and logical manner by using mathematical representations” in grade 2 of lower secondary schools. We will also discuss prerequisites necessary for students to willingly and autonomously engage in these activities.

#### **○ Activities to explain whether or not a lottery is fair by using probability**

These activities are mathematical activities for grade 2 content, ”D. Making Use of Data,” (1)-b. Suppose there is a lottery with two winning tickets among five, and two students will be drawing them. The purpose is to explain ”whether or not there is a difference in the likelihood of winning depending on who draws the first ticket and who draws the second one”, using the ideas of probability. In the process, help students know what types of judgments can be made based on probability so that they can make use of them in examination of uncertain phenomena.

For this purpose, before doing these activities and through doing activities, teach students to determine probability empirically by conducting experiments many times or by counting possible outcomes in simple cases.

Based on those studies, set up activities for students to explain whether or not the lottery is fair. First, by actually conducting the experiment several times, have students predict, ”the first person has a better chance,” ”the second person has a better chance,” or ”they are equally likely to win.” Then, calculate the probability for the first person and for the second person by counting all possible outcomes using a tool like a tree diagram. Then, have them explain that the lottery is fair because the probability of winning is the same for each player. For those students who can calculate the probability but unable to explain, facilitate them to review the meaning of probability. Also, help them check the relationship between probability and conducting experiments many times.

## [Grade3]

### A. Numbers and Algebraic Expressions

- (1) To understand the square roots of positive numbers, and to be able to represent and consider by using the square roots.
  - (a) To understand the necessity and meaning of square roots of positive numbers.
  - (b) To calculate simple expressions which include square roots of positive numbers.
  - (c) To represent and process by using square roots of positive numbers in concrete situations.

[Terms and Symbols]

radical sign, rational number, irrational number,  $\sqrt{\quad}$

In grade 1 of lower secondary schools, the range of numbers treated in mathematics is expanded to include positive and negative numbers. Students are expected to understand the necessity and the meaning of positive and negative numbers, as well as be able to carry out the four arithmetic operations with positive and negative numbers.

In grade 2, students' understanding of numbers is deepened through the study of algebraic expressions with letters, equations, functions, and probability.

In grade 3, since rational numbers are not sufficient to solve quadratic equations and to find the length of a side using the Pythagorean theorem, the range of numbers is expanded to include irrational numbers. By introducing square roots, we can express numbers that could not have been expressed, for example, the length of diagonal of a square with the side length of 1 unit as  $\sqrt{2}$ . The aims are to help students understand the necessity and the meaning of square roots of positive numbers and to enable students to carry out simple calculations and represent and process using square roots in concrete situations.

## The necessity and the meaning of square roots

In grade 1, the range of numbers is expanded to include positive and negative numbers, and students are expected to understand the necessity and the meaning of positive and negative numbers. To expand the range of numbers means that we can now express those numbers that could not have been expressed and make them the object of our thoughts. In daily life, there are quantities that cannot be represented by rational numbers, such as the length of a diagonal of a 1m square. To represent these numbers, we need new numbers. It is also necessary that students understand that, while thinking about the inverse of squaring a number, there are numbers to which a square of another rational number is equal. Through such study, help students to understand the necessity of square roots of positive numbers.

In general, the values of  $x$  which satisfies  $x^2 = a$  ( $a > 0$ ) are called square roots of  $a$  and represented as  $\sqrt{a}$  or  $-\sqrt{a}$ .  $\sqrt{\quad}$  is a symbol for new types of numbers, and by using  $\sqrt{\quad}$ , we can represent concisely and clearly those numbers that could not have been fully represented. Just as  $\pi$  is used to represent the ratio of circumference,  $3.14\dots$ , we will represent numbers using  $\sqrt{\quad}$  as a symbol. It is important to help students know the merits of the symbol and use them correctly. Students are to understand that a positive number  $a$  has two square roots, positive and negative, and that represents a positive square root. The square root of 0 is 0.

Square roots of positive numbers, for example,  $\sqrt{2}$  and  $\sqrt{5}$ , are a new type of numbers that cannot be expressed as fractions. They are different from the numbers we have studied so far, rational numbers, and they are called irrational numbers. By expanding the numbers to include irrational numbers, it becomes possible to solve quadratic equations and to find unknown lengths using the Pythagorean theorem. By understanding these merits, students can deepen their understanding of the merits of mathematics further. Students are also expected to understand that, using terms, rational and irrational numbers, numbers can be categorized into two groups by considering whether or not they can be represented as a fraction.

Square roots of positive numbers can be obtained by using a calculator with a  $\sqrt{\quad}$  key. However, to help students understand the meaning of square roots, it is necessary for students to determine the approximate values by using the following method:

By calculating  $1.4^2$  and  $1.5^2$  using a calculator, and comparing them to 2, they can discover that  $1.4 < \sqrt{2} < 1.5$ . Then, using a similar approach, students can calculate a more and more accurate approximation of  $\sqrt{2}$ . To have students actually experience this is important in order to deepen their understanding of the meaning of square roots of positive numbers. It is also important for them to know about the mathematical method of approximating unknown numbers step by step.

## Calculation of expressions involving square roots

In arithmetic with expressions involving square roots of positive numbers, commutative and associative properties still hold.

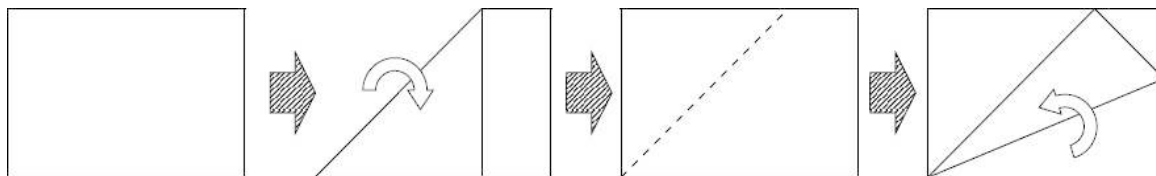
Multiplication and division of square roots of positive numbers can be performed based on  $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$  and  $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$  ( $a > 0, b > 0$ ), respectively. It is important that students experience validating the result of  $\sqrt{2} \times \sqrt{3} = \sqrt{6}$  by calculating  $1.414 \cdots \times 1.732 \cdots$  on a calculator to see that the product,  $2.449 \cdots$  is very close to the value of  $\sqrt{6}$ .

On the other hand, the cases of addition and subtraction are different from those of multiplication and division. For example, it is not the case that  $\sqrt{2} + \sqrt{3} = \sqrt{2+3}$ . To show that  $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$  is not true, we only need to show a counterexample. To explain whether something is valid by proving or invalid by indicating a counterexample is not limited to the instruction of proofs in geometry.

Moreover, expressions like  $\sqrt{2} + 1$  and  $\sqrt{2} + \sqrt{3}$  are numbers that cannot be simplified any further, and each of them represents one specific irrational number. This is similar to the fact that each of the expressions like  $a + 1$  and  $a + b$  represents one specific number. With addition and subtraction involving square roots of positive numbers, we generally try to represent the results in the simplified form, and the thinking involved is similar to that of calculation with algebraic expressions with letters. We should concentrate on the calculation that are commonly needed in the process of solving quadratic equations and the use of the Pythagorean theorem, and avoid unnecessarily complicated calculations which may be pointless.

## Representing and processing with square roots

Square roots are not only the objects of calculation, but they also expand the range of numbers we can use to represent and process concrete situations. For example, the various A-size papers we use for printing has their dimensions in the ratio of  $1 : \sqrt{2}$ . By folding a sheet as shown in the figure, we can verify that the ratio of the two adjacent sides can be represented using a square root of a positive number.



Or, suppose you are given circles with radii of 2 cm and 4 cm. By processing with square roots, we can determine the radius of a circle whose area is equal to the sum of the area of the two circles. It is necessary that students can examine phenomena more deeply by representing and processing with square roots of positive numbers in concrete situations.

(2) To be able to expand and factor simple polynomial expressions using letters, and to develop students' abilities to transform algebraic expressions and read their meanings according to the purpose.

(a) To perform multiplication of monomial and polynomial expressions, and division of a polynomial expression by a monomial expression.

(b) To perform multiplication of simple linear expressions and expand and factor simple algebraic expressions using the following formulas:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

(c) To grasp and explain numbers and quantities and the relationships of numbers and quantities through algebraic expressions using letters.

[Terms and Symbols]

factor

[Handling the Content]

(1) In connection with items like (2) under “ A. Numbers and Algebraic Expressions ” in “ Content, ” factoring natural numbers into the product of prime numbers should be dealt with.

In grade 2 of lower secondary school, students' have the abilities to identify quantitative relationships in phenomena and represent them in algebraic expressions with letters or to interpret algebraic expressions in contexts. In addition, students are expected to master addition and subtraction of simple polynomials and multiplication and division of monomials. They have also studied that algebraic expressions with letters can be used to grasp and explain quantities and quantitative relationships, as well as transforming simple algebraic expressions according to the purpose.

In grade 3, building upon these studies, multiplication of monomials and polynomials, dividing polynomials by monomials, and multiplication of simple linear expressions will be taught. In addition, expanding and factoring polynomials will be discussed, and using this idea, students are expected to extend their ability to transform algebraic expressions using the formulas and to interpret algebraic expressions.

### **Multiplication of monomials and polynomials and dividing polynomials by monomials**

Students have already learned to multiply and divide monomials, to multiply polynomials by numbers, and to divide polynomials by numbers. In grade 3, multiplication of monomials and polynomials, for example,  $2a \times (3a - 5b)$ , and dividing polynomials by monomials, for example,  $4x(4x^2 + 6x) \div 2x$  will be taught.

In particular, since multiplication of monomials and polynomials is a prerequisite for the study of multiplication of polynomials, which is the central focus of grade 3, it is important to help students to develop fluency.

## Multiplication of linear expressions and expansion and factoring of polynomials

It is important that students can multiply linear expressions based on their understandings of commutative, associative, and distributive properties, not just as procedural manipulations.

When multiplying linear expressions such as expanding  $(a + b)(c + d)$ , if we consider  $a + b$  as  $M$ , we can think of this multiplication as  $M(c + d)$  and use what we have learned about multiplying monomials by polynomials and easily calculate.

The following formulas will be taught.

$$\begin{aligned}(a + b)^2 &= a^2 + 2ab + b^2 \\(a - b)^2 &= a^2 - 2ab + b^2 \\(a + b)(a - b) &= a^2 - b^2 \\(x + a)(x + b) &= x^2 + (a + b)x + ab\end{aligned}$$

These formulas are frequently used in future study. Make sure that students understand the meaning of the formulas and the merits of using them. Students should be able to process expressions efficiently.

These formulas are the formulas for factoring if we look at it from a reverse perspective. Factoring is a way to transform algebraic expressions. However, consider the formula  $a^2 - b^2 = (a + b)(a - b)$ . With this formula, we can easily calculate  $13^2 - 12^2 = (13 + 12)(13 - 12) = 25$ . Moreover, if we consider 25 as  $5^2$ , then, we can find the three sides of a right triangle, 5, 12, and 13. Students are expected to understand that the study of factoring includes, in addition to processing algebraic expressions, interpretations of those expressions as this example has shown.

Algebraic expressions used here should be those that are commonly used in explaining with algebraic expressions and the study of quadratic equations and the Pythagorean theorem. Avoid the pointless practice of unnecessarily complicated expressions.

## Grasping and explaining with algebraic expressions with letters

The formulas for expanding/factoring polynomials are frequently used in explaining why properties of numbers and geometrical figures hold true by using algebraic expressions with letters and also solving quadratic equations. Therefore, it is important for students to be able to use these formulas efficiently and transform algebraic expressions purposefully and interpreting them. Building on the instruction in grade 2, enable students to grasp and explain quantities and quantitative relationships using algebraic expressions with letters and to deepen their understanding of the merits and the necessity of algebraic expressions with letters. For example, steps of explaining, "if you add 1 to the product of two consecutive even numbers, the result is a square of an odd number," will be as follows:

- ① Express the smaller even number as  $2n$  where  $n$  is a natural number. Then, the larger even number will be expressed as  $2n + 2$ .
- ② "Add 1 to the product of two consecutive even numbers" means to calculate  $2n(2n + 2) + 1$ .
- ③ We want to show that the result of this calculation is "a square of an odd number." Therefore, we try to transform  $2n(2n + 2) + 1$  into a form  $(\text{odd number})^2$ .

By clarifying the direction of thinking, we can then explain the specific process of transforming the algebraic expression and show that "if you add 1 to the product of two consecutive even numbers, the result is a square of an odd number." To explain means not only being able to write the explanation but also being able to communicate the explanation in the way that can be easily understood by others.

Also, in this problem, by reflecting on the transformation of the algebraic expression,  $2n(2n + 2) + 1 = (2n + 1)^2$ , we may notice that  $2n + 1$  is the odd number in between the two consecutive even numbers,  $2n$  and  $2n + 2$ . Therefore, from this transformation, we can interpret a relationship, "if we add 1 to the product of two consecutive even numbers, the result is the square of the odd number in between the two even numbers." This idea relates to the content in grade 2, "B. Geometrical Figures," which states, "by reading a proof, we discover a new property."

## Prime factorization of natural numbers

As discussed in, "Handling the Content" (1), factoring natural numbers into prime numbers is taught in conjunction with the content of, "A. Numbers and Algebraic Expressions" (2). This is because the prime factorization of natural numbers is equivalent to factoring of polynomials.

In elementary school mathematics, students learned about the properties of natural numbers from the perspectives such as even and odd numbers, factors, multiples, the greatest common factor, and the least common multiple. They have encountered prime numbers in the process of investigating factors.

Here, students are expected to understand that natural numbers greater than 1 can be sorted into those numbers whose factors are 1 and the numbers themselves, and those with more factors - that is, prime numbers and non-prime numbers.

Non-prime numbers can be expressed as the product of several natural numbers using their factors. If any of the factors are not prime numbers, then they can be expressed as a product of their factors. When this process is repeated several times, eventually we will have a product of prime numbers. This is the idea of prime factorization, and it is unique for each number. It is important that students can understand the uniqueness through concrete examples and experiences such as factoring the same number in different ways and comparing the results.

- (3) To understand quadratic equations, and to be able to consider by using quadratic equations.
  - (a) To understand the necessity and meaning of quadratic equations and the meaning of their solutions.
  - (b) To solve quadratic equations by factoring them and transforming them to the squared expressions.
  - (c) To know the solution formula for quadratic equations and to solve quadratic equations by using the formula.
  - (d) To make use of quadratic equations in concrete situations.

### Handling the Content

- (2) In connection with (3) under "A. Numbers and Algebraic Expressions" in "Content", quadratic equations with real number solutions should be dealt with.
- (3) In connection with (3)-b under "A. Numbers and Algebraic Expressions" in "Content", the quadratic equations  $ax^2 = b$  (where  $a$  and  $b$  are rational numbers) and the quadratic equations  $x^2 + px + q = 0$  (where  $p$  and  $q$  are integers) should be dealt with. The teaching of how to solve quadratic equations by factoring them should deal mainly with the quadratic equations where the formulas shown in (2)-b under "A. Numbers and Algebraic Expressions" in "Content" can be used. In addition, the teaching of how to solve the quadratic equations by transforming them to the squared expressions should deal mainly with the quadratic equations where the coefficient of  $x$  is an even number.

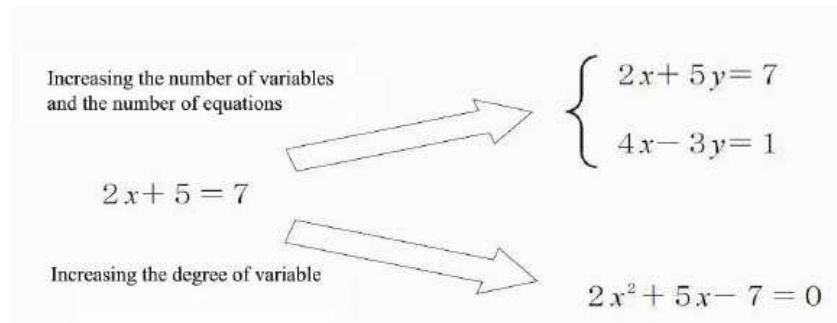
In grade 1 of lower secondary school, linear equations with one variable, and in grade 2, linear equations with two variables by relating them to linear equations with one variable, are taught.

In grade 3, enable students to solve quadratic equations and make use of them in concrete problem solving situations. Enable students to use equations in many more problem solving situations than before.

## The need and the necessity of quadratic equations and the meaning of their solutions

In concrete problem solving situations, quadratic equations will make it easier to find their solutions. Suppose the area of a rectangle obtained by extending a side of a square by 1 cm and shortening another side by 1 cm is  $24\text{cm}^2$ . The length of the side of the original square cannot be determined by using linear equations or simultaneous equations discussed previously. Quadratic equations are also necessary to use the Pythagorean theorem to find a missing length. Help students understand the necessity of quadratic equations through concrete problem solving situations like these.

If we consider simultaneous linear equations with two variables as something created by increasing the number of variables and the number of equations, then quadratic equations can be considered as equations obtained by increasing the degree of variable in linear equations with one variable.



Focusing on the number or degree of letters may serve as a chance to recognize that there are other new types of equations. It is important that students can experience the broadening of equations in general by understanding the meaning of quadratic equations.

Moreover, by considering quadratic equations from this perspective and by thinking about their solutions, students may be able to anticipate the solution's process by the following reasoning: since we found the solutions for simultaneous linear equations with two variables, "by eliminating one letter to make use of linear equations with one variable," we may be able to solve quadratic equations, "by reducing the degree to make use of linear equations with one variable."

The meaning of solutions for quadratic equations is fundamentally the same as those of linear equations with one variable, studied in grade 1, and simultaneous linear equations with one variable studied in grade 2. However, it should be noted that there are two solutions to quadratic equations in general.



## Solving quadratic equations by factoring and by completing the square

In general, there are two ways to solve quadratic equations of the form,  $ax^2 + bx + c = 0$ .

- ① By factoring, transform the given equation into the form of a product of two linear expressions. Then, use the property, "If  $AB = 0$ , then either  $A = 0$  or  $B = 0$ ."
- ② By transforming the equation into the form,  $x^2 = k$ , then use the idea of square roots.

The method ① is useful when the left hand side of the quadratic equation,  $x^2 + px + q = 0$ , can be factored so that the equation can be transformed into the form  $(x - a)(x - b) = 0$ . In this method, quadratic equations are solved by relating them to previously learned linear equations with one variable. The study of this method is also an opportunity for students to re-examine what it means to solve an equation by relating it into linear equations with one variable. It is also an opportunity to re-affirm the conditions for a product to be 0. It is also possible to learn from this method that quadratic equations have two solutions in general.

In method ①, the left hand side of quadratic equation,  $(x - a)(x - b) = 0$ , is a product of linear expressions. To determine the solutions from this equation, we use the fact that, "If  $AB = 0$ , then  $A = 0$  or  $B = 0$ ." Therefore, students need to understand the meaning of the logical connective, "or." Furthermore, by discussing the meaning of, "or," help students to think more coherently.

However, the left hand side of quadratic equation,  $x^2 + px + q = 0$ , cannot always be factored easily. Thus, this method is applicable to limited types of quadratic equations.

Method ② is related to the idea of determining square roots. It plays an important role as a general solution method for quadratic equations.

Quadratic equations in the form of,  $ax^2 = k$ , in other words, in quadratic equations with no  $x$  term, we can determine the solutions directly by using the idea of square roots. Even quadratic equations in the form of,  $x^2 + px + q = 0$ , may be solved using the idea of square roots by completing the square. That is, even when quadratic equations cannot be easily factored, we can solve them by using method ②.

However, in some cases, transforming the expression using method ② may not be easy. Therefore, the discussion of method ② should mainly involve quadratic equations with an even number coefficient of  $x$  terms, like  $x^2 + 4x - 7 = 0$ , so that students can understand that quadratic equations can be solved by completing the square. As for quadratic equations with odd number coefficient of  $x$  terms may be discussed in conjunction with the study of the solution formula discussed next.

## Knowing the formula for the solutions and solving the quadratic equations

Using method ② above, we can solve quadratic equations whether or not we can factor them. However, the process of ② involves a rather complicated transformation of expressions, and they are not too useful in the contexts where quadratic equations are used to solve problems. To eliminate the need for repetitious process of complicated manipulation, the quadratic formula was derived. Knowing why the formula for the solutions was derived is the first step in learning the quadratic formula.

It is difficult to understand the quadratic formula if the focus was solely on procedural transformations of expression as the derivation of the formula for quadratic equations,  $ax^2 + bx + c = 0$  involves rational expressions and square roots of expressions with letters. Therefore, the emphasis will be on understanding the process of deriving the quadratic formula by completing the square of quadratic equations with numerical coefficients.

When using the quadratic formula, students should not just substitute the values of coefficients in the formula. It is important for students to understand that the solutions of quadratic equations in the form,  $ax^2 + bx + c = 0$ , are determined by the values of the three coefficients,  $a$ ,  $b$ , and  $c$ , that is, the solutions can be obtained by arithmetic manipulation of coefficients. Moreover, for those quadratic equations that can be solved by factoring, students should know that they could use the solutions obtained from the quadratic formula to factor the equation. In that way, they can know the relationship between the two solution methods discussed above.

## Using quadratic equations

The aim here is for students to be able to use quadratic equations in concrete situations. Students are expected to know that, the problems that could not be solved by using what they had learned previously can be solved with quadratic equations. In that way, students understand that equations can be used in problem solving more broadly. This understanding can be deepened in the context of using the Pythagorean theorem, too.

What is important is for students to be able to solve concrete problems using quadratic equations. The emphasis should be placed on setting up the equations. It is important that appropriate support is provided to those students who understand the problem but cannot set up equations.

Moreover, when dealing with problems related to our daily life, quadratic equations obtained involve complicated numerical values. In those situations, the use of calculators and other tools should be considered. Students sometimes lose the number sense of the solutions obtained through the quadratic equations, and some of them will not notice that the solutions may not be realistic in the situation. By recognizing this issue, when students are using the quadratic equation to solve concrete problems, it is necessary to place more emphasis on checking the appropriateness of the solutions obtained in the context than when linear equations or simultaneous equations were used to solve problems.

## B. Geometrical Figures

- (1) To verify the properties of geometrical figures based on the facts like the conditions for similar triangles, to develop the ability to think and represent logically, and to be able to consider by using the properties of similar geometrical figures.
  - (a) To understand the meaning of the similarity of plane figures and the conditions for similar triangles.
  - (b) To verify logically the basic properties of geometrical figures based on the facts like the conditions for similar triangles.
  - (c) To find out and verify the properties of ratio of line segments related to parallel lines.
  - (d) To understand the meaning of the similarity of basic solids, as well as the relationships between the scale factor, the ratio of areas and the ratio of volumes of similar geometric figures.
  - (e) To make use of the properties of similar geometrical figures in concrete situations.

[Terms and Symbols]

$\infty$

One of the purposes of examining geometrical figures using mathematical reasoning is to organize previously learned properties of geometrical figures to build a logical structure. To do so, congruence and similarity are important concepts. In grade 2 of lower secondary school, students learned to verify logically the basic properties of triangles and parallelograms using the congruence conditions for triangles.

In grade 3, students are expected to deepen their understandings of the necessity and the meaning of mathematical reasoning and their methods as they logically verify the properties of geometrical figures by using ideas such as the similarity conditions for triangles. They are to extend their abilities of examining and representing logically. Moreover, additional aims are for students to understand the meaning of the similarities of basic solids and determine the measures of geometrical figures using the properties of similar figures.

### Meaning of similarity

In elementary school mathematics, students learned about reduced and enlarged drawings through observation and construction of geometrical figures in grade 6. In other words, students are expected to understand two geometrical figures that are the same shape through the study of reduced and enlarged drawings. Based on this, in lower secondary school mathematics, what it means for triangles and other polygons to have the same shape will be further clarified.

It is necessary to enrich the image of similar figures by drawing scale drawings of different ratios so that students can understand the meaning of similarity.

Moreover, the words "enlarge" and "reduce" can mean manipulating one figure to create a new figure such as "enlarge figure A to draw figure B," or "reduce figure A to draw figure B."

However, "similarity" is a word that represents a concept relating two geometrical figures as in "figures A and B are similar to each other."

Two figures are considered similar when one of the following is true:

- ① When one figure is enlarged or reduced, it will become congruent with the other.
- ② The ratios of corresponding segments are constant and the corresponding angles' measurements are equal.
- ③ They can be placed in the position of similarity after appropriate transformations.

Definition ① is the definition of similarity corresponding to the definition of congruence based on the motion of geometrical figures, "If one figure can be moved to overlap the other figure completely, then the two figures are congruent." This definition is easy to understand in the introduction of drawing similar figures. It can be applied to figures including curves, and the correspondences with the original figure are clear. If we use this as the definition of similarity, ② may be considered as a property of similar figures.

Definition ② is important as the foundation for proving the similarity of figures composed of straight segments. Using this definition, we can easily proceed with deductive reasoning. However, it is removed from the methods of actually drawing similar figures, and it may be difficult to identify corresponding sides or corresponding angles.

In contrast to the definition of congruent figures, "figures that can be made to overlap completely," definition ③ means similar figures "may be overlapped when they are looked at from a particular point." This definition may be applied to figures with curves. However, it should be noted that a figure might need to be flipped in order to place it into the position of similarity with another figure.

No matter which definition is used, it is necessary to consider both the ease of understanding for students and the lack of logical contradiction.

### Conditions for similarity of triangles

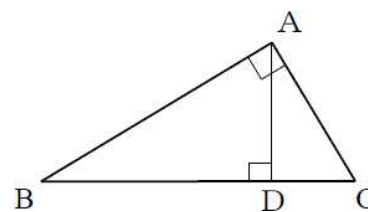
As the conditions for similarity of triangles, the following three will be taught:

Two triangles are similar under each condition.

- The ratios of all three pairs of corresponding sides are equal.
- The ratios of two pairs of corresponding sides are equal and the measures of the corresponding angles in between the two sides are equal.
- The measures of two pairs of corresponding angles are equal.

These conditions should be compared and contrasted to the congruence conditions of triangles studied in grade 2. Students should understand these conditions intuitively at first, and then develop a more logical understanding as the study progresses.

Then, position these conditions as a possible basis of deductive reasoning so that students can verify the basic properties of geometrical figures logically using the similarity conditions. For example, students may prove the following logically using the similarity conditions: Given a right triangle ABC with the right angle at A. Let D be the point of intersection of a perpendicular segment drawn from A to the hypotenuse BC. Then both  $\triangle DBA$  and  $\triangle DAC$  are similar to  $\triangle ABC$ .



### Properties of ratio of line segments related to parallel lines

Through observation and manipulation, help students discover properties of ratios of line segments related to parallel lines. Then, help students understand that those properties may be deduced from properties of parallel lines and the similarity conditions of triangles. It is possible to treat the midpoint theorem here as a special case of ratios of line segments related to parallel lines. Then, based on this theorem, help students to think about new properties of geometrical figures, for example, if you connect the midpoints of the four sides of a quadrilateral, the resulting quadrilateral is a parallelogram.

## Relationship between the ratio of similarity (scale factor) and the ratio of area, and the ratio of volume

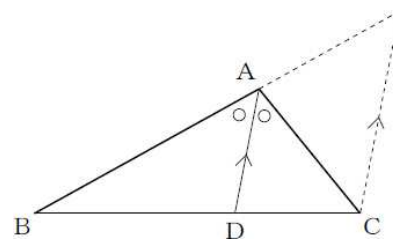
Based on the meaning of similarity of plane figures, by analogy, help students understand the similarity of basic solids such as cubes, cuboids, prisms, pyramids and spheres.

In general, the ratios of corresponding segments of similar figures are constant. In addition, the measures of corresponding angles are equal. The ratio of corresponding segments is the ratio of similarity (scale factor). In similar solids, corresponding faces are similar and the scale factor for the faces is the same as the scale factor of the solids. In similar plane figures, the ratio of the lengths of corresponding segments are equal to the scale factor, but the ratio of areas is not the same as the scale factor. Rather, the ratio of areas of similar figures is the square of the scale factor, and the ratio of volumes of similar solids is the cube of the scale factor. Students are to understand these relationships.

In addition, help students be able to determine the area or volume of similar figures by knowing the scale factor and the area or the volume of one figure by using the relationship between the scale factor and the ratio of area and the ratio of volume.

### Making use of properties of similar figures

In the situations where students are making use of properties of similar figures, it is necessary that students identify similar triangles within a given figure so that they can think about the ratio of corresponding segments and positional relationships. For example, consider the proof for the following statement: If  $D$  is the point of intersection of the angle bisector of angle  $A$  in  $\triangle ABC$  and the side  $BC$ , then,  $AB : AC = BD : CD$ . In this situation, similar triangles may not be identified in the figure as it is. It is important to guide students so that they can make, for example, a triangle similar to  $ABD$  through trial and error, keeping in mind the positional relationships of  $AB$ ,  $AC$ ,  $BD$ , and  $CD$ . The above property to be proved may be derived by generalizing the property studied in grade 2, "The angle bisector of the angle opposite from the base of an isosceles triangle is the perpendicular bisector of the base," by replacing "isosceles triangle" by general triangles and by actually measuring different parts of the triangle.



As an example, the use of similarity in our daily life is a map. A map is a reduced drawing, and one can determine the distance by using a map, without actually having to go to the locations. Another example is a blue print of electronic devices. Those blue prints are enlarged figures. Even those parts that are very small can be designed accurately by enlarging the drawings. It is important for students to identify and examine those situations where the idea of similarity is being used. We can also consider making measurements that cannot be directly measured, such as the height of a tree or the distance between two trees where there is an obstacle such as a pond in between them. It is possible to organize a lesson in which students will identify distances and angles that can be measured directly first, draw a reduced drawing based on those measurements, then, finally, determine the necessary height or distance using the reduced drawing. These types of activities are incorporated in the study of reduced and enlarged drawings in elementary school mathematics. Therefore, it is necessary to help students feel that the degree of use has been deepened by the study of similarity.

With respect to the ratio of the volumes of similar solids, a possible lesson might investigate the value of two items that are packaged in boxes that are similar. By knowing the scale factor, students can determine the ratio of the volumes. Then, using the ratio of the volumes and the ratio of their prices, students can determine which is a better buy.

- (2) Through activities like observation, manipulation and experimentation, find out and understand the relationships between inscribed angles and its central angle in a circle, and be able to consider by using the relationships.
- (a) To understand the meaning of the relationships between inscribed angles and its central angle in a circle, and to know that the relationships can be proven.
- (b) To make use of the relationships between inscribed angles and its central angle in concrete situations.

#### Handling the Contents

- (4) In connection with (2) under “ B. Geometrical Figures ” in “ Content ”, the converse of the theorem of inscribed angles should be dealt with.

Circles, along with lines, are one kind of familiar geometrical figures. Students have been learning about circles from a mathematical perspective starting in elementary school. For example, in elementary school mathematics, center, radius, diameter, ratio of circumference, and area of circles are taught. In lower secondary school mathematics, tangent lines are taught in grade 1.

In grade 3 of lower secondary school, students examine the relationship between inscribed angles and central angles through mathematical reasoning. Students are to deepen their understanding of circles, and learn to make use of the relationship between central angles and central angles in concrete situations.

#### The meaning of the relationship between inscribed angles and central angles

Between inscribed angles and central angles, there is the relationship, “in a circle, the measure of the inscribed angle is a half of the measure of the central angle on the same arc”. It is important to help students discover and examine this relationship through observation, manipulation, and experimentation.

Based on this relationship between inscribed angles and central angles, students can discover the relationship, “the measures of all inscribed angles on the same arc are constant”. This discovery is then summarized as the inscribed angle theorem.

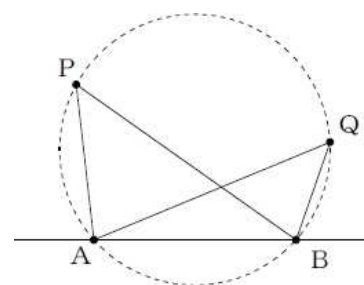
#### To know that the relationship between inscribed angles and central angles can be proven

In order for students to feel the need and the merits of proofs, it is important for them to experience proving something new that they did not know previously or proving something that they were unsure of.

Help students discover that, in a given circle, measures of inscribed angles on the same arc are equal, through observation, manipulation, and experimentation. Then, by examining the inscribed angles in relationship to the central angle, they can prove the property of inscribed angles.

Through the study of inscribed angles, we can increase students’ interests toward mathematics. For example, students can use computers to investigate the relationship between the measures of inscribed angles and the central angle as a point is moved around a circle. They may conjecture that the measure of the central angle is a half of the measure of the inscribed angles, and the think about why that is the case.

As for the relationship between inscribed angles and central angles, we can prove that the measure of an inscribed angle is a half of the central angle on the same arc by using the following properties: the base angles of isosceles triangles are equal in their measures, and the measure of an exterior angle is equal to the sum of measures of the two opposite interior angles. The study to learn that the relationship between inscribed angles and central angles aims to help students understand the merits of proofs, not understanding the need for sorting the positional relationships between inscribed angles and the central angle in the process.

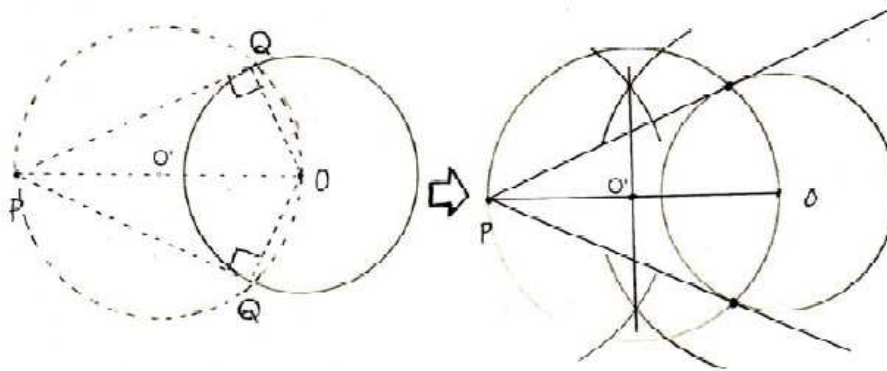


As for the converse of the inscribed angle theorem, “If points P and Q are on the same side of line AB and  $\angle APB$  and  $\angle AQB$ , then points A, B, P, and Q are on the same circle,” it is important that students can make use of it.

### Making use of the relationship between inscribed angles and central angles

As an example of the use of the relationship between inscribed angles and central angles, we can consider the problem of drawing a tangent line to circle  $O$  from point  $P$ , which lies outside of the circle.

Suppose you draw a tangent line  $PQ$  to circle  $O$ . Since a tangent line is perpendicular to the radius through the point of tangency,  $\angle OQP = 90^\circ$ . Thus, to locate the point of tangency,  $Q$ , you draw a circle,  $O'$ , with  $OP$  as the diameter. Then, the point of intersection of  $O$  and  $O'$  will be point  $Q$  we need. This is because the central angle on arc  $OP$  of circle  $O'$  is  $\angle OO'P$ , which is  $180^\circ$ . Thus, by the inscribed angle theorem,  $\angle OQP$ , which is an inscribed angle on arc  $OP$ , should be  $90^\circ$ .



In this way, drawing of a tangent line using the relationship between inscribed angles and central angles is also an opportunity for constructing something with a good prospect in mind, then explaining the validity of the construction coherently and making the basis of reasoning clear.

As an example of making use of the relationship between inscribed angles and central angles, we can think of locating the center of a circle using a rectangle. If you place the right angle at a vertex of the rectangle on the circle, points where the two sides of the angle intersect the circle will be the end points of a diameter. If we repeat the process again, we can find a second diameter, and where the two diameters intersect will be the center of the circle. This method is using the property of inscribed angles and central angles. By examining this situation, students can also understand the carpenter's tool called A Carpenter's Square.

- (3) Through activities like observation, manipulation and experimentation, find out and understand the Pythagorean theorem, and be able to consider by using the theorem.
- (a) To understand the meaning of the Pythagorean theorem, and to know that the theorem can be proven.
  - (b) To make use of the Pythagorean theorem in concrete situations.

The Pythagorean theorem expresses the relationship among the three sides of a right triangle, and it is one of the most important theorems in mathematics. When teaching the theorem, it is desired that students can appreciate the beauty of the theorem that expresses the relationship of the three sides of a right triangle, instead of simply treating it as an extension of proving various properties of geometrical figures. This theorem is used in a wide range of situations, including the well-known applications of the theorem used in surveying.

## Meaning of the Pythagorean theorem

As stated above, the Pythagorean theorem expresses the relationship among the three sides of a right triangle. It also expresses that there is a specific relationship among the area of the three squares constructed on the sides of a right triangle. Therefore, the Pythagorean theorem can be considered as a theorem expressing the relationship of lengths and the relationship among areas. In other words, the study of the Pythagorean theorem is an opportunity to discuss geometrical figures and numbers/algebraic expressions in a unified manner.

In the introduction of the Pythagorean theorem, it is important to pique students' interests by discussing its historical background and anecdotes such as how ancient Egyptians used knotted rope in surveying and the ancient Greek mathematician, Pythagoras summarized it as a theorem.

It is also effective to have students draw right triangles on a grid paper and examine the area of squares built on their sides so that students can discover the Pythagorean theorem through observation, manipulation, and experimentation.

## To know that the Pythagorean theorem can be proven

There are many known proofs of the Pythagorean theorem - some are geometrical while others are algebraic. However, students might think some proofs as too technical. Therefore, discuss those proofs that are appropriate for students' interest, and focus on helping students to know that the theorem can be proven.

It is also possible in this process to use computers and information network to gather data to use in the study.

As for the converse of the Pythagorean theorem, that is, "if the three sides of a triangle,  $a$ ,  $b$ , and  $c$ , are such that  $a^2 + b^2 = c^2$ , then the triangle is a right triangle," avoid getting too deep into its proof. Rather, help students to focus on the fact that whether or not a triangle is a right triangle is based on the relationship of the lengths of its three sides.

## Using the Pythagorean theorem

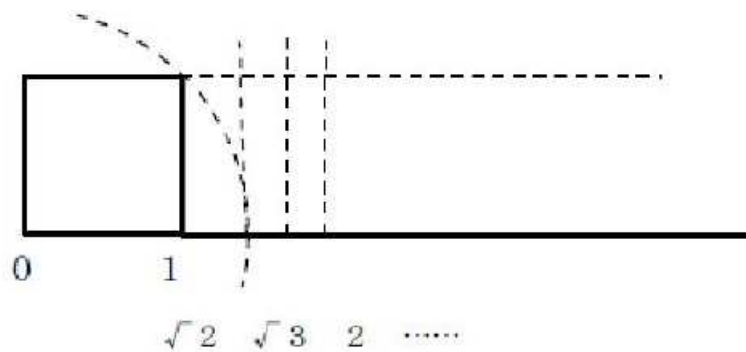
Situations where the Pythagorean theorem is used include determining the distance between two points on a coordinate plane, determining the length of a diagonal of a rectangle or the height of a cone. In these situations, we can determine the desired measurements without having to actually measure them by using the Pythagorean theorem. In these ways, the Pythagorean theorem is often used in many places where we examine measurements in plane and space figures. Even when a right triangle is not obvious, by locating an appropriate right triangle, or even creating one, we can determine the lengths of segments. We can consider these situations as opportunities to deepen spatial sense and to foster the ability to decompose and compose geometrical figures.

In our daily life, the Pythagorean theorem is used, for example, to determine the distance between two points on a map that are located at different altitudes, or to determine the range of sight from a point above ground such as at the top of a building or from a satellite. The Pythagorean theorem may be used to determine the values without actually measuring them.



What is important is for students to use the Pythagorean theorem even in space. Also it is important to experience idealizing and simplifying the problematic situations in the real life so that they can be considered as objects of mathematics and to draw figures so that may be necessary to solve those problems. Students are expected to understand that the answers obtained from idealized or simplified problems may be limited because of the process of idealizing or simplifying real situations.

The distances obtained by using the Pythagorean theorem will often involve square roots. Those are opportunities for students to experience the need of square roots. For example, as a method of constructing segments to represent  $\sqrt{2}$  or  $\sqrt{3}$ , something like what is shown below is used which uses the Pythagorean theorem. Using this method, students can construct segments to represent square roots of positive numbers.



Moreover, for example, when determining the distance between two points on a map with different altitudes, instead of using the square roots as the answers, express them using approximations so that the answers are more realistic. This is one instance where students can express the need of significant digits studied in grade 1 of lower secondary school.

## C. Functions

- (1) Through finding out two numbers/quantities in concrete phenomena and exploring their changes and correspondence, understand the function  $y = ax^2$ , and develop the ability to find out, represent and think about functional relationships.
  - (a) To know that there are some phenomena which can be grasped as the function  $y = ax^2$  in concrete phenomena.
  - (b) To understand the function  $y = ax^2$  by interrelating their tables, algebraic expressions and graphs.
  - (c) To grasp and explain concrete phenomena by using the function  $y = ax^2$ .
  - (d) To understand that in various concrete phenomena there are some functional relationships.

Direct and inverse proportional relationships were taught in grade 1 of lower secondary school, and linear functions were learned in grade 2. In both cases, students' abilities to identify and represent functional relationships have been gradually raised through the examination of changes and correspondences of two quantities in concrete phenomena.

In grade 3, as before, through investigation of changes and correspondences of two quantities in concrete phenomena, students will examine function,  $y = ax^2$ . At the same time, students' understanding of functions will be deepened by making connections among tables, algebraic expressions, and graphs and also by examining rates of change and characteristics of graphs. Through these studies, students' abilities to identify and represent functional relationships will be further developed.

Furthermore, by discussing the fact that there are many functional relationships in our daily life and in society that cannot be grasped by the functions that have been previously studied, enrich the content of the study of functions in lower secondary school and lay the foundation for future studies.

### Phenomena and function, $y = ax^2$

By identifying two quantities,  $x$  and  $y$ , in concrete phenomena and examining their changes and correspondences, student are to know that there is a relationship like the following:

- When the value of  $x$  becomes  $m$  times as much, the value of  $y$  becomes  $m^2$  times as much.

Furthermore, if we make a table and examine the values of  $x^2$  corresponding to  $x$  and values of  $y$ , their ratios are constant. Therefore, we can conclude that, “ $y$  is directly proportional to the square of  $x$ ”. In other words, students are to understand that when a quantity is directly proportional to the square of another variable, we can represent the relationship as  $y = ax^2$  where  $a$  is the constant. Students are to know that there are phenomena that can be grasped with function,  $y = ax^2$ .

## **Table, algebraic expression, and graph of function, $y = ax^2$ and their relationships**

As with other functions studied up to this point, examine the function  $y = ax^2$  by using a table to capture the way function changes and use a graph to understand the characteristics of the changes and correspondences of the function. In addition to capturing the characteristics of the function  $y = ax^2$ , deepen students understanding by making connections among those representations as was the case with the prior study of functions.

Of the functions that can be represented with algebraic expressions, this is only the second time the graph of the function turns out to be a curve – the first one was in the inverse proportional relationships studied in grade 1 of lower secondary school. Students will be able to tell that the graph will not be a straight line since the rate of change is not constant. In this way, students can think by connecting the characteristics of the graph and the table. In addition, the fact that the graph changes, the direction of change (increase/decrease) around the origin can be understood by relating it to the algebraic expression,  $y = ax^2$ , in particular about the fact that  $x$  is being squared. In the same way, the direction and the width of the opening of the graph is understood by relating them to the sign and the absolute value of the proportion constant,  $a$ . In this manner, as it was done so in grades 1 and 2, it is important to make connections among tables, algebraic expressions, and graphs to help students understand characteristics of changes and correspondences of functions in grade 3. As for the investigation of the ratios of change of functions, the purpose is not to just memorize the methods of calculating the ratio. Rather, the real purpose is to deepen students' understanding of function,  $y = ax^2$ , and to know the purpose of ratio of change in examining functions and how to perceive graphs.

### **To grasp phenomena using function, $y = ax^2$**

It is important to study functions in the context of concrete phenomena and situations. Some of the concrete phenomena involving function,  $y = ax^2$ , observed around us include the following: the motion of rolling objects studied in science, the braking distance of a car, the shape of the path of water in a water fountain, and satellite antenna. It is also possible to consider square numbers obtained by lining up counters in the shape of a square or to investigate function,  $y = ax^2$ , in the context of the problems involving area or volume. By grasping and explaining these phenomena using function  $y = ax^2$ , students can deepen their abilities to identify, represent, and examine functional relationships.

As was the case in grades 1 and 2, when grasping and explaining phenomena using function,  $y = ax^2$ , it is important to idealize and simplify the phenomena so that they can be perceived as function,  $y = ax^2$ . For example, when investigating the relationship between the speed of a car and its braking distance, we ignore the distance traveled before the brake actually starts working so that the braking distance can be considered as directly proportional to the square of the speed. Then, we can predict the braking distance at the given speed and explain why. In such a situation, as was the case in grade 2, compare the results from the actual experiments and the predicted values. Then, through activities of explaining and communicating, think about the reasons for the differences and devise a better way of making predictions.

It is necessary to set up occasions intentionally where students will explain their ideas using mathematical representations in order to extend their abilities of grasping and explaining phenomena. It is important that students can experience the deepening of examining phenomena by selecting appropriate representations from tables, algebraic expressions, and graphs, or by comparing their own expressions with others.

### **Various phenomena and functions**

Based on the previous studies, functions different from direct and inverse proportional relationships, linear functions, and function,  $y = ax^2$ , are taught in grade 3. For example, consider the pricing system for public transportation system or postal services. Even in those situations, which are difficult to represent using algebraic expressions, relationships between two quantities may be examined by using tables and graphs as before and their characteristics may be made clear. Through these experiences, deepen students' understanding of functional relationships - if one of the two quantities that are changing simultaneously is fixed, the other quantity is also fixed. Students are to develop the disposition to incorporate the idea of functions in examination of phenomena. Those experiences should become the foundation for future learning.

## D. Making use of Data

(1) Through selecting samples out of a population and exploring their trends by using a computer and other means, be able to understand that it is possible to read trends in the population.

(a) To understand the necessity and meaning of a sample survey.

(b) To carry out sample surveys in simple cases, and grasp and explain trends in the population.

[Terms and Symbols]

complete count survey

In lower secondary school mathematics, the idea of collecting data purposefully, organizing it, and identifying trends in data using histograms and representative values are taught in grade 1. In grade 2, probability is taught through activities used to investigate trends in the likelihood of uncertain events by conducting experiments multiple times.

Based on these studies, methods of collecting data from a sample and the fact that the trends of the population may be identified by investigating trends in the samples are taught in grade 3.

### The necessity and the meaning of sampling

In grade 1, with the assumption that a complete set of data can be gathered, students learned to identify trends in the data using histograms and representative values. However, in many cases in daily life or in society, we can collect data from only a portion of the population for a variety of reasons. For example, the collecting of data from all adults in order to poll their opinions on various social issues is unrealistic economically and is too time consuming. Or, in order to check the safety of food products, we cannot open all packages. In those situations, it is necessary to think about how much we can say about the whole population based on data from a portion of the population. Thus, the idea of sampling was developed. Help students deepen their understanding of the necessity and meaning of sampling, perhaps by comparing it to the complete count survey

### To carry out sample surveys in simple cases

Here, the ideas of random sampling and estimating the population trends from samples are taught. To understand these ideas, it is necessary to have students actually conduct sample surveys.

Since we are dealing with sample surveys, the population must be fairly large, but care should be taken so that it will not be too complicated to actually conduct sample surveys. In addition, in order to evaluate the results of sample surveys, it is necessary that the characteristics of the population are either known or determinable.

When sampling from a population, we must be careful to sample without any bias so that the sample will reflect the characteristics of the population accurately. In other words, it is necessary to select a sample so that the probability of any data from the population is selected is equal to each other - that is, random sampling. Students are to understand that, by using random numbers, it is possible to conduct random sampling.

For example, let's think about determining the total number of words included in an English-Japanese dictionary. Suppose this dictionary has 980 pages. By using a die or a computer, we can generate a random number chart with numbers 1 through 980. Then, we can select a certain number of pages randomly. By counting the number of words on those selected pages and calculating their mean, we can estimate the total number of words in the dictionary. A dictionary typically indicates the total number of words included in it, and students can compare the estimates with the actual number of words. Help students deepen their understanding of random sampling by having them conduct random sampling with different number of samples (number of pages), or by comparing the result to non-random sampling such as using the first 10 pages of the dictionary. From those experiences, students can experience that the risk of large error is minimized when random sampling is used to estimate trends in the population.

## **To grasp and explain trends of the population**

Deepen students' understanding of sample surveys by grasping and explaining trends of the population through conducting sample surveys.

In sample surveys, it is difficult to make any definitive judgment about the population. In cases where sample surveys are used in real life situations, to compensate for this problem, it is common to quantify the probability of error and report it. However, since this is only the introductory treatment of sample surveys, the aim is to help students understand experientially that predictions and judgments based on sample surveys may involve errors through experimentations and other activities.

When students make predictions or judgments, the emphasis should be on what they used as the basis for their explanations. Help them develop a common understanding of appropriateness of methods and conclusions through activities such as communication activities.

For example, consider the question, "how much do 200 grade 3 students at my lower secondary school sleep a day?" In this case, we can anticipate the following activities:

- ① Clarify what "how much sleep a day" (how much you slept last night, or the average sleep time over the last week) means and develop a survey.
- ② Select a sample of students and conduct the survey.
- ③ Organize the data.
- ④ Based on the sample survey, predict and explain the amount of sleep for all students.

In this case, "explanation" in ④ includes not only the explanation for the prediction but also the explanation of the process described in ① through ③. Moreover, based on the explanation, conduct a whole class discussion on the appropriateness of the sampling processes and the predictions. In similar ways, enable students to interpret the results and explanations of sample surveys correctly by having students actually conduct sample surveys and grasp and explain trends in the population. For example, help students to pay attention to such issues as, if there is any bias in location or in groups in the samples or that questions on the survey are not too leading. It is important for students to not be confused by information about uncertain phenomena by actually conducting sample surveys and grasping and explaining trends in the population.

## **Using tools such as computers**

As was the case in grade 1, tools such as computers should be used to improve the efficiency of data processing when a large number of data must be organized or the data include large numbers or numbers from fractional parts. In addition, they may be used to produce random numbers needed for sampling. In those situations, it may be easier for students to intuitively understand that any group from the population may be selected without any intention if tools like dice are used as well. It is also possible to collect data or investigate various sample surveys and their results through various information networks such as the intranet. In those situations, it is necessary to discuss with students the reliability of the information. It is also important that students can consider the reliability of data as they consider their own predictions and judgments.

## [Mathematical Activities]

- (1) In learning each content of “ A. Numbers and Algebraic Expressions, ” ” B. Geometrical Figures, ” ” C. Functions ” , and “ D. Making Use of Data ” , and in learning the connection of these contents, students should be provided opportunities for doing mathematical activities like the following:
- (a) Activities for finding out and developing the properties of numbers and geometrical figures based on previously learned mathematics
  - (b) Activities for making use of mathematics in daily life and society
  - (c) Activities for explaining and communicating to each other in an evidenced, coherent and logical manner by using mathematical representations

In grade 2, in learning the content of each domain and in learning connections among them, opportunities for students to engage in the following types of activities were provided: activities for finding out and developing the properties of numbers and geometrical figures based on previously learned mathematics; activities for making use of mathematics in daily life and society; and, activities for explaining and communicating to each other in an evidenced, coherent and logical manner by using mathematical representations. As students engage in these mathematical activities willingly and autonomously, it is aimed that students master the basic and fundamental knowledge and skills, that they improve their ability to think, make judgments, represent, and experience the joy and purpose of learning mathematics.

In grade 3, these fundamental ideas will continue to be emphasized. The reason the three types of activities discussed are the same as those discussed in grade 2, is that mathematical activities should be continuously enriched through grades 2 and 3.

The grade 3 content of, ”Role of mathematical activities” (pp 83) and ” Engaging students in mathematical activities” (pp. 82) do not change from those of grades 1 and 2.

### **a. Activities for finding out and developing the properties of numbers and geometrical figures based on previously learned mathematics**

In grade 3, continuing what was started in grade 2, it is necessary to further enrich ”activities for finding out and developing the properties of numbers and geometrical figures based on previously learned mathematics.” Here are some examples of ”activities for explaining and communicating each other in an evidenced, coherent and logical manner by using mathematical representations” in grade 3 of lower secondary schools. We will also discuss prerequisites necessary for students to willingly and autonomously engage in these activities.

○Activities used to make clear the reasons behind various methods for quick calculations and to think about new methods for quick calculations

In the activities for grade 3 content, "A. Numbers and Algebraic Expressions," (2)-c, the purpose is to think about the reasons behind various methods for quick calculations, for example, "how to mentally calculate the product of two 2-digit numbers whose tens digits are the same and the sum of the ones digits is 10" (Figure 1), using algebraic expressions with letters and think about new quick calculation methods and the reasons behind them. In the process, students should also learn the merits of representing generally using algebraic expressions with letters and purposefully transforming them, as well as interpreting those expressions. This learning can in turn be made use of in the later study of equations and properties of geometrical figures.

$$\begin{array}{r} 74 \\ \times 76 \\ \hline 5624 \\ \hline \end{array}$$

$7 \times (7+1)$        $4 \times 6$

Figure 1

As a precursor, teach them, through doing activities, about "how to mentally calculate the square of 2-digit numbers with 5 as the ones digit" (Figure 2) and examine why the method works using algebraic expressions with letters. In that process, help students understand that a natural number with a as the tens digit and b as the ones digit may be represented as  $10a + b$ , and its square may be calculated by using a multiplication formula and transformed purposefully. This may also become an opportunity for students to think if there is any other quick calculation method.

$$\begin{array}{r} 35 \\ \times 35 \\ \hline 1225 \\ \hline \end{array}$$

$3 \times (3+1)$        $5 \times 5$

Figure 2

Based on these studies, provide opportunities for students to engage in activities to generalize "square of numbers with 5 as the ones digit" into "numbers whose ones digit add up to 10," to conjecture the results of the generalized statement using specific numbers, and to clarify the reason behind "how to calculate mentally the product of two 2-digit numbers with the same tens digit and the sum of their ones digits is 10" using the algebraic expressions with letters. For those students who cannot clarify the reason behind the quick calculation method, help them to think about the purpose of transforming algebraic expressions with letters - that is what form we would like to obtain.

$$\begin{array}{r} 35 \\ \times 35 \\ \hline 1225 \\ \hline \end{array}$$

$3 \times (3+1)$        $5 \times 5$

Figure 3

Furthermore, think about other quick calculation methods by changing the conditions. For example, from the above example, students might think that there may be a quick calculation method, "how to mentally calculate the product of two 2-digit numbers whose ones digits are the same and the sum of their tens digits is 10" (Figure 3). They should use specific numbers to develop a conjecture, and then clarify the reasons behind the method.

**b. Activities for making use of mathematics in daily life and society**

In grade 3, continuing what was started in grade 2, it is necessary to further enrich "activities for making use of mathematics in daily life and society." Here are some examples of "activities for making use of mathematics in daily life and society" in grade 3 of lower secondary schools. We will also discuss prerequisites necessary for students to willingly and autonomously engage in these activities.

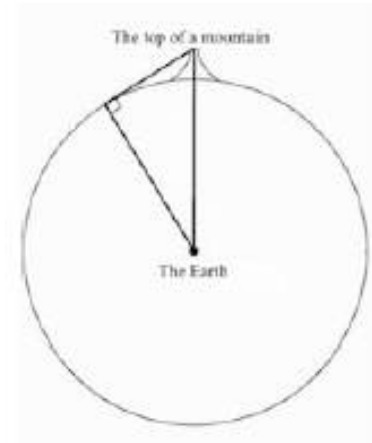
○Activities to determine the distances that may be difficult to directly measure using the Pythagorean theorem

These activities are for grade content, "B. Geometrical Figures," (3)-b. The purpose is to determine distances that cannot be measured directly, for example, how far you can see from the top of a mountain, based on such ideas as the Pythagorean theorem and the properties of tangent lines to a circle. In the process, help students understand the merits of idealizing and simplifying to consider an object as a geometrical figure, for example, by assuming there is no obstacle or that the Earth is a sphere. Students are to make use of such processes.

As a prerequisite for these activities, enable students to use the Pythagorean theorem to determine the length of the third side of a right triangle when the other two sides are given or to determine the distance between two points in space by imagining a right triangle. In addition, teach, through doing activities, the idea of idealizing and simplifying concrete situations.

Based on these studies, students are to plan how to determine the distance one can see from the top of a mountain. That is, they must organize the known information, draw a diagram representing the problem situation, determine what additional information is needed to determine the distance one can see from the top of a mountain, and clarify what previously learned ideas may be used. Then, following this plan, collect necessary information and determine the distance by using such ideas as the Pythagorean theorem and properties of tangent lines. In the process of calculation, enable students to use approximate distance using calculators and other tools. For those students who cannot draw an appropriate diagram, suggest that they draw a picture showing the relationship between the observer and the points that he/she can see from a third person's perspective.

In addition, it is also important to prepare items such as photographs of the mountain taken from a distance so that the results may be verified. From these activities, help students to experience that features such as distance may be determined mathematically without having to actually go to the specific location.





### c. Activities for explaining and communicating each other in an evidenced, coherent and logical manner by using mathematical representations

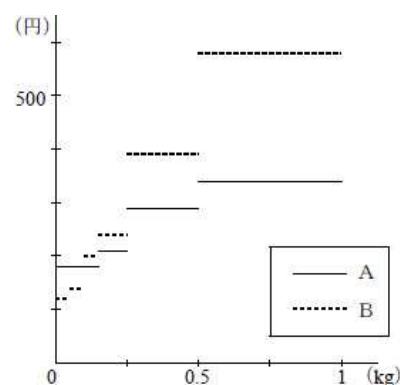
In grade 3, continuing what was started in grade 2, it is necessary to further enrich "activities for explaining and communicating each other in an evidenced, coherent and logical manner by using mathematical representations." Here are some examples of "activities for explaining and communicating each other in an evidenced, coherent and logical manner by using mathematical representations" in grade 3 of lower secondary schools. We will also discuss prerequisites necessary for students to willingly and autonomously engage in these activities.

#### ○Activities used to identify functional relationships in various phenomena and explain the characteristics of changes and correspondences

These activities are for Grade 3 content, "C. Functions," (1)-d. The purpose is to grasp functional relationships from our surroundings, such as the fares for public transportation systems or postal services, and to explain the characteristics of changes and correspondences by representing them in tables and graphs.

As a prerequisite, teach, through doing activities, the idea of grasping the characteristics of functional relationships from tables, algebraic expressions, and graphs, as well as interpreting and explaining what those characteristics mean as quantitative relationships in the concrete phenomena.

Through these studies, provide students opportunities to engage in activities used to explain the characteristics of functional relationships found in various phenomena. For example, have students investigate the relationships between the distance and the fare on a public transportation system or the weight and the shipping cost. Students are to clarify that the cost is a function of distance or weight, and are to explain the characteristics of changes and correspondences by representing the relationships in tables and graphs. Or, students can compare different modes of transportation or methods of shipping and determine which may be most economical for what conditions by using graphs. For those students who think functional relationships can always be represented as an algebraic expression, re-affirm that two quantities that are changing simultaneously are in a functional relationships if whenever one quantity is fixed, the other quantity is also fixed to one value. Make it clear that graphs for these relationships will be a collection of segments arranged like steps, not a continuous line or curve, and explain that these segments mean that for a certain segment, the cost is the same. For those students who cannot graph the relationships, suggest that they plot specific combinations on the coordinates and grasp an approximate shape of the graph.



the relationships between the distance and the fare of two modes of transportation

## Chapter 3. Syllabus Design and Handling the Content

”Chapter 3 Syllabus Design and Handling The Content” of the previous COS, 6 items of considerations were discussed. In the current revision, in view of the purposes of the chapter, consideration points are organized into the following four items: ”Consideration Points for Designing a Syllabus,” ”Consideration Points for Handling the Content in Chapter 2,” ”Consideration Points for Instruction of Mathematical Activities,” and ”Project-Based Learning and its Position.”

### 1. Consideration Points for Designing a Syllabus

#### (1) In regard to the contents to be taught in each grade

- (1) Some of the contents for each of the grades can be dealt with lightly, to the extent that this treatment does not hinder the achievement of the objectives for the grade in Section II, with the instructional content being provided in upper grades. In addition, the teaching can be carried out by adding some content from upper grades, to the extent that this treatment does not deviate from the objectives for the grade.

Teaching lower secondary school mathematics requires the designing of appropriate syllabi (lesson plans) to match the ability of students. In order to make use of teachers’ creativity, it is important that a certain amount of flexibility is provided as they design lesson plans.

Therefore, this statement permits teachers to treat the contents to be taught flexibly across grades as long as the adjustments do not hinder achieving the objectives for the grade - for example, changing the sequence of the contents across grades, providing reviews from the previous grade, or incorporating a part of the content from the later grade.

#### (2) In regard to setting up opportunities to relearn

- (2) In order to ensure reliable learning on the part of the students, when teaching new content, consideration should be given to intentionally take up related content that has already been taught once again and provide opportunities to relearn the content.

In the COS, content that has been discussed previously is not discussed again later as a general rule. However, in actual teaching, it is sometimes effective to broaden and deepen students’ understanding if previously discussed topics are intentionally treated again when a new content is taught. For example, while teaching the rate of change in the study of linear functions in grade 2, discuss the inverse proportional relationships discussed in grade 1. Such a discussion can solidify students’ understanding of the ways the quantities change and understand the shapes of graphs, and it will also help them understand that there are functions for which the rate of change is not constant.

In this way, providing opportunities to relearn does not simply mean to increase opportunities to review. Care should be taken to set up these opportunities appropriately.

### **(3) In regard to the relationship with moral education**

(3) Based on the objectives of moral education listed in Sections I-2 of Chapter 1, “General Provisions” and in Section I of Chapter 3, “Moral Education”, instructions concerning the content listed in Section II of Chapter 3, “Moral Education” should be given appropriately. The instructions should be in accordance with the characteristics of mathematics and should be related to the period for moral education.

In Section I-2 of Chapter 1 of the COS, “General Provisions,” it is stated that, “moral education in schools will proceed with the class period of moral education serving as the cog, but appropriate instruction should also take place in other subject areas and in the integrated study periods reflecting the unique characteristics of subjects, while considering students’ developmental levels”.

Thus, this statement indicates that ideas related to moral education should be taught when appropriate in mathematics as well.

Instruction related to moral education in mathematics should include considerations for learning activities and dispositions toward learning and the influence of teachers’ attitudes and actions. In addition, appropriate instruction should be conducted while clearly paying attention to the relationships between the objectives of mathematics and moral education as shown below.

In mathematics, the objectives have been set as the following: through mathematical activities, help students deepen their understandings of fundamental concepts, principles and rules regarding numbers, quantities, geometrical figures and so forth; to help students acquire the way of mathematical representation and processing; to develop their ability to think and represent phenomena mathematically; to help students enjoy their mathematical activities and appreciate the value of mathematics; and to foster their attitude toward making use of the acquired mathematical understanding and ability for their thinking and judging.

Helping students develop their ability to think and represent phenomena mathematically also contributes to developing students’ ability to make moral judgments. Also, fostering the attitude toward making use of the acquired mathematical understanding and the ability for thinking and judging contributes to fostering the attitude to devise better ways for living and studying.

Next, it is necessary to consider the connection mathematics has with the cog of moral education, the classroom time of moral education. There are occasions where using the teaching materials from a mathematics lesson in the class period of moral education is effective. Also, when using the teaching materials from the period of moral education in mathematics lessons, think about ways to make use of what was taught in the class period of moral education. Therefore, it is important during the planning of annual instruction plans for mathematics to consider the relationships of the overall plans of moral education and the relationships among contents and the allocation of time so that they can support their mutual learning.

## 2. Consideration Points for Handling the Content in Chapter 2

### (1) Terms and symbols

- (1) Terms and symbols indicated under the content for each grade in Section II have the purpose of clarify the range and the extent of the content dealt with in each grade. In teaching of these terms and symbols, consideration should be given to dealing with them in close connection with the content for each grade.

This statement is about how terms and symbols in mathematics learning and also in the COS are discussed.

### ① Mathematics learning and terms and symbols

#### Importance of terms and symbols

Terms and symbols concisely express socially agreed upon contents, and by using them appropriately, we can think more easily and communicate more effectively. In mathematics, symbols play significant roles. Since symbols are abstract, they are easy to manipulate and can be used more generally. By developing a symbol system, we can manipulate procedurally without being bound by specific meanings and can proceed with our thinking more efficiently.

By being accustomed to the ways symbols and terms are used in mathematics, students can think more accurately, appropriately, and efficiently, which in turn may contribute to the development of society.

Mathematical terms and symbols may be divided into three types. First are those terms and symbols about the objects of mathematical thought. The second type of terms and symbols are about manipulations and operations on those objects. The third type of terms and symbols are about the relationships among the objects. By understanding the functions of mathematical terms and symbols, students may deepen their understanding of mathematics, and they can think more accurately and efficiently.

#### Points of consideration on teaching of terms and symbols

It is necessary to teach mathematical terms and symbols through the study of specific content in each domain so that students can fully understand the meaning of terms and symbols and the content. It is also necessary to help students grasp the merits of using terms and symbols, that is, conciseness, clarity, and appropriateness. We must avoid making the study of terms and symbols separate from the specific content and just formal instruction. We must consider things like the following while teaching terms and symbols:

There are terms that are used differently in mathematics than in our everyday situations. For example, we may sometimes encounter situations in our daily life that the phrases, direct and inverse proportional relationships, are used to mean the relationship where one quantity increases as the other also increases, or one quantity decreases while the other increases. However, in mathematics, what is important is the ratio of the two quantities. Teachers must be conscious of those situations where there is a gap between mathematical and daily uses of terms.

### ② Terms and symbols in the Course of Study

Terms and symbols used in mathematics instruction can be summarized as follows:

- a. The terms and symbols whose meaning and usage are necessary to mathematics learning
- b. The terms and symbols related to contents that are necessary to clarify the treatment of content
- c. The terms and symbols used to discuss content
- d. The terms and symbols that are obviously treated in connection to specific content even though they may not be used to discuss the content.

The above terms and symbols, a. and b. are discussed in the COS while c. and d. are omitted because when content is discussed, those terms and symbols related to the content will be naturally included in the discussion.

For example, grade 1 content, "A. Numbers and Algebraic Expressions," (1), positive and negative signs and the absolute value symbols are examples of type (a), while "natural numbers" and "terms" are type (b) terms and symbols. Type (c) terms and symbols will include "positive numbers," "negative numbers," and "arithmetic operations," while "plus" and "minus" are type (d). In the COS, the terms and symbols shown in each grade do not suggest that instruction of those will complete in that grade and "students are expected to use them." Rather, they indicate that starting in that grade, those terms and symbols are beginning to be used. Therefore, it is necessary to consider helping students extend their ability to use those terms and symbols starting from that grade level and beyond.

## **(2) Use of computers, information networks and other tools**

(2) In teaching the content of each area, consideration should be given to properly using tools like the soroban (Japanese abacus), calculators, computers, and information and communication networks as needed in order to improve the learning results. This should especially be taken into account for the instructional content related to numerical calculations, as well as in teaching through activities like observation, manipulation and experimentation.

In lower secondary school mathematics, , tools like computers and information networks may be used in one of the following ways: as tools for calculation, as instructional materials, or making use of information and communication networks. In other words, the purpose is not to teach students about computers and information networks, rather, they are used as tools to teach mathematics. In grades 1 and 3, with respect to content, "D. Making Use of Data," (1), it is specifically stated to "using computers," however, it is necessary to examine how these tools may be used in teaching of other contents and actively incorporate them.

The phrase "properly using" indicates the need for considering the development of "media literacy" so that students can make appropriate judgments and respond as they use information networks such as the internet to collect information or communicate with others.

### **① As calculation tools**

Let us consider the soroban, calculators, computers, etc., as calculation tools. It is certainly necessary that students master basic computational ability; however, when dealing with problems involving complicated calculations, the use of calculators can make learning much more effective. In particular, when calculating area or volume involving relatively large numbers or decimal numbers; or processing the quantitative data obtained from observations, manipulations, or experimentations; instead of spending much time on calculation, secure sufficient time to think and explain by actively using calculators. In those situations, it is necessary to teach students to not just copy the results from the calculators but to compare them with estimated results, and how detailed the values should be.

It is also possible to consider using graphing calculators, which combine portability of calculators and simple features of computers. With graphing calculators, it is possible to investigate how changes in coefficients of function expressions will influence their graphs continually, or determine the solutions of equations easily. By utilizing the features of those calculators, for example, it becomes possible to represent the relationships among tables, algebraic expressions, and graphs organically as students learn about functions. It is also possible to collect data from moving objects using a motion detector or to solve equations obtained in the process of solving problems in our daily life or in society easily.

## ② As instructional materials

Computers used as instructional materials will serve not only as a tool for teachers to modify and improve their teaching methods but also as a tool for students to make their learning through observations, manipulation, and experimentation more effective and to experience the joy of mathematical activities. Examples of how computers may be used in "D. Making Use of Data" were already discussed in Chapter 2. However, they may be used in other domains. For example, in "A. Numbers and Algebraic Expressions", computers may be utilized to help students master calculations with algebraic expressions by providing them with supplementary problems appropriate for their individual needs. Or, in "B. Geometrical Figures", suppose a problem includes a statement, "connect the mid-points of two sides". Teachers can display and dynamically change the shape using a computer so that students can understand that even though the shapes are different, the ratios of the lengths stay constant or they may discover other properties of geometrical figures. In teaching of the domain, "C. Functions", computers may be used to show the shapes of graphs more accurately by plotting points with much finer intervals of  $x$  values, or to trace the point on a graph as the corresponding value of  $x$  changes. It is also possible to change the conditions of problems freely to further develop ideas. For example, we can investigate graphs of the linear equation,  $y = ax + b$ , by fixing the value of  $a$  and by changing the value of  $b$ , or by fixing the value of  $b$  and changing the value of  $a$ . Computers and other tools should be used as necessary in project-based learning activities to make them more effective. It is necessary to consider the use of computers in those situations where students may discover mathematical properties.

By considering the content of instruction, the mode of instruction may be adjusted flexibly. For example, in some cases, a lesson can take place in a computerized classroom where each student can work at a computer terminal, while in other cases a single laptop computer equipped with a projector may be used as a presentation device in a regular classroom.

## ③ Use of information and communication networks

Among the uses of these tools as instructional materials, the use of information and communication networks such as the internet should be actively pursued while making their purposes clear. For example, it is effective to use them instead of just relying on encyclopedias and other reference materials while searching for information that may deepen students' understanding of contents, for example, various proofs of the Pythagorean theorem, historical information about mathematics such as wasan (Japanese style mathematics) and problems posed tablets dedicated to temples and shrines, and statistical data. It is also possible for students from different parts of the country to pose problems to each other or communicate their solutions using e-mail, bulletin boards, or video conferencing. Such activities may raise their interest in learning mathematics. In these situations, it is required that teachers make clear the purpose of the activities. It is also necessary that teachers consider the idea of media literacy as they have students gather information or solve problems using these tools.

### 3. Consideration Points for Instruction of Mathematical Activities

#### (1) Enjoy mathematical activities and experience the significance and the necessity of learning mathematics

- (1) Mathematical activities should be enjoyable for students, and opportunities should be created in order for them to realize the significance of learning mathematics and the necessity of mathematics.

Helping students enjoy mathematical activities is included in the objectives of the lower secondary school mathematics, and it should be considered while teaching through mathematical activities. As it was stated in Section 1 of Chapter 2, the joy of mathematical activities is not limited to simply enjoying the activities themselves but also must include the joy arising from intellectual growth. It is important that students can enjoy mathematical activities, engage in those activities autonomously while anticipating the joy of their own intellectual growth, and carry out the activities with confidence. Moreover, based on these experiences, considerations should be given so that students can ask about the significance and the necessity of learning mathematics to themselves and find their own answers.

#### (2) To engage in mathematical activities with foresight and to reflect on them

- (2) Opportunities should be given to students for finding out one's own problems to be solved, making a plan for solving them, implementing the plan, and then evaluating and improving the results.

Mathematical activities will in general take place in the form of problem solving. Considerations should be given so that students will engage in the process with some foresight. Appropriate opportunities should be set up so that students will discover their own problems based on mathematics that they have learned previously instead of just solving problems given by their teachers. It is important that students will not engage in the activities haphazardly but rather plan what they need to do to solve problems. It is also important that students can follow their plans, selecting appropriate strategies such as trials and errors, collect data, observe, manipulate or experiment in order to reach conclusions. It is necessary to enable students to engage in mathematical activities with foresight so that they can engage in mathematical activities autonomously. Moreover, even when the results are different from what was expected, reflecting on and evaluating their own activities may provide chances for them to improve the results or discover new problems to be solved. Having such experiences is important in fostering students' autonomy.

#### (3) To share the results of mathematical activities

- (3) Opportunities should be given to students to share among others the results of mathematical activities by looking back at the process of mathematical activities, and then making and presenting their reports on the activities

Because the instruction through mathematical activities emphasizes not only the results but also the processes, provide opportunities for students to reflect on the processes and share their results with each other perhaps by having them present reports. Although "presenting reports" may suggest that a large amount of time is needed, it is also possible, for example, to have students summarize their results briefly in 1-page reports after they complete their activities. What to be shared should be more than just the results and may also include such things as their own ideas, challenges they faced while doing the process and the satisfaction they felt in the process even if the results turned out to be incorrect.

What is important is that students can share ideas and thoughts related to (1) and (2) above so that they can make use of them in the future.

## 4. Project-Based Learning and its Position

4. The “problem situation-based learning” is a type of learning which solves the problems found out through such means as synthesizing the content in different areas, or making the connection of them with everyday phenomena and the learning of other school subjects. It is designed with the aim of encouraging initiatives for mathematical activities on the part of the students, as well as fostering their abilities to think, to judge, express themselves and so forth. The implementation of “problem situation-based learning” should be properly positioned within the syllabus for each grade.

The current revision of the COS states that the problem situation-based learning (also described as “project-based learning” elsewhere in this document) should be appropriately positioned in the instruction plan in each grade by reflecting on their purposes and considering students’ development.

### (1) Aim of Project-Based Learning

The aim of Project-Based Learning is to deepen students mathematical ways of viewing and thinking by having them autonomously identify and solve problems identified as they integrate the contents of the domains, “A. Numbers and Algebraic Expressions”, “B. Geometrical Figures”, “C. Functions”, and “D. Making Use of Data”, or make connections to phenomena from their daily life or their studies in other subject areas. Therefore, it is important that students’ engagement in mathematical activities is facilitated through Project-Based Learning so that students can not only experience the joy but also raise their ability to think, judge and express themselves. These are also important in everyday lessons as well; however, in regular lessons, because instruction typically takes place along the specific content domains, the problems also tend to focus on the contents of the specific domain. Thus, students tend to consider the contents of different domains as unrelated to each other. In contrast, Project-Based Learning involves students in problem solving integrating contents of different domains. Through these studies, help students to experience the usefulness of mathematics more deeply and extend their ability to solve problems further.

### (2) Essential factors of tasks

In order to achieve the aim discussed above, Project-Based Learning that “integrates the contents of the domains, or makes connections to phenomena from their daily life or their studies in other subject areas” is to be implemented.

In those cases, it is very important how the task is posed to the students. It is necessary that tasks in Project-Based Learning must be something that students can actively engage in with interest and persist until their conclusions.

For that purpose, tasks should possess the potential for students to enjoy mathematical activities and to experience the merits of mathematics, as well as essential factors such as the following:

- a. The tasks that will make it possible for each student to utilize his or her own ideas and creativity and to willingly persist until their conclusions.
- b. The tasks that will make it possible for each student to anticipate the results on their own.
- c. The tasks that will allow various mathematical ways of viewing and thinking.
- d. The tasks whose solutions may allow students to think about extensions as they reflect on them.

Moreover, in order to realize students’ autonomous learning, it is important that students are given opportunities to tackle tasks that are collected and organized in their everyday lessons, daily life, or during Project-Based Learning.



### (3) Project-Based Learning and everyday lessons

Project-Based Learning is to be “properly positioned within the syllabus for each grade”. It will be difficult to implement Project-Based Learning envisioned above if everyday lessons are teacher-centered dissemination of knowledge and facilitate autonomous learning during Project-Based Learning. Therefore, it is required that even in everyday lessons instruction based on mathematical activities centered around problem solving that will facilitate students’ autonomous learning should be established and enhanced. That is, learning through problem solving and Project-Based Learning are not mutually independent. Project-Based Learning may be positioned as the continuation of the study of the content of various domains or as opportunities to solve problems and tasks identified by integrating the content of various domains or making connections to daily phenomena or the content of other subject matters.

Thus, it is important that, through Project-Based Learning, “autonomous learning” and “fostering of mathematical ways of viewing and thinking” are further facilitated.

Attitudes, dispositions, and ways of viewing and thinking developed through Project-Based Learning can be useful in the future lessons.

Project-Based Learning will also provide opportunities for teachers to engage in *kyozaikenkyu* (researching instructional materials) and to improve their teaching methods. Thus, it is desired that teachers further engage in their own Project-Based Learning.

中学校学習指導要領解説数学編作成協力者（五十音順）

Committee

（職名は平成20年6月末日現在）

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